General Theory of Vibrations of Cylindrical Tubes

PART—V : UNCOUPLED FLEXURAL VIBRATIONS OF OPEN TUBES†

by

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Summary

Based on the simplicity in satisfying free-edge boundary condition, open tubes vibrating in flexural mode are classified as type C and type D. Compared to type D tubes, type C tubes have relatively simpler free-edge boundary condition. An alternate formulation is developed for type D tubes in which the satisfaction of the free-edge condition is as simple and direct as in type C tubes.

Analysis of type C tubes is very similar to that of closed tubes. In this paper we study the flexural vibration characteristics of a simply-supported lipped-I-section which is a typical open tube of type D.

Additional Notation‡

a, b	Typical cross-sectional dimensions of the tube
K_u^4	$= \dfrac{\omega^2 \rho L^4 A}{E B_{xx}}$
k_{ji}^2	$= B_{xx}/A$
P	$= a/b$
Q	$= L/a$
$\lambda_1^2, \lambda_2^2, \lambda_3^2$	Roots of the polynomial (see Eqs. (5.20) and (5.37))
μ_6^2; ν_4^2, ν_5^2	Defined by Eq. (5.31)

†This forms part of a Thesis entitled "Vibration studies of some basic aircraft structural components" by A. V. Krishna Murty approved for the award of the Degree of Doctor of Philosophy in the Faculty of Engineering, Indian Institute of Science, Bangalore.

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‡In addition to the notations used in Part I.

5.0 Introduction

For open tubes of doubly symmetric or doubly antisymmetric cross-sections we have uncoupled flexural and torsional vibrations; but each mode, generally, involves considerable warping motion also. Owing to the considerable influence of shear lag effects, beam theories[2,4]† are, most often, inadequate. In this report we use the theory proposed by the authors[32] in Part I to study the flexural vibration characteristics of open tubes.

As different from the analysis of closed tube the analysis of open tube involves satisfaction of free-edge boundary condition namely

$$\frac{\partial w}{\partial s} + \frac{dx}{ds} \frac{du}{dz} = 0$$

The end conditions on warp are ... (5.1)

$$\text{either } w = 0 \quad \text{or} \quad \frac{\partial w}{\partial z} = 0$$

If $\frac{dx}{ds} = 0$ at the free-edges, Eqs. (5.1) involve w only. Hence it is easy to choose admissible functions for the method of section 1.6*. Such tubes are classified as type C. Neverthless if $\frac{dx}{ds} \neq 0$ at open edges it is not easy to get a suitable admissible function to satisfy free edge boundry condition. Such tubes are classified as type D (see Fig. 5.1). A modification in the governing equations, renders the application of the method of section 1.6 to type D tubes, as easy and straightforward as for type C tubes; these modifications are presented in section (5.2).

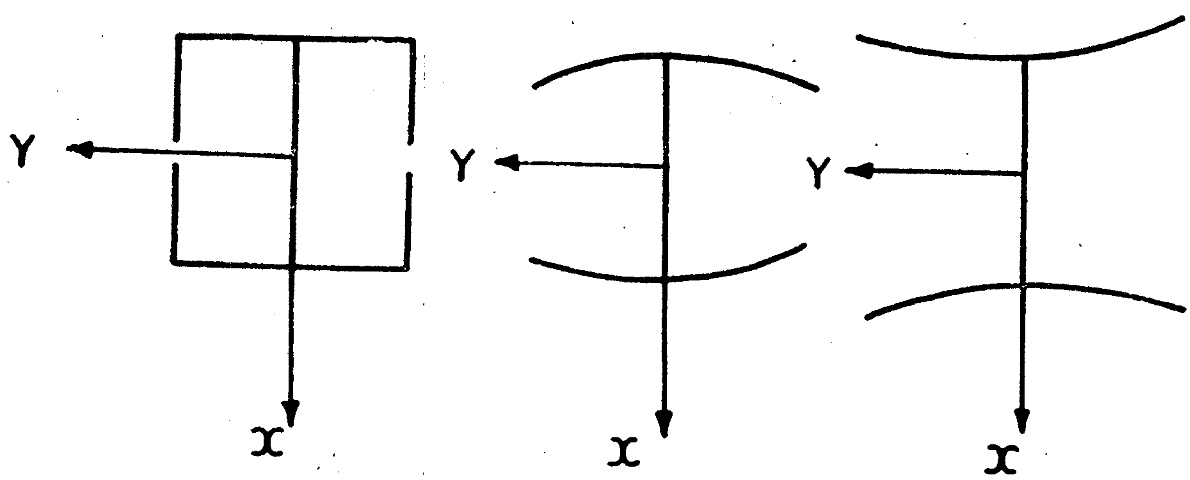

Fig. 5.1 : Open Tubes of Type D

In this paper we study the natural vibration characteristics of a simply-supported open tube of lipped I section, representative of type D tubes, using second order approximation equations.

* Sections and equations referred as (1.) can be seen in Part I.

† References are given in Part I

‡ Sections and equations referred as (3.) can be seen in Part III.

5.1 Governing equations for open tubes of type C—rigorous formulation.

These are the same as those for closed tubes. Use is made of the zero shear strain condition at the free edges (section (3.1))*. Equilibrium equations are

$$\frac{d^2u}{dz^2} + k_u^2 u = - \frac{1}{S_{xx}} \frac{d}{dz} \oint \frac{\partial w}{\partial s} \frac{dx}{ds} t\, ds \qquad \dots (5.2)$$

$$k^2 \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial s^2} + k_s^2 w = - \frac{d^2x}{ds^2} \frac{du}{dz}.$$

The boundary conditions at each end are

$$\text{either } u = 0 \text{ or } \frac{du}{dz} + \frac{1}{S_{xx}} \oint \frac{\partial w}{ds} \frac{dx}{ds} t\, ds = 0$$

$$\text{either } w = 0 \text{ or } \frac{\partial w}{\partial z} = 0 \qquad \dots (5.3)$$

and

$$\frac{\partial w}{\partial s} = 0 \text{ at free edges} \qquad \dots (5.4)$$

As mentioned earlier, the analysis of open tubes of type C is similar to that of closed tubes and these will not be considered here further. Although these equations are applicable to open tubes of D also, it is necessary to modify them in order to facilitate the use of the method of solution of section 1.6. Hence, in the rest of the paper, we deal with open tubes of type D only.

5.2 Governing equations for open tubes of type D—rigorous formulation.

As mentioned earlier, it is necessary to select an expression for w satisfying

$$\frac{\partial w}{\partial s} + \frac{dx}{ds} \frac{du}{dz} = 0 \text{ at open edges}$$

and $\qquad \dots (5.5)$

$$\text{either } w = 0 \text{ or } \frac{\partial w}{\partial z} = 0 \text{ at each and}$$

so that one can use the method of solution suggested in section 1.6. It is clear from the above, the selection of such function is not possible unless we modify the equations since the first of Eqs. (5.5) involves u also.

However, it may be mentioned here, that if one proposes to use Rayleigh-Ritz method, the formulation presented for open tubes of type C may be used for open tubes of type D also.

The difficulty in selecting a suitable function for w can be avoided by effecting transformation of the governing equations using the relationship

$$w = - x \frac{du}{dz} + w_1 (z, s) \qquad \dots (5.6)$$

* Sections and equations referred as (3.) can be seen in Part III.

because in this case, u will be eliminated from the first of Eqs. (5.5). The first term of Eq. (5.6) may be recognised as the one associated with the elementary bending theory.

Substituting Eq. (5.6) in the second of Eqs. (5.2), we have the second of Eqs. (5.8) namely

$$\bar{z} = \frac{\partial^2}{\partial z^2}\left\{ k^2 \left(-x\frac{du}{dz} + w_1 \right)\right\} + \frac{\partial^2 w_1}{\partial s^2} + k_s^2 \left\{ -x\frac{du}{dz} + w_1 \right\} = 0 \qquad \ldots (5.7a)$$

Substituting Eq. (5.6) in the first of Eqs. (5.2) and using the condition that

$$\oint \bar{z}\, x\, ds = 0 \qquad \ldots (5.7b)$$

one obtains the first of Eqs. (5.8). Thus the equations of equilibrium are

$$k^2 B_{xx}\frac{d^4u}{dz^4} + k_s^2 B_{xx}\frac{d^2u}{dz^2} - k_s^2 A\, u = k^2 \frac{d^2}{dz^2}\oint \frac{\partial w_1}{\partial z} zt\, ds + k_s^2 \frac{d}{dz}\oint w_1\, x\, t\, ds$$

$$k^2 \frac{\partial^2 w_1}{\partial z^2} + \frac{\partial^2 w_1}{\partial s^2} + k_s^2 w_1 = k^2 x\left(\frac{d^3u}{dz^3} + k_w^2 \frac{du}{dz}\right); \qquad \ldots (5.8)$$

the boundary conditions at each end are (these are obtained from Eqs. (5.3) by following the same method used to get Eqs. (5.8) from Eqs. (5.2))

$$\text{either } u = 0 \text{ or } k^2 B_{xx}\frac{d^3u}{dz^3} + k_s^2 B_{xx}\frac{du}{dz} - k^2\frac{d}{dz}\oint \frac{\partial w_1}{\partial z} xt\, ds - k_s^2 \oint w_1 xt\, ds = 0$$

$$\text{either } u' = 0 \text{ or } B_{xx}\frac{d^2u}{dz^2} - \oint \frac{\partial w_1}{\partial z} xt\, ds = 0 \qquad \ldots (5.9)$$

and

$$\frac{\partial w_1}{\partial s} = 0 \text{ at free edges} \qquad \ldots (5.10)$$

Cross-sectional constants in Eqs. (5.8) to (5.10) are defined as

$$B_{xx} = \oint x^2 t\, ds$$

$$A = \oint t\, ds \qquad \ldots (5.11)$$

Now if we choose the expression for w_1 to satisfy Eq. (5.10) we are left with four boundary conditions in all, namely Eqs. (5.9). Substituting this expression for w_1 in the first of Eq. (5.8) and solving for u, we find, that the solution involves four additional arbitrary constants and these can be determined using (5.9). Thus, we have generated expressions for u and w_1 satisfying all boundary conditions and also the first equation of Eqs. (5.8). As suggested in section 1.6, either the error in the second equation of Eqs. (5.8) can be minimised, or the orthogonality properties can be made use of to generate simultaneous equations, truncating which the eigen values and eigen vectors can be computed.

5.3 Open tubes of type D—first order approximation

The governing equation in this case is obtained by putting $w_1 = 0$ in Eqs. (5.8) and using Eq. (5.7b) as

$$k^2 B_{xx} \frac{d^4u}{dz^4} + k_s^2 B_{xx} \frac{d^2u}{dz^2} - k_s^2 Au = 0 \qquad \ldots (5.12)$$

and the boundary conditions at each end are (Eqs. (5.9))

$$\text{either } u = 0 \text{ or } k^2 B_{xx} \frac{d^3u}{dz^3} + k_s^2 B_{xx} \frac{du}{dz} = 0 \qquad \ldots (5.13)$$

$$\text{either } u' = 0 \text{ or } k^2 B_{xx} \frac{d^2u}{dz^2} = 0$$

Eq. (5.12) can also be written as

$$u^{iv} + k_w^2 u'' - K_u^4 u = 0 \qquad \ldots (5.14)$$

and the boundary conditions as

$$\text{either } u = 0 \text{ or } u''' + k_w^2 u' = 0 \qquad \ldots (5.15)$$

$$\text{either } u' = 0 \text{ or } u'' = 0$$

It can be readily seen that Eq. (5.14) is the well known Rayleigh's equation for vibration of beams. This involves neglect of transverse shear effect. Hence, these equations can give good results only if transverse shear effect is small.

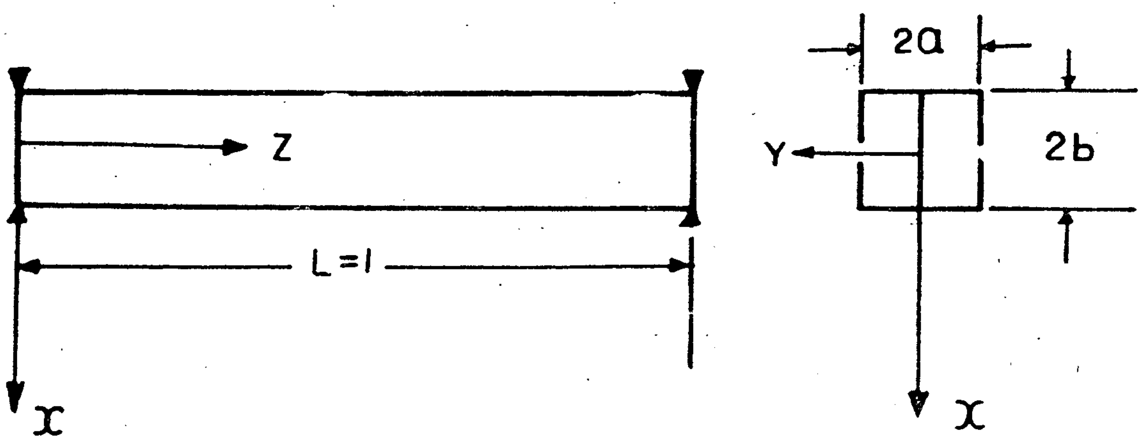

Fig. 5.2: A Simply Supported Tube of I Section

5.4 A simply supported open tube of type D—first order approximation equations.

Fig. (5.2) shows a simply supported open tube of type D. The cross sectional constants pertaining to this section are

$$B_{xx} = \oint x^2 t \, ds = 2b^2 t \, (b + 2a)$$

$$A = \oint t \, ds = 2(2a + 3b)t \qquad \ldots (5.16)$$

and $\quad w = - x \, u'$

The boundary conditions in this case are

$$u\,(0) = u''\,(0) = 0 \qquad \ldots (5.17)$$
$$u\,(l) = u''\,(l) = 0$$

The solution of Eq. (5.14) is ... (5.18)

$$u = A_1 \sin \lambda_1 z + A_2 \cos \lambda_1 z + A_3 \sinh \lambda_2 z + A_4 \cosh \lambda_2 z \qquad \ldots (5.19)$$

where $-\lambda_1^2$ and λ_2^2 are roots of the quadratic equation

$$\xi^2 + k_w^2 \, \xi - K_u^4 = 0 \qquad \ldots (5.20)$$

Satisfaction of the boundary conditions, in a nontrivial case requires

$$\sin \lambda_1 \sinh \lambda_2 = 0$$

$$\text{or} \qquad \lambda_1 = m\pi \, ; \quad m = 1,2,3, \ldots \infty \qquad \ldots (5.20a)$$

and the mode shapes are

$$u = A_1 \sin m \, \pi \, z$$
$$w = -A_1 \, xm \cos m\pi z \qquad \ldots (5.21)$$

Let

$$\frac{k_w^2}{K_u^4} = \frac{B_{xx}}{A} = k_{Li}^2 \qquad \ldots (5.22)$$

Using Eq. (5.22) and substituting

$$\xi = - \lambda_1^2 = m^2 \pi^2$$

one obtains

$$K_u^{*4} = \frac{K_u^4}{m^4 \pi^4} = \frac{1}{1 + m^2 \, \pi^2 \, k_{Li}^2} \qquad \ldots (5.23)$$

Neglecting longitudinal inertia we have

$$K_u^{*4} = 1 \qquad \ldots (5.24)$$

Some numerical results are discussed in section (5.7).

5.5 Open tubes of type D — second order approximation

The appropriate expression for w in the second order approximation is

$$w = -x\frac{du}{dz} - \bar{w}_{2,x}\Psi_x \qquad \dots (5.25)$$

$\bar{w}_{2,x}$ has to be obtained from the relation (section (1.13))

$$\bar{w}_{2,x} = -\iint x\,ds\,ds \qquad \dots (5.26a)$$

The constants of integration are evaluated from zero strain condition at free edges namely

$$\frac{d\,\bar{w}_{2,x}}{ds} = 0 \qquad \dots (5.26b)$$

Substituting Eq. (5.25), into the second of Eqs. (5.2), we have

$$-\bar{z} = \frac{\partial^2}{\partial z^2}\left\{k^2\left(-x\frac{du}{dz} - \bar{w}_{2,x}\Psi_x\right)\right\} - \frac{d^2\bar{w}_{2,x}}{ds^2}\Psi_x$$

$$+ k_s^2\left\{-x\frac{du}{dz} - \bar{w}_{2,x}\Psi_x\right\} = 0 \qquad \dots (5.27a)$$

Instead of Eq. (5.27a), we choose to satisfy

$$\oint \bar{z}x\,ds = 0$$

$$\oint \bar{z}\bar{w}_{2,x}\,ds = 0 \qquad \dots (5.27b)$$

Substituting Eq. (5.25) in the first of Eqs. (5.2) and using Eq. (5.27b), we have the equilibrium equations as

$$k^2B_{xx}\left(\frac{d^4u}{dz^4} + k_w^2\frac{d^2u}{dz^2}\right) + k^2\bar{L}_{xx}\left(\frac{d^3\Psi_x}{dz^3} + k_w^2\frac{d\Psi_x}{dz}\right) - k_s^2Au = 0$$

$$k^2\bar{L}_{xx}\left(\frac{d^3u}{dz^3} + k_w^2\frac{du}{dz}\right) + k^2\bar{\bar{L}}_{xx}\left(\frac{d^2\Psi_x}{dz^2} + k_w^2\frac{d\Psi_x}{dz}\right) - L_{xx}\Psi_x = 0$$

(5.28)

and the boundry conditions at each end are (procedure is the same as that for getting Eqs. (5.28) from Eqs. (5.2))

$$\text{either } u = 0 \text{ or } k^2B_{xx}\left(\frac{d^3u}{dz^3} + k_w^2\frac{du}{dz}\right) + k^2\bar{L}_{xx}\left(\frac{d^2\Psi_x}{dz^2} + k_w^2\Psi_x\right) = 0$$

$$\text{either } u' = 0 \text{ or } B_{xx}\frac{d^2u}{dz^2} + \bar{L}_{xx}\frac{d\Psi_x}{dz} = 0 \qquad \dots (5.29)$$

$$\text{either } \Psi_x = 0 \text{ or } \bar{L}_{xx}\frac{d^2u}{dz^2} + \bar{\bar{L}}_{xx}\frac{d\Psi_x}{dz} = 0$$

The following notation is adopted in Eqs. (5.28) and (5.29)

$$B_{xx} = \oint x^2 t\, ds$$

$$A = \oint t\, ds$$

$$\bar{L}_{xx} = \oint x\, \bar{w}_{2,x}\, t\, ds \qquad \ldots (5.30)$$

$$\bar{\bar{L}}_{xx} = \oint \bar{w}^2_{2,x}\, t\, ds$$

$$L_{xx} = \oint \left(\frac{d\bar{w}_{2,x}}{s}\right)^2 t\, ds$$

Introducing the notation

$$\upsilon_4^2 = \frac{\bar{L}_{xx}}{B_{xx}}\ ; \quad \mu_6^2 = \frac{L_{xx}}{k^2\, \bar{L}_{xx}}\ ; \qquad \ldots (5.31)$$

$$\upsilon_5^2 = \frac{\bar{\bar{L}}_{xx}}{\bar{L}_{xx}}\ ; \quad K_u = \frac{k_s^2 A}{k^2 B_{xx}}\ ;$$

Eqs. (5.28) become

$$u^{iv} + k_w^2 u'' + \upsilon_4^2 \left(\psi_x''' + k_w^2 \psi_x \right) - K_u^4\, u = 0 \qquad \ldots (5.32)$$

$$u''' + k_w^2 u' + \upsilon_5^2 \left(\psi_x'' + k_w^2 \psi_x \right) - \mu_6^2 \psi_x = 0$$

While the boundary conditions at each end are

$$\text{either } u = 0 \quad \text{or } u''' + k_w^2 u' + \upsilon_4^2 \left(\psi_x'' + k_w^2 \psi_x \right) = 0$$

$$\text{either } u' = 0 \quad \text{or } u'' + \upsilon_4^2 \psi_x' = 0 \qquad \ldots (5.33)$$

$$\text{either } \psi_x = 0 \quad \text{or } u'' + \upsilon_5^2 \psi_x' = 0$$

Combining the two equations (5.32), we have

$$\{ D^6 + a_1 D^4 + a_2 D^2 + a_3 \}\ (u \text{ or } \psi_x) = 0 \qquad \ldots (5.34)$$

where

$$a_1 = 2k_w^2 - \frac{\mu_6^2}{\upsilon_5^2 - \upsilon_4^2}$$

$$a_2 = k_w^4 - \frac{\mu_6^2}{\upsilon_5^2 - \upsilon_4^2} k_w^2 - \frac{\upsilon_5^2}{\upsilon_5^2 - \upsilon_4^2} K_u^4 \qquad \ldots (5.35)$$

$$a_3 = \frac{\mu_6^2 - \upsilon_5^2 k_w^2}{\upsilon_5^2 - \upsilon_4^2} K_u^4$$

From Eqs. (5.34)

$$u = A_1 \sin \lambda_1 z + A_2 \cos \lambda_1 z + A_3 \sinh \lambda_2 z + A_4 \cosh \lambda_2 z + A_5 \sinh \lambda_3 z + A_6 \cosh \lambda_3 z$$

$$\psi_x = A_1' \sin \lambda_1 z + A_2' \cos \lambda_1 z + A_3' \sinh \lambda_2 z + A_4' \cosh \lambda_2 z + A_5' \sinh \lambda_3 z + A_6' \cosh \lambda_3 z \quad \ldots (5.36)$$

where $-\lambda_1^2$, λ_2^2, λ_3^2 are the roots of the cubic equation (Eq. (5.34))

$$\xi^3 + a_1 \xi^2 + a_2 \xi + a_3 = 0 \quad \ldots (5.37)$$

in writing Eq. (5.36), it is assumed that λ_1^2, λ_2^2 and λ_3^2 are positive. As the present interest is limited to assessing the influence of shear lag on the natural frequencies associated with primarily transverse motion, additional frequencies arising out of negative λ_2^2 and λ_3^2 are excluded from the present discussion. It is obvious that all the arbitrary constants in Eqs. (5.36) are not independent for they have to satisfy any one of Eqs. (5.32) also. Satisfying the first of Eqs. (5.32) one finds

$$A_1 = -\frac{\lambda_1 \upsilon_4^2 (-\lambda_1^2 + k_w^2)}{\lambda_1^4 - \lambda_1^2 k_w^2 - K_u^4} A_2'$$

$$A_2 = \frac{\lambda_1 \upsilon_4^2 (-\lambda_1^2 + k_w^2)}{\lambda_1^4 - \lambda_1^2 k_w^2 - K_u^4} A_1'$$

$$A_3 = \frac{\lambda_2 \upsilon_4^2 (\lambda_2^2 + k_w^2)}{\lambda_2^4 + \lambda_2^2 k_w^2 - K_u^4} A_4'$$

$$A_4 = \frac{\lambda_2 \upsilon_4^2 (\lambda_2^2 + k_w^2)}{\lambda_2^4 + \lambda_2^2 k_w^2 - K_u^4} A_3' \quad \ldots (5.38)$$

$$A_5 = \frac{\lambda_3 \upsilon_4^2 (\lambda_3^2 + k_w^2)}{\lambda_3^4 + \lambda_3^2 k_w^2 - K_u^4} A_6'$$

$$A_6 = \frac{\lambda_3 \upsilon_4^2 (\lambda_3^2 + k_w^2)}{\lambda_3^4 + \lambda_3^2 k_w^2 - K_u^4} A_5'$$

5.6 Cross sectional constants of I section –second order approximation

Since the tube is doubly symmetric, for flexural vibrations in the plane of XOZ, warp is antisymmetric about y-axis and symmetric about x-axis. Hence it is sufficient if we consider the region ABCD for obtaining $\bar{w}_{2,x}$. The expression for x = x (s) is

$$
\begin{aligned}
x &= -s \qquad \text{in } 0 \le s \le b \\
&= -b \qquad \text{in } b \le s \le a + b \\
&= s - (a + 2b) \qquad \text{in } (a + b) \le s \le a + 2b
\end{aligned} \qquad \text{... (5.39)}
$$

$\bar{w}_{2,x}$ is obtained from (Eq. (5.26a))

$$\bar{w}_{2,x} = -\iint x \; ds \; ds \qquad \text{... (5.40)}$$

and using the conditions

$$\left(\frac{d\bar{w}_{2,x}}{ds} \right)_{\text{at } A} = \left(\bar{w}_{2,x} \right)_{\text{at } D} = 0 \qquad \text{... (5.41)}$$

as well as the conditions of continuity of $\bar{w}_{2,x}$ and $\frac{d\bar{w}_{2,x}}{ds}$ at corners,

$$\bar{w}_{2,x} = -\frac{s^3}{6} + \frac{ab}{2}\left(a + 3b \right) + b^3 ; \quad \text{in } 0 \le s \le b$$

$$= -\frac{b}{2}\left(s-b \right)^2 - \frac{b^2}{2}\left(s-b \right) + \frac{ab}{2}\left(a+3b \right) + \frac{5}{6} b^3 ;$$

$$\text{in } b \le s \le a + b$$

$$= -\frac{b}{2}\left(s-b \right)^2 + \frac{(s-b)^3}{6} - \left(ab + \frac{b^2}{2} \right)\left(s-b \right) + ab^2 + \frac{5b^3}{6} ;$$

$$\text{in } a + b \le s \le a + 2b ; \qquad \text{...(5.42)}$$

variation of $\bar{w}_{2,x}$ over the section is shown in Fig. (5.3).

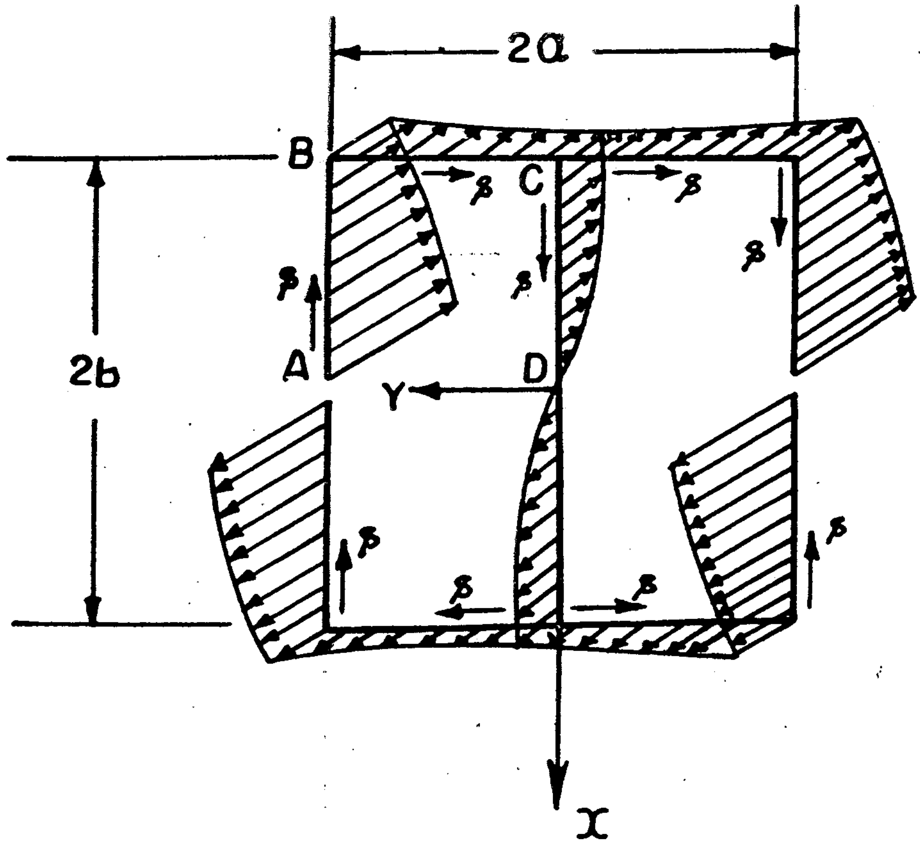

Fig. 5.3 : $\bar{w}_{2,x}$ in I Section

Using Eqs (5.42) and (5.39) in Eqs. (5.30) one has

$$B_{xx} = b^2t\ (4a + 3b)$$

$$\bar{C}_{xx} = -\ \frac{bt}{15}\left(10a^4 + 20a^3b + 30a^2b^2 + 25ab^3 + 3b^4 \right)$$

$$\bar{L}_{xx} = \frac{b^2t}{2520}\left(1344a^5 - 3990a^4b + 5740a^3b^2 - 315a^2b^3 + 721ab^4 + 60b^5\right)$$

$$L_{xx} = \frac{b^2t}{30}\left(20a^4 + 40a^3b + 120a^2b^2 + 30ab^3 + 3b^4 \right)$$

$$A = 2(2a + 3b)\ t \qquad \ldots\ (5.43)$$

Considering the case of $a/b = 1$, and using the notation $L/a = Q$ we have

$$\upsilon_4^2 = -\ \frac{88}{105Q^2}$$

$$\upsilon_5^2 = -\ \frac{1065}{880} \qquad \ldots\ (5.44)$$

$$\mu_6^2 = -\ \frac{40}{231k^2Q^2}$$

5.7 A simply supported tube of I section – second order approximation

The boundary conditions in this case are (Fig. 5.2)

$$u(0) = u''(0) = \Psi''_x(0) = 0 \quad \dots \quad (5.45)$$

$$u(1) = u''(1) = \Psi'_x(1) = 0 \quad \dots \quad (5.46)$$

Appropriate expressions for u and Ψ_x are given in Eqs. (5.36) and (5 38). Satisfaction of Eq. (5.45) yield

$$A_2 = A_4 = A_6 = A'_1 = A'_3 = A'_5 = 0 \quad \dots \quad (5.47)$$

Satisfying Eqs. (5.46) one finds for nontrivial solution

$$\sin \lambda_1 \ \sinh \lambda_2 \ \sinh \lambda_3 = 0$$

or

$$\lambda_1 = m\pi ; \quad m = 1,2,3, = \infty \quad \dots \quad (5.48)$$

Substituting

$$\xi = -\lambda_1^2 = -m^2\pi^2$$

in Eq. (5.37), neglecting longitudinal inertia $\left(k_w^2 = 0 \right)$, one obtains

$$K_u^{*4} = \frac{K_u^4}{m^4\pi^4} = 1 - \frac{m^2\pi^2\upsilon_4^2}{m^2\pi^2\upsilon_5^2 + \mu_6^2} \quad (5.49)$$

Figs. (5.4), (5.5) and (5.6) show the variation of K_u^{*4} with length. Each figure includes three cases namely :

(i) bending alone considered (Eq. (5.24));

(ii) bending and longitudinal inertia considered (Eq. (5.23));

(iii) bending and shear lag considered (Eq. (5.49)).

These reveal that the effects of longitudinal inertia and shear lag are small in long tubes, but these effects are large in short tubes.

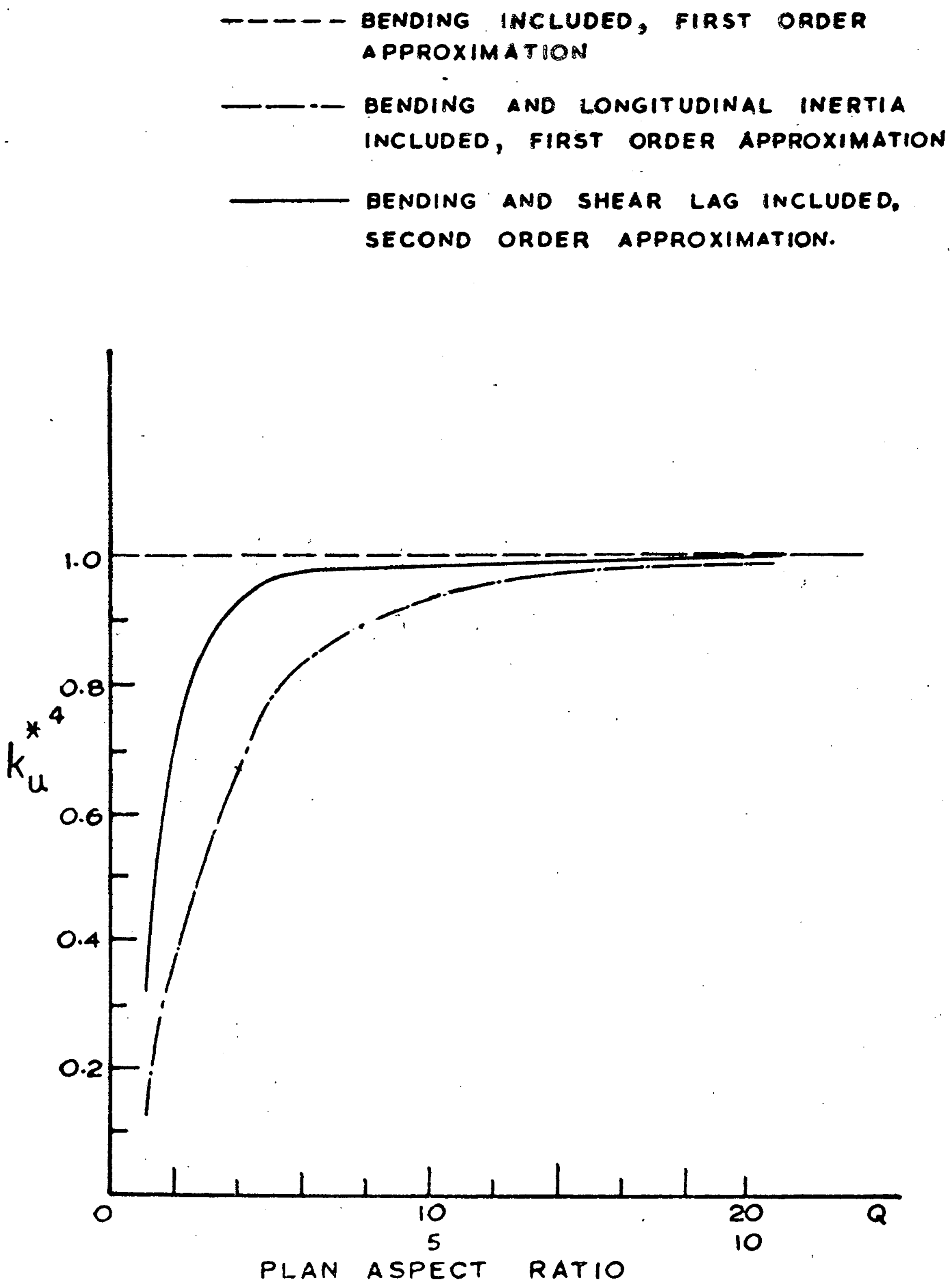

Fig. 5.4 : Influence of Plan Aspect Ratio on the Frequency Parameter for a Simply Supported Beam of I Section with a/b=1, Fundamental

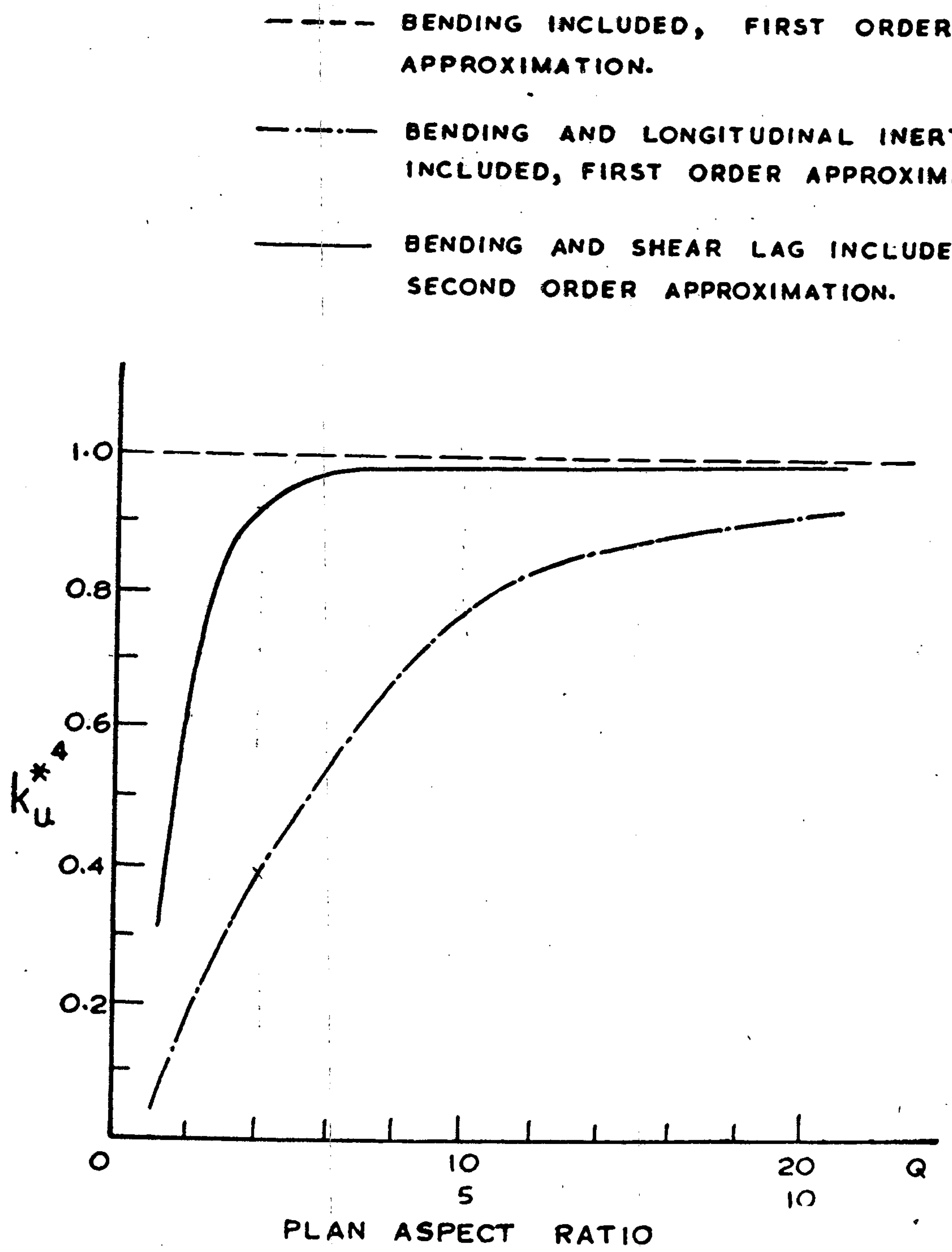

Fig. 5.5 : Influnce of Plan Aspect Ratio on the Frequency Parameter for a Simply Supported Tube of I Section with a/b=1 Second Mode

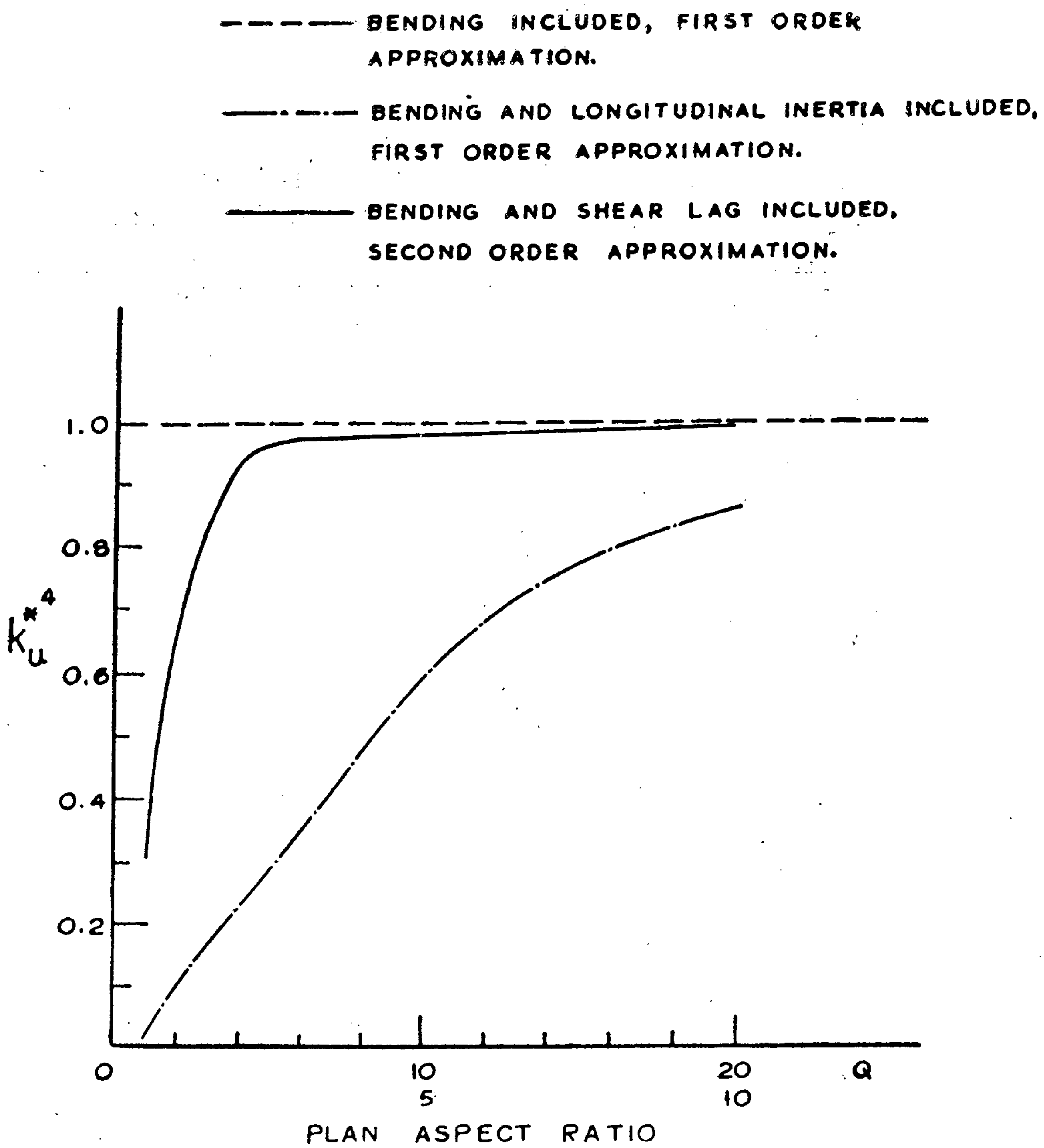

Fig. 5.6: Influence of Plan Aspect Ratio on the Frequency Parameter for a Simply Supported Tube of I Section with a/b=1, Third Mode

5.8 Conclusions

In this paper, problems of flexural vibrations of open tubes are discussed. Based on the simplicity in satisfaction of free-edge boundary condition open tubes in flexural mode of vibrations are classified as type C and D tubes. The free-edge boundary condition of type D tubes is relatively more complicated than type C tubes. A modified formulation is developed for type D tubes in which this difficulty is overcome. Using the second order approximation equations of this modified formulation a typical type D tube—a lipped I section, is analysed for flexural mode of vibration. The results show, at least second order approximations must be used in order to obtain reasonable estimates of natural frequencies of short open tubes.

Acknowledgements :

The authors are grateful to Prof. A Kameswara Rao for many valuable discussions.