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Displacement and strain fields due to axially symmetric sources in an elastic half-space in welded contact with another elastic half-space

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Abstract

Closed form analytical expressions for the displacement and strain components due to five axially symmetric sources, namely, a vertical force, a vertical dipole, a centre of dilatation, a tensile dislocation on a horizontal fault and a compensated linear vector dipole (CLVD) in an elastic half-space in welded contact with another elastic half-space are obtained. The effect of rigidity contrast of the two half-spaces is investigated numerically. The elastic field due to these axially symmetric point sources in two welded half-spaces is compared with the corresponding field in a half-space and in a homogeneous unbounded medium.

(Keywords : centre of dilatation/displacement/half-space/strain/tensile dislocation)

Introduction

Mogi[1] used a centre of dilatation in an elastic half-space to interpret the ground deformation produced in volcanic areas. This model is often called Mogi's model and has been used very extensively since then. Bonafede *et al.*[2] obtained an analytical solution for the displacement field due to a centre of dilatation in a viscoelastic half-space. The results obtained were applied to the volcanic area of Campi-Flegrei. It was shown that the consideration of viscoelasticity allows the same deformation to take place with pressure values in the magma chamber than are required by purely elastic models. Singh *et al.*[3] used four additional axially symmetric sources, namely, a vertical force, a vertical dipole, a tensile dislocation on a horizontal fault and a CLVD in an elastic half-space to model the ground deformation in volcanic areas and made a comparison of displacement and strain fields due to these forces with the corresponding fields due to a centre of dilatation.

Static deformation of an elastic half-space by surface loads has been reviewed by Farrel[4]. The corresponding review for shear and tensile faults has been given by Okada[5]. Yang and Davis[6] obtained closed-form analytic expressions for the displacements, strains and stresses due to a tensile rectangular crack in an elastic half-space. In volcanic areas, centri-symmetric inflation of the summit is often accompanied by intrusion into the rift zones. The intrusion gives surface displacements expected as a result of dike or sill emplacement. Other examples of surface deformation resulting from tabular intrusion include surface tilts which occur at the time of artificially induced hydrofracture in boreholes in order to simulate oil reservoirs. Inversion of measured deformation can give an idea about the geometry and position of the intrusion.

Though, at present, the half-space model is considered adequate for modelling surface deformation, the welded half-spaces model is useful for considering the effect of internal boundaries. This model brings into focus the effect of structural discontinuity, but ignores the effect of free surface. The main advantage of this model is that one can derive a closed form analytical solution, which is not possible for more complex models, e.g., a layer over a half-space model.

Rongved[7] derived closed form algebraic expressions for the Papkovich-Neuber displacement potentials for an arbitrary point force acting in an infinite medium consisting of two elastic half-spaces in welded contact. Heaton and Heaton[8] used the expressions for the Papkovich-Neuber potentials derived by Rongved[7] to obtain the displacements produced by point forces and point force couples embedded in two welded half-spaces. However, Heaton and Heaton[8] made the simplifying assumption that the two half-spaces are Poissonian. Therefore, their results cannot be used to determine the effect of Poisson's ratio on the deformation field. Kumari *et al.*[9] generalized the results of Heaton and Heaton[8] to the case in which Poisson's ratios for the two half-spaces are arbitrary.

Tinti and Armigliato[10] obtained an analytical solution for a single force in two welded half-spaces; but their final results are rather cumbersome, involving complicated functions of the elastic constants of the two media. Singh *et al.*[11] obtained the displacements and stresses due to a single force acting at an arbitrary point of a two phase medium consisting of two homogeneous, isotropic and perfectly elastic half-spaces in welded contact. The force may be acting either parallel to or perpendicular to the interface. The results obtained are valid for arbitrary values of the Poisson's ratio of the two media and for arbitrary source and observer locations. Deformation of two welded half-spaces due to inclined shear and tensile point dislocations and a centre of dilatation has been investigated by Singh *et al.*[12]

In the present study, we obtain the displacement and strain fields caused by five axially symmetric sources, namely, a vertical force, a vertical dipole, a centre of dilatation, a tensile dislocation on a horizontal fault and a compensated linear vector dipole (CLVD) in an elastic half-space in welded contact with another elastic half-space. The effect of the rigidity contrast of the two half-spaces is investigated numerically. The elastic field due to a source in two welded half-spaces is compared with the corresponding field in a half-space and in a homogeneous unbounded medium.

Theory

Consider a homogeneous, isotropic and perfectly elastic half-space ($z > 0$, medium 1) with Lame constants λ_1, μ_1 in welded contact with another homogeneous, isotropic and perfectly elastic half-space ($z < 0$, medium 2) with Lame constants λ_2, μ_2 (Fig. 1). A point source is located at the point (0, 0, c) of medium 1. The expressions for the displacements and strains for various axially symmetric point sources have been obtained by using the results of Kumari *et al.*[9] and Singh *et al.*[11-12]. The displacement components are continuous at the horizontal interface $z = 0$ because of the imposed boundary conditions that the two half-spaces are in welded contact. It is found that the horizontal strain e_{rr} is also continuous at $z = 0$. However, the vertical strain e_{zz} is not continuous at $z = 0$.

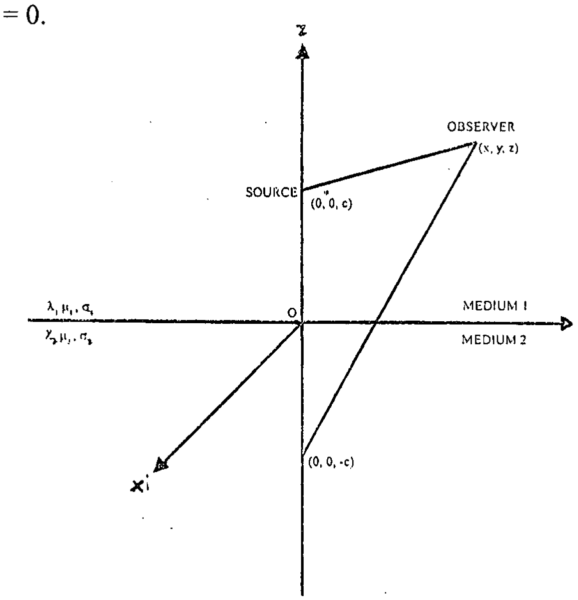

Fig. 1 – Geometry of a point source in two welded half-spaces.

We have used the following notation in these expressions :

$$k_1 = 3 - 4\,\sigma_1,\ \ k_2 = 3 - 4\,\sigma_2,\ \ m = \mu_2/\mu_1,$$

$$A = \frac{1 - m}{1 + mk_1},\quad B = \frac{k_2 - mk_1}{k_2 + \mathrm{m}}\ \ \text{and}$$

$$R = (x^2 + y^2 + c^2)^{1/2} = (r^2 + c^2)^{1/2}, \qquad (1)$$

where σ_1 (σ_2) is the Poisson's ratio of medium 1 (medium 2).

Vertical force : For a vertical point force of magnitude F_3, the radial and the vertical components of the displacement and strain at the interface ($z = 0$) are given by (Singh *et al.*[11])

$$u_r = \frac{-F_3\, r}{8\pi\mu_1\,(k_1 + 1)}$$

$$\left[\frac{2(1 + Ak_1)c}{R^3} + \frac{Ak_1^2 - B}{R\,(R + c)}\right], \qquad (2a)$$

$$u_z = \frac{F_3}{8\pi\mu_1 (k_1 + 1)}$$

$$\left[\frac{Ak_1^2 + 2k_1 + Bc}{R} + \frac{2(1 + Ak_l)c^2}{R^3}\right], \quad (2b)$$

$$e_{rr} = \frac{-F_3}{8\pi\mu_1 (k_1 + 1)} \left[\frac{2(1 + Ak_l)c}{R^3} \left\{1 - \frac{3r^2}{R^2}\right\} + \right.$$

$$\left.\frac{Ak_1^2 - B}{R(R + c)} \left\{1 - \frac{r^2}{R^2} - \frac{r^2}{R(R + c)}\right\}\right], \quad (2c)$$

$$e_{(1)zz} = \frac{-F_3 c}{8\pi\mu_1 (k_1 + 1)}$$

$$\left[\frac{Ak_1^2 + B - 2 (1 + A) (k_l - 2) - 2AK_1}{R^3}\right.$$

$$\left. + \frac{6c^2 \{A(k_1 - 2) - 1\}}{R^5}\right], \quad (2d)$$

$$e_{(2)zz} = \frac{F_3 c}{8\pi\mu_2 (k_1 + 1)}$$

$$\left[\frac{(1 - A)k_1 + (1 - B)k_2 + 2 (A + B - 2)}{R^3}\right.$$

$$\left. - \frac{6c^2 \{1 - A\}}{R^5}\right], \quad (2e)$$

where u_r is the displacement in the radial (horizontal) direction, u_z is the displacement in the vertical (up) direction, e_{rr} is the horizontal strain and $e_{i(zz)}$ is vertical strain in medium i ($i = 1, 2$).

Vertical dipole : For a vertical dipole of moment F_{33}, we have (Singh *et al.*[12])

$$u_r = \frac{F_{33}\, r}{8\pi\mu_1 (k_1 + 1)} \left[\frac{Ak_1^2 - 2Ak_1 - B - 2}{R^3}\right.$$

$$\left. + \frac{6(1 + Ak_1)c^2}{R^5}\right], \quad (3a)$$

$$u_z = \frac{-F_{33}\, c}{8\pi\mu_1 (k_1 + 1)}$$

$$\left[\frac{Ak_1^2 + 2k_1 (1 - 2A) + B - 4}{R^3} + \frac{6(1 + Ak_1)c^2}{R^5}\right], \quad (3b)$$

$$e_{rr} = \frac{F_{33}}{8\pi\mu_1 (k_1 + 1)} \left[\frac{Ak_1^2 - 2Ak_1 - B - 2}{R^3}\right.$$

$$\left.\left\{1 - \frac{3r^2}{R^2}\right\} + \frac{6(1 + Ak_1)c^2}{R^5} \left\{1 - \frac{5r^2}{R^2}\right\}\right], \quad (3c)$$

$$e_{(1)zz} = \frac{F_{33}}{8\pi\mu_1 (k_1 + 1)}$$

$$\left[\frac{2(1 + A) (k_1 - 2) - Ak_1^2 - B + 2Ak_1}{R^3}\right.$$

$$+ \frac{3c^2 \{2 (1 + A) (5 - k_1) + Ak_1^2 + B + 6A - 8Ak_1\}}{R^5}$$

$$\left. + \frac{30\{A(k_1 - 2) - 1\}c^4}{R^7}\right], \quad (3d)$$

$$e_{(2)zz} = \frac{F_{33}}{8\pi\mu_2 (k_1 + 1)}$$

$$\left[\frac{(1 - A) (k_1 - 2) + (1 - B) (k_2 - 2)}{R^3} \left\{1 - \frac{3c^2}{R^2}\right\}\right.$$

$$\left. + \frac{6(1 - A)c^2}{R^5} \left\{3 - \frac{5c^2}{R^2}\right\}\right], \quad (3e)$$

Centre of dilatation : A centre of dilatataion is equivalent to three equal mutually orthogonal dipoles. If the moment of each of these dipoles is M_0, we obtain (Singh *et al.*[12])

$$u_r = \frac{C_0\, r}{4\mu_1} \left[\frac{1 + Ak_1}{R^3}\right], \quad (4a)$$

$$u_z = -\frac{-C_0\, c}{4\mu_1} \left[\frac{1 + Ak_1}{R^3}\right] = -\left(\frac{c}{r}\right) u_r, \quad (4b)$$

$$e_{rr} = \frac{C_0\,(1+Ak_1)}{4\mu_1 R^3}\left[1-\frac{3r^2}{R^2}\right], \quad (4c)$$

$$e_{(1)zz} = \frac{C_0\,(2A-Ak_1+1)}{4\mu_1 R^3}\left[1-\frac{3c^2}{R^2}\right], \quad (4d)$$

$$e_{(2)zz} = \frac{C_0\,(1-A)}{4\mu_2 R^3}\left[1-\frac{3c^2}{R^2}\right], \quad (4e)$$

where

$$C_0 = \frac{(k_1-1)}{\pi(k_1+1)}\,M_0$$

is the intensity of the centre of dilatation.

Compensated linear vector dipole : A compensated linear vector dipole (Knopoff and Randall[13]) consists of three mutually orthogonal dipoles with moments in the ratio (-1, -1, 2). If the principal dipole of moment $2M$ is vertical, we find

$$u_r = \frac{-Mr}{8\pi\mu_1\,(k_1+1)}\left[\frac{4(1+Ak_1)+k_1\,(2-Ak_1)+3B}{R^3} - \frac{18(1+Ak_1)c^2}{R^5}\right], \quad (5a)$$

$$u_z = \frac{-Mc}{8\pi\mu_1\,(k_1+1)}\left[\frac{k_1\,(Ak_1+4)+3B-10\,(1+Ak_1)}{R^3} + \frac{18(1+Ak_1)c^2}{R^5}\right], \quad (5b)$$

$$e_{rr} = \frac{-M}{8\pi\mu_1\,(k_1+1)}\left[\frac{4\,(1+Ak_1)+k_1\,(2-Ak_1)+3B}{R^3}\left\{1-\frac{3r^2}{R^2}\right\} - \frac{18c^2(1+Ak_1)}{R^5}\left\{1-\frac{5r^2}{R^2}\right\}\right], \quad (5c)$$

$$e_{(1)zz} = \frac{-M}{8\pi\mu_1\,(k_1+1)}\left[\frac{Ak_1^2+3B}{R^3}\left\{1-\frac{3c^2}{R^2}\right\} + \frac{2\,(5+4A-2k_1-3Ak_1)}{R^3} + \frac{12c^2\,(6Ak_1+k_1-11A-7)}{R^5} + \frac{90c^4(1-Ak_1+2A)}{R^7}\right] \quad (5d)$$

$$e_{(2)zz} = \frac{-M}{8\pi\mu_2\,(k_1+1)}\left[\frac{(A-1)\,(k_1-4)+(B-1)\,(3k_2-6)}{R^3}\left\{1-\frac{3c^2}{R^2}\right\} - \frac{18(1-A)c^2}{R^5}\left\{3-\frac{5c^2}{R^2}\right\}\right], \quad (5e)$$

Tensile dislocation : A tensile dislocation U_0 on a horizontal planer element of area ds is equivalent to a vertical dipole of moment $(\lambda+2\mu)\,U_0\,ds$ plus two mutually orthogonal horizontal dipoles of moment $\lambda U_0 ds$ each (Ben-Menahem and Singh[14]). The corresponding deformation is found to be

$$u_r = \frac{U_0\,ds\,r}{4\pi(k_1+1)}\left[\frac{(1-B)-(1-A)\,k_1}{R^3} + \frac{6(1+Ak_1)c^2}{R^5}\right], \quad (6a)$$

$$u_z = \frac{U_0\,ds\,c}{4\pi(k_1+1)}\left[\frac{(1-B)-(1-A)\,k_1}{R^3} - \frac{6(1+Ak_1)c^2}{R^5}\right], \quad (6b)$$

$$e_{rr} = \frac{1}{4\pi(k_1+1)}\left[\frac{(1-B)-(1-A)\,k_1}{R^3}\right.$$

$$\left.\left\{1-\frac{3r^2}{R^2}\right\}+\frac{6(1+Ak_1)c^2}{R^5}\left\{1-\frac{5r^2}{R^2}\right\}\right] \quad (6c)$$

$$e_{(1)zz}=\frac{1}{4\pi(k_1+1)}\left[\frac{(2-k_1)(A-1)+(1-B)}{R^3}\right.$$

$$+\frac{3c^2\left\{(2-k_1)(1+5A)+B+5\right\}}{R^5}$$

$$\left.+\frac{30c^4\{A(k_1-2)-1\}}{R^7}\right] \quad (6d)$$

$$e_{(2)zz}=\frac{1}{4\pi m(k_1+1)}\left[\frac{(1-A)+(1-B)(k_1-2)}{R^3}\right.$$

$$\left.\left\{1-\frac{3c^2}{R^2}\right\}+\frac{6c^2(1-A)}{R^5}\left\{3-\frac{5c^2}{R^2}\right\}\right] \quad (6e)$$

Numerical Results

We define dimensionless epicentral distance D, dimensionless radial displacement U, dimensionless vertical displacement W and dimensionless radial strain E by the relations

$$D=\frac{r}{c},\; U=\frac{P}{c}u_r,\; W=\frac{P}{c}u_z,\; E=Pe_{rr}$$

where P is a dimensionless constant for each source, chosen in such a manner that $W=1$ at $r=0$.

Vertical force : From equations (2a) - (2c), we find

$$U=\frac{-D}{(Ak_1^2+2Ak_1+2k_1+B+2)(1+D^2)^{1/2}}$$

$$+\left[\frac{2(1+Ak_1)}{1+D^2}+\frac{Ak_1^2-B}{1+(1+D^2)^{1/2}}\right], \quad (7a)$$

$$W=\frac{1}{(Ak_1^2+2Ak_1+2k_1+B+2)(1+D^2)^{1/2}}$$

$$\left[(Ak_1^2+2k_1+B)+\frac{2(1+Ak_1)}{(1+D^2)}\right], \quad (7b)$$

$$E=\frac{-1}{(Ak_1^2+2Ak_1+2k_1+B+2)(1+D^2)^{1/2}}$$

$$\left[\frac{2(1+Ak_1)(1-2D^2)}{(1+D^2)^2}+\frac{Ak_1^2-B}{1+(1+D^2)^{1/2}}\right.$$

$$\left.\left\{\frac{1}{(1+D^2)}-\frac{D^2}{\{1+(1+D^2)^{1/2}\}(1+D^2)^{1/2}}\right\}\right], \quad (7c)$$

$$P=\frac{8\pi\mu_1(k_1+1)c^2}{F_3(Ak_1^2+2Ak_1+B+2+2k_1)} \quad (7d)$$

Vertical dipole : From equations (3a) - (3c), we have

$$U=\frac{-D}{(Ak_1^2+2Ak_1+2k_1+B+2)(1+D^2)^{3/2}}$$

$$\left[\left(Ak_1^2-2Ak_1-B-2\right)+\frac{6(1+Ak_1)}{(1+D^2)}\right], \quad (8a)$$

$$W=\frac{1}{(Ak_1^2+2Ak_1+2k_1+B+2)(1+D^2)^{3/2}}$$

$$\left[(Ak_1^2-4Ak_1+2k_1+B-4)+\frac{6(1+Ak_1)}{1+D^2}\right], \quad (8b)$$

$$E=\frac{-1}{(Ak_1^2+2Ak_1+2k_1+B+2)(1+D^2)^{5/2}}$$

$$\left[(Ak_1^2-2Ak_1-B-2)(1-2D^2)\right.$$

$$\left.+\frac{6(1+Ak_1)(1-4D^2)}{1+D^2}\right], \quad (8c)$$

$$P=\frac{-8\pi\mu_1(k_1+1)c^3}{F_{33}(Ak_1^2+2Ak_1+2k_1+B+2)} \quad (8d)$$

Centre of dilatation : From equations (4a) - (4c), we find

$$U=\frac{-D}{(1+D^2)^{3/2}}, \quad (9a)$$

$$W = \frac{1}{(1+D^2)^{3/2}} = -\frac{U}{D} \qquad (9b)$$

$$E = \frac{(2D^2-1)}{(1+D^2)^{5/2}}, \qquad (9c)$$

$$P = \frac{-4\mu_1 c^3}{C_0(1+Ak_1)} \qquad (9d)$$

Compensated linear vector dipole : From equations (5a) - (5c), we have

$$U = \frac{D}{(Ak_1^2 + 8Ak_1 + 4k_1 + 3B + 8)(1+D^2)^{3/2}} \left[(4Ak_1 + Ak_1^2 + 2k_1 + 3B + 4) - \frac{18(1+Ak_1)}{1+D^2}\right], \qquad (10a)$$

$$W = \frac{1}{(Ak_1^2 + 8Ak_1 + 4k_1 + 3B + 8)(1+D^2)^{3/2}} \left[(Ak_1^2 - 10Ak_1 + 4k_1 + 3B - 10) + \frac{18(1+Ak_1)}{1+D^2}\right], \qquad (10b)$$

$$E = \frac{1}{(Ak_1^2 + 8Ak_1 + 4k_1 + 3B + 8)(1+D^2)^{5/2}} \left[(4Ak_1 - Ak_1^2 + 2k_1 + 3B + 4)(1-2D^2) - \frac{18(1+Ak)(1-4D^2)}{(1+D^2)}\right], \qquad (10c)$$

$$P = \frac{-8\pi\mu_1(k_1+1)c^3}{M(Ak_1^2 + 8Ak_1 + 4k_1 + 3B + 8)} \qquad (10d)$$

Tensile dislocation : From equations (6a) - (6c), we find

$$U = \frac{-D}{(5Ak_1 + k_1 + B + 5)(1+D^2)^{3/2}} \left[(Ak_1 - k_1 - B + 1) + \frac{6(1+Ak_1)}{(1+D^2)}\right], \qquad (11a)$$

$$W = \frac{-1}{(5Ak_1 + k_1 + B + 5)(1+D^2)^{3/2}} \left[(Ak_1 - k_1 - B + 1) - \frac{6(1+Ak_1)}{(1+D^2)}\right], \qquad (11b)$$

$$E = \frac{-1}{(5Ak_1 + k_1 + B + 5)(1+D^2)^{5/2}} \left[(Ak_1 - k_1 - B + 1)(1-2D^2) + \frac{6(1+Ak_1)(1-4D^2)}{1+D^2}\right], \qquad (11c)$$

$$P = \frac{-4\pi(k_1+1)c^3}{U_0 ds\,(5Ak_1 + k_1 + B + 5)}, \qquad (11d)$$

For numerical computations, we have assumed that the two half-spaces are Poissonian so that $\sigma_1 = \sigma_2 = 1/4$, $k_1 = k_2 = 2$.

Fig. 2 shows the variation of the vertical displacement (uplift) with the epicentral distance for four values of the rigidity contrast $m = 0$, 1/2, 1,2. The case $m = 0$ corresponds to a uniform half-space and the case $m = 1$ that to a uniform whole space. For $m = 0$ [Fig, 2 (a)], the vertical displacement is positive (i.e. uplift) for all epicentral distances for a vertical force, a centre of dilatation and a tensile dislocation. However, for a vertical dipole and a CLVD, the vertical displacement vanishes at $D = 2.24$ and 1.61, respectively. For $m = 1/2$ [Fig. 2(b)], the vertical displacement is positive for all epicentral distances for a vertical force, a centre of dilatation and a tensile dislocation. However, for a vertical dipole and a CLVD, the vertical displacement vanishes at $D = 3.74$ and 2.15, respectively. For $m = 1$ [Fig. 2 (c)], the vertical displacement is positive for all epicentral distances for a vertical force, a centre of dilatation, a vertical dipole and a tensile dislocation. However, for a CLVD, the vertical displacement vanishes at $D = 2.83$. For $m = 2$ [Fig. 2(d)], the vertical displacement is positive for all epicentral distances for a vertical force, a centre of dilatation, a vertical dipole and a tensile dislocation. However, for a CLVD, the vertical displacement vanishes at $D = 5.92$ (not shown in the figure).

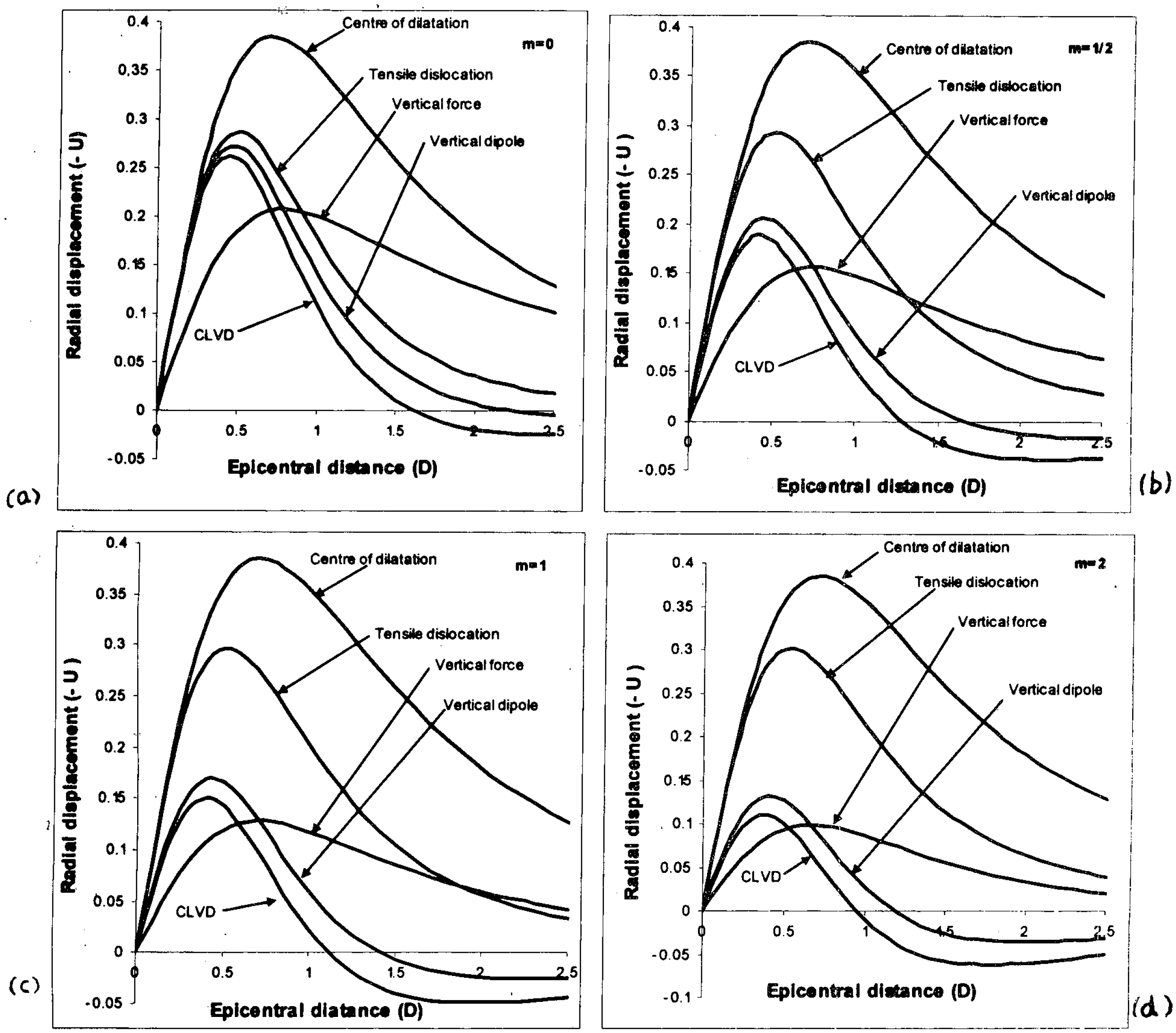

Fig. 2 (a, b, c & d) – Variation of the dimensionless vertical displacement (uplift, W) with epicentral distance. (a) $m = 0$, corresponds to a half-space with traction-free boundary; (b) $m = 1/2$ (c) $m = 1$; corresponds to a uniform infinite medium; (d) $m = 2$. For each source, the source strength is so normalised as to make $W = 1$ at $r = 0$.

Fig. 3 shows the variation of the radial (horizontal) displacement with epicentral distance for four values of the rigidity contrast m = 0, 1/2, 1,2. For all these values of m, the radial displacement does not change sign for a vertical force, a centre of dilatation and a tensile dislocation. However, it vanishes for a vertical dipole and a CLVD at different values of D (depending upon m) given in Table 1.

Table 1 – Dimensionless epicentral distance (D) at which the horizontal displacement changes sign for a vertical dipole and a CLVD for different values of m.

	m			
	0	1/2	1	2
Vertical dipole	2.24	1.66	1.41	1.18
CLVD	1.61	1.28	1.12	0.95

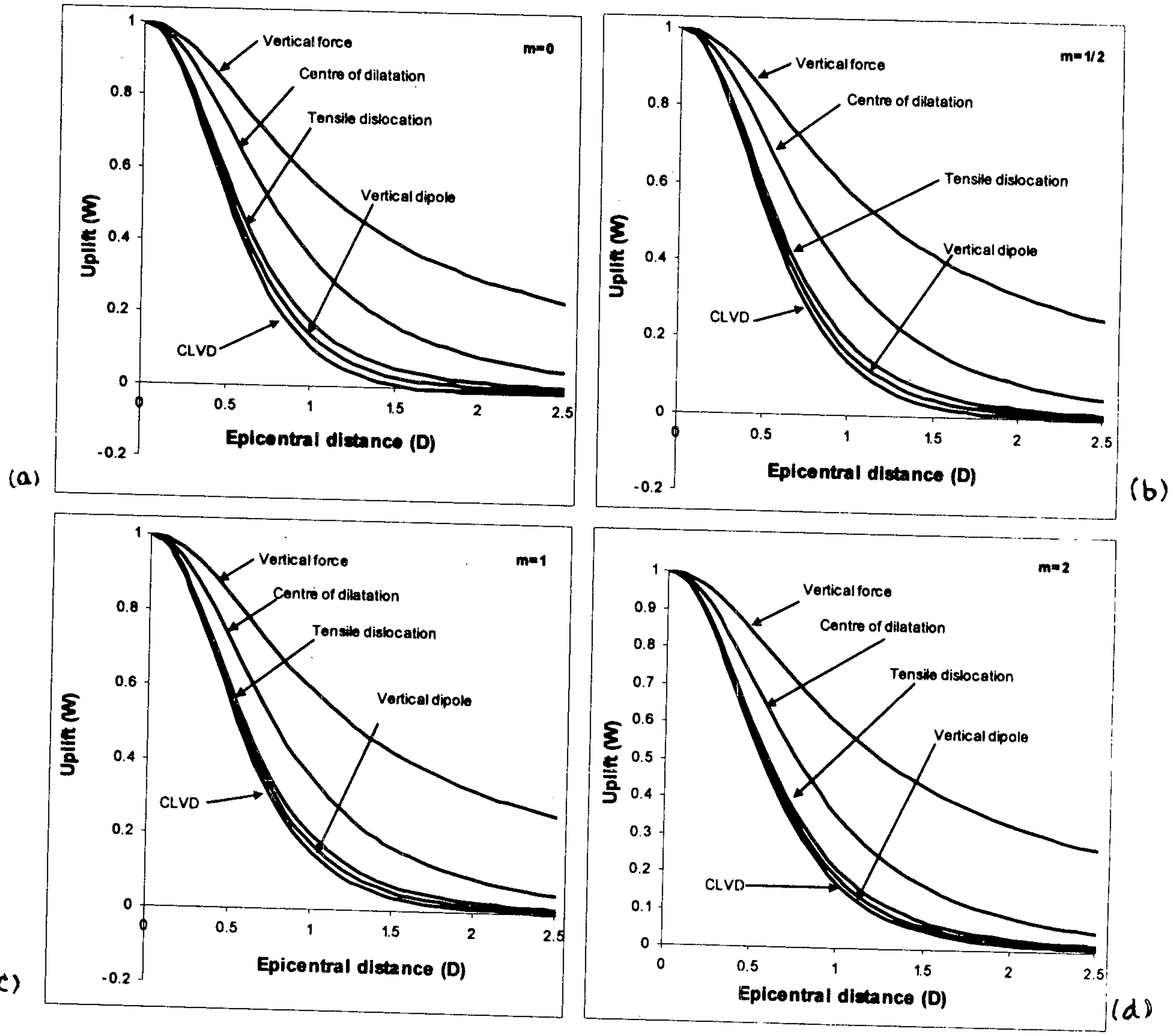

Fig. 3 (a, b, c & d) – Variation of the dimensionless horizontal (radial) displacement ($-U$) with epicentral distance. (a) $m = 0$; (b) $m = 1/2$; (c) $m = 1$; (d) $m = 2$.

We note that the value of the epicentral distance at which the radial displacement vanishes decreases as the value of the rigidity contrast $m = \mu_2/\mu_1$ increases. Table 2 gives the maximum values of the radial (horizontal) displacement U for the five sources for various values of m and the epicentral distances at which it is attained.

Table 2– The maximum value of the radial displacement U and the epicentral distance $D = r/c$ at which it is attained for the five sources ($W_{max} = 1$, $\sigma_1 = \sigma_2 = 1/4$).

Source	$m = 0$		$m = 1/2$		$m = 1$		$m = 2$	
	U_{max}	D	U_{max}	D	U_{max}	D	U_{max}	D
Centre of dilatation	0.385	0.71	0.385	0.71	0.385	0.71	0.385	0.71
Tensile dislocation	0.286	0.50	0.293	0.51	0.297	0.52	0.301	0.53
Vertical dipole	0.273	0.47	0.207	0.45	0.170	0.43	0.132	0.41
CLVD	0.262	0.45	0.189	0.42	0.150	0.40	0.110	0.38
Vertical force	0.207	0.79	0.156	0.74	0.128	0.71	0.099	0.66

Fig. 4 shows the variation of the radial (horizontal) strain with the epicentral distance.

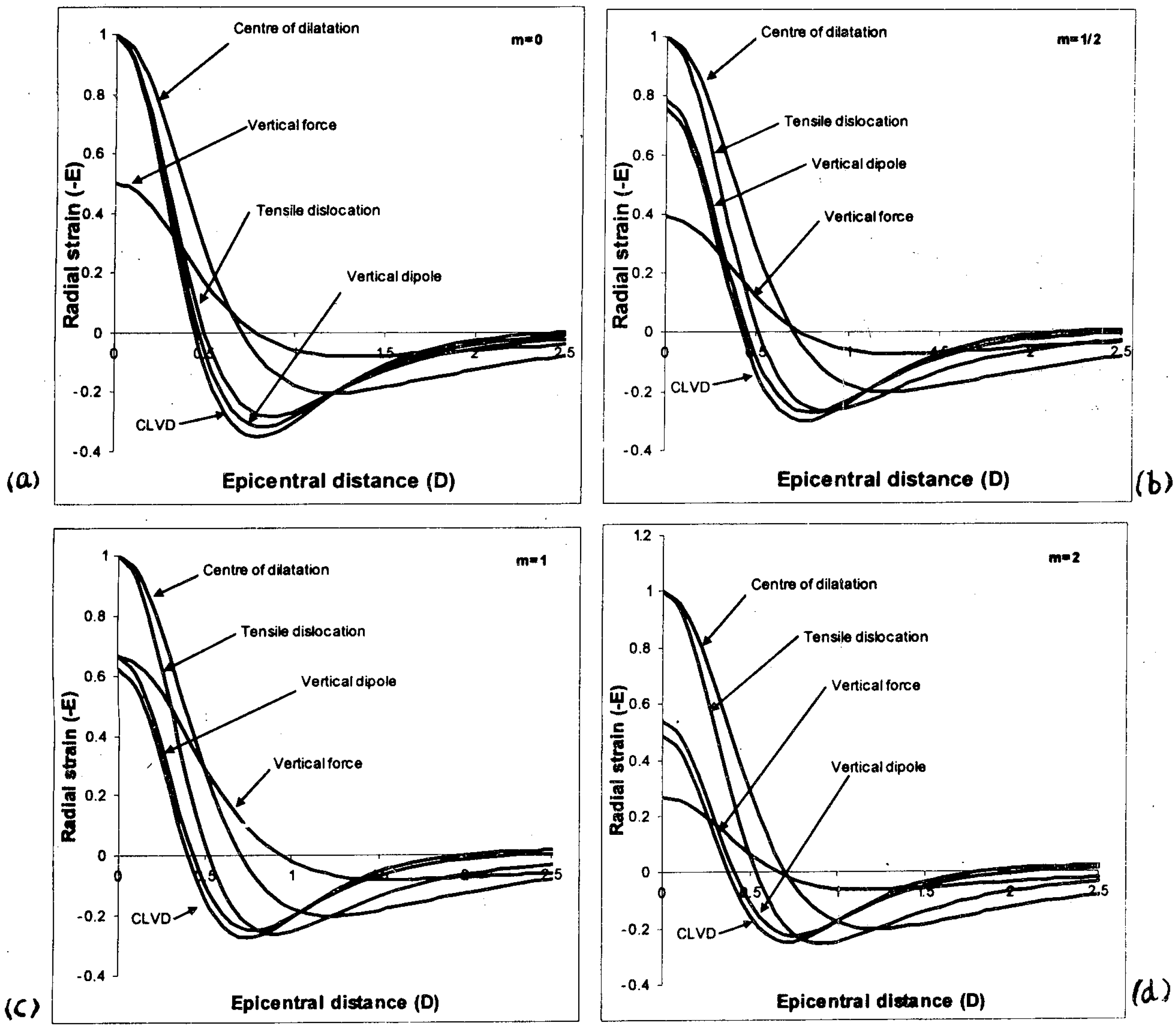

Fig. 4 (a, b, c & d) – Variation of the horizontal (radial) strain ($-E$) with epicentral distance, (a) $m = 0$; (b) $m = 1/2$; (c) $m = 1$; (d) $m = 2$.

Discussion and Conclusions

(a) Analytical expressions for the displacement and strain components due to five axially symmetric sources, namely, a vertical force, a vertical dipole, a centre of dilatation, a tensile dislocation on a horizontal fault and a compensated linear vector dipole (CLVD) in an elastic half-space in welded contact with another elastic half-space have been obtained. Numerical results presented show the variation of the uplift, horizontal displacement and horizontal strain with epicentral distance. The sources considered in the paper serve as useful models to describe various geophysical phenomenon. A centre of dilatation has been used very extensively to model spherical inflation of magma (see, e.g., Mogi[1]). Yang and Davis[6] used a tensile dislocation to represent the dyke model of a volcanic source. Spall is a widely observed phenomenon accompanying underground nuclear explosions. Day and McLaughlin[15] have shown that the spall can be represented either by a vertical force or by a tensile dislocation on a horizontal fault. An atmospheric nuclear explosion can be modelled by a vertical force (see, e.g., Carpenter[16]). Tensile failure under high fluid pressure can be modelled by a force system consisting of a double couple and a CLVD (Julian[17]).

(b) The elastic field due to these axially symmetric point sources in two welded half-spaces is compared with the corresponding field in a uniform half-space and in a homogeneous unbounded medium.

(c) When the two half-spaces have identical elastic properties (i.e., when the source is placed in a homogeneous unbounded medium),

$k_1 = k_2, m = 1 , A = 0 , B = 0 , T = 1.$

Similarly, when the source is placed in a uniform half-space with a traction-free boundary,

$m = 0 , A = 1 , B = 1 , T = 0.$

It has been verified that in the particular case of a uniform half-space our results coincide with the expressions given by Singh *et al.*[3].

(d) Out of the five axially symmetric source models considered, the centre of dilatation is the most efficient and the vertical force is the least efficient in generating the radial (horizontal) displacement. In particular, the spherical inflation model is more efficient in generating the radial displacement than the dyke model.

(e) The elastic field for a spherical inflation model is independent of the rigidity contrast of the two half-spaces. However, for a dyke model, the elastic field does depend upon the rigidity contrast.

(f) Singh *et al.*[3] have shown that for the spherical inflation model (centre of dilatation) and the dyke model (tensile dislocation) in a half-space, the vertical displacement is an uplift at all epicentral distances. The present study reveals that the same is true even for two half-spaces in welded contact.

(g) The radial displacement is outwards for the spherical inflation model as well as for the dyke model both for a uniform half-space and for two welded half-spaces.

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