MODELING OF PLATE FLEXURE SATISFYING NORMAL STRESS CONDITIONS AT THE PLATE SURFACES

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Abstract

Modeling of plate flexure, based on cubic variation of inplane displacements and quartic variation of the normal displacement has been discussed. Displacement functions satisfy the zero shear stress condition at top and bottom surfaces of the plate. The normal stress condition at the surfaces of the plate and the elemental equilibrium equations are enforced through the use of the Lagrangian multipliers in the variational procedure used. Several new models are available as special cases in this formulation. An illustrative example of a simply-supported plate subjected to sinusoidal loading is included to indicate relative merits of the special cases.

1. Introduction

Plate theory attempts to provide a two-dimensional representation of an essentially three-dimensional phenomenon. The classical theory of plate flexure, based on Kirchhoff's assumption, predicts the inplane stresses well. The transverse shear and normal stresses are usually estimated by integrating local equilibrium equations. Unfortunately this results in a violation of the constitutive relations as the transverse shear and normal strains are considered to be zero in the classical theory. Higher order models make provision for non-zero transverse shear and normal strains. Here we consider the displacement based higher order models wherein, one starts the displacement field in a series form, in terms of thicknesswise coordinate as [1]

$$\bar{U}_i\ (x,y,z) = \sum z^j\ U_{ij}\ (x,y)$$

Retaining a finite number of terms in the expansion and invoking variational principles/physical equilibrium conditions, the governing equations and boundary conditions are set up. In this approach the transverse shear and normal stresses can be estimated using the constitutive relations; unfortunately such estimates violate the boundary conditions on stresses at the surfaces of the plate. More recently Krishna Murty [2], Levinson [3] gave the expressions for displacements, wherein the zero shear stress condition at the top and bottom surfaces of the plate are also satisfied. So far there appears to be no attempt to formulate a plate theory satisfying the normal stress conditions at the top and bottom surfaces of the plate and this paper is an attempt in this direction.

With the growing use of fiber reinforced plastic laminates in engineering applications and the importance of interlaminar stresses in such laminates, there is a new spur in developing new plate theories [4-11]. The interlaminar normal stress is often a crucial design parameter for the laminates, in particular when there are free edges. Development of laminated plate theory, with provision for direct estimation of interlaminar normal stress is indeed complex. Basic studies related to isotropic plates are useful in identifying the direction for the development of laminated plate models.

Primarily two basic approaches may be identified for modeling plates satisfying the normal stress condition. A straight forward approach, wherein all stresses and strains are formulated in terms of chosen displacement field, represents the classical consistent approach. This formulation is simple, popular and is readily amenable for complicated problems. In the second approach, transverse shear and normal stresses are obtained by integrating local equilibrium equations after substituting the estimates to in-plane stresses from a displacement based theory [9,11]. In this paper, we consider the

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first approach, to develop theories of flexure satisfying the normal stress condition.

One way of satisfying normal stress conditions at the surfaces of the plate in a displacement based theory of plate flexure is the use of Lagrangian multipliers. Unfortunately, it introduces a new difficulty namely, the governing equations deduced following the variational procedure, do not correspond to the physical equilibrium equations. Therefore, it becomes necessary to introduce additional constraints representing elemental equilibrium equations through Lagrangian multipliers to achieve satisfaction of these equilibrium equations also. In the case of plate flexure problem, a total of four Lagrangian multipliers will be necessary.

The purpose of this paper is to present a study, regarding modeling of plate flexure satisfying normal stress condition at the surface of the plate.

2. Formulation

A typical plate and the coordinate system are shown in Fig.1. The plate is subjected to normal loading at top and bottom surfaces $(\sigma_z)_{\xi=\pm 1} = \pm Q$, and the shear loading on the plate surfaces is taken to be zero. In Ref. [12], a plate theory has been formulated based on the displacement field.

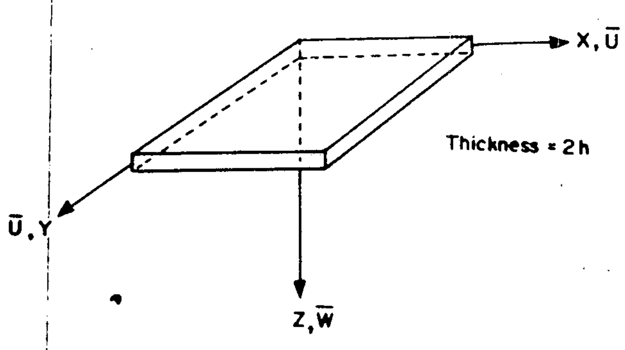

Fig. 1 A typical Plate

$$\bar{U}(x,y,z) = -z\ W,x - pu$$

$$\bar{V}(x,y,z) = -z\ W,y - pv$$

$$\bar{W}(x,y,z) = W - p'w \qquad (1)$$

Where W,u,v, w are functions of the Cartesian coordinates, x and y only and denotes differentiation with respect to ξ.

$$p = \xi\left(1 - \frac{1}{3}\xi^2\right) \qquad (2)$$

and $\xi = z/h$, where the thickness of the plate is 2h. This displacement field satisfies the zero shear stress conditions on the surfaces $\xi = \pm 1$ of the plate. But the normal stress conditions on these faces are not satisfied.

Appendix A contains a simple model satisfying all conditions at $\xi = \pm 1$. It may be noted here, that although in this model, all boundary conditions are satisfied and the governing equations are variationally consistent, the transverse shear could not be estimated accurately. The reason for this debacle, is traced to be the fact that the governing equations deduced from the variational process do not correspond to the elemental equilibrium equations, leaving errors in elemental equilibrium. Hence it is imperative to make provision for the satisfaction of elemental equilibrium conditions, by treating them as additional conditions to be satisfied by introducing more Lagrangian multipliers. Keeping in view that there are three elemental equilibrium conditions namely the transverse and two rotational equilibrium conditions in addition to the normal stress conditions at the top and bottom faces of the plate and recognising that zero shear stress conditions are already satisfied, it is clear, that a viable model must have, at least five independent variables to describe the displacement field as,

$$\bar{U}(x,y,z) = -z\ W,_x - pu$$

$$\bar{V}(x,y,z) = -z\ W,_y - pv$$

$$\bar{W}(x,y,z) = W - p'w - qw_1 \qquad (3)$$

$$\text{Where } q = \xi^2(1 - \xi^2) \qquad (2b)$$

At this stage, for the sake of simplicity, it is convenient to restrict the presentation to the strip of an infinite plate, infinitely long in the y-direction, with uniform boundary conditions along the edges x = constant. In such a case $\bar{V} = 0$ and variations of all quantities with respect to y are zero. Thus the displacement field reduces to

$$\bar{U} = -z\ W,_x - pu$$

$$\bar{W} = W - p'w - qw_1 \qquad (4)$$

The expressionss for the non-zero strains in terms of displacements become