

Mini-jet Total Cross-sections and Overlap Functions through Bloch-Nordsieck Summation¹

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Abstract

Predictions for total inelastic cross-sections for photon induced processes are discussed in the context of the QCD-inspired minijet model. Large theoretical uncertainties exist, some of them related to the parton distributions of hadrons in impact parameter space. A model for such distribution is presented, based on soft gluon summation. This model incorporates (the salient features of distributions obtained from) the intrinsic transverse momentum behaviour of hadrons. Under the assumption that the intrinsic behaviour is dominated by soft gluon emission stimulated by the scattering process, the b-spectrum becomes softer and softer as the scattering energy increases. In minijet models for the inclusive cross-sections, this will counter the increase from σ_{jet} .

The impact of parton scattering on the rise of inclusive cross-sections with energy was suggested by Cline and Halzen [1], after such rise was first observed in proton-proton collisions at ISR. Minijet models were put forward to describe quantitatively the further rise at higher energy [2, 3, 4] and eikonal minijet models [5, 6, 7, 8] were subsequently developed to include an increasing number of partonic collisions in QCD resulting from the rapid rise in gluon densities. Recent measurements of photo- and hadro-production

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total cross-sections [9, 10] in energy regions where QCD processes dominate, confirmed the rising trend and have been confronted with theoretical predictions obtaining varying degrees of success [11].

The simplest mini-jet model [3] was written as

$$\sigma_{inel}(s) = \sigma_{soft} + \sigma_{jet}(p_{tmin}, s) \quad (1)$$

with the rise controlled only by an energy dependent parameter, namely p_{tmin} , which regulated the Rutherford scattering divergence in the QCD jet-cross-section. The lack of unitarity of this model was amended in the eikonized mini-jet model, where inelastic hadron-hadron cross-sections are written as

$$\sigma_{inel} = \int d^2\vec{b}[1 - e^{-n(b,s)}]. \quad (2)$$

Here the average number of collisions at impact parameter b is given by

$$n(b, s) = A(b)\sigma(s) \quad (3)$$

with fixed p_{tmin} . In this version, the excessive QCD rise, controlled by $p_{tmin}(s)$ in the non-unitarized models, was softened through eikonization and introduction of the overlap function $A(b)$, which thus became a key ingredient of all models with a QCD component. This function, which describes matter distribution in impact parameter space, in most applications [5] has been assumed to be the Fourier transform of the product of hadronic form factors of the colliding particles. In other models [12], a gaussian shape has been preferred, thus relating $A(b)$ to the intrinsic transverse momentum distribution of partons in the colliding hadrons. In either case, detailed information on $A(b)$ relies on parameters to be determined case by case. However, while direct measurements of the EM form factors are available for nucleons and pseudoscalar mesons, experimental information regarding photons or other hadrons such as vector mesons is lacking. The same observation applies to the intrinsic- p_t interpretation for the spatial distribution of partons in vector mesons or photons. Thus the extension of this model to photonic cross-sections unveils one of its main drawbacks, viz. lack of a fundamental description of parton b -distributions. To reduce the uncertainties in the QCD description of the rise of the inelastic cross-section, and allow this model to graduate to QCD respectability, it is mandatory to arrive at a QCD description of the overlap function $A(b)$.

For the case of photonic processes, there are further uncertainties related to the hadronic behaviour of photons. The model has been adapted to photonic processes by writing the inelastic cross-section as

$$\sigma_{ab}^{inel} = P_{ab}^{had} \int d^2\vec{b}[1 - e^{-n(b,s)}] \quad (4)$$

where P_{ab}^{had} gives the probability that both colliding particles a, b be in a hadronic state[13] and $n(b, s) = n_{soft}(b, s) + n_{hard}(b, s)$. Here $n_{soft}(b, s)$ contains the non-perturbative part of the cross-section from which the factor P_{ab}^{had} has already been factored out and the hard,

QCD contribution to the average number of collisions at a given impact parameter \vec{b} is given by

$$n_{hard}(b, s) = A_{ab}(b) \frac{1}{P_{ab}^{had}} \sigma_{ab}^{jet} \quad (5)$$

σ_{ab}^{jet} is the hard part of the cross-section. We then have

$$P_{\gamma p}^{had} = P_{\gamma}^{had}, P_{\gamma\gamma}^{had} = (P_{\gamma}^{had})^2; \quad P_{pp}^{had} = 1.$$

The predictions of the eikonalised mini-jet model [14] for photoproduction processes therefore depend on 1) the assumption of one or more eikonals 2) the hard jet cross-section $\sigma_{ab}^{jet} = \int_{p_{tmin}} \frac{d^2\hat{\sigma}}{dp_t^2} dp_t^2$ which in turn depends on the minimum p_t above which one can expect perturbative QCD to hold, viz. p_{tmin} , and the parton densities in the colliding particles a and b , 3) the soft cross-section σ_{ab}^{soft} 4) the overlap function $A_{ab}(b)$, usually written as

$$A_{ab}(b) = \frac{1}{(2\pi)^2} \int d^2\vec{q} \mathbf{F}_a(q) \mathbf{F}_b(q) e^{i\vec{q}\cdot\vec{b}} \quad (6)$$

where \mathbf{F} is the Fourier transform of the b-distribution of partons in the colliding particles and 5) last, but not the least, P_{ab}^{had} .

To study the parameter dependence of this model, one can restrict attention to a single eikonal : more eikonal terms although improving the fits, de facto introduce new sets of parameters and very much reduce the predictivity of the model. The hard jet cross-sections have been evaluated in LO perturbative QCD using two different photonic parton densities DG [15] and GRV [16]. The dependence of σ_{ab}^{jet} on p_{tmin} for DG densities is given in Ref. [11]. Clearly this dependence is strongly correlated with the parton densities used. Here we shall only show the results for eikonalised mini-jet cross-sections using GRV densities. Further, for the purposes of this note, we try to estimate $\sigma_{soft}^{\gamma\gamma}$ from $\sigma_{soft}^{\gamma p}$ which in turn is determined by a fit to the photoproduction data. For $\gamma\gamma$ collisions, we use the Quark Parton Model suggestion $\sigma_{soft}^{\gamma\gamma} = \frac{2}{3} \sigma_{soft}^{\gamma p}$.

In the original use of the eikonal model, the overlap function $A_{ab}(b)$ of eq. 6 was obtained using for \mathbf{F} the electromagnetic form factors and thus, for photons, a number of authors [17, 18] have assumed for \mathbf{F} the pion pole expression, on the basis of Vector Meson Dominance (VMD). As mentioned, another possibility is that the b-space distribution of partons in the photon is the Fourier transform of their intrinsic transverse momentum distributions. For protons this has been assumed to correspond to a gaussian shape. For photons, the perturbative part [19] of the intrinsic transverse momentum has been suggested to correspond to the functional expression

$$\frac{dN_{\gamma}}{dk_t^2} = \frac{1}{k_t^2 + k_o^2} \quad (7)$$

Recently this expression was verified by the ZEUS [20] Collaboration, with $k_o = 0.66 \pm 0.22 \text{ GeV}$. It is interesting to notice that for photonic collisions the overlap function will have the same analytic expression for both ansätze for \mathbf{F} : the VMD inspired pion form

factor or the intrinsic transverse momentum; the only difference being that the former corresponds to a fixed value of $k_0 = 0.735 \text{ GeV}$ whereas the latter allows to vary the value of the parameter k_0 . Thus both possibilities can be easily studied by simply changing k_0 appropriately. The overlap function, which for proton-proton collisions would be given by

$$A_{pp}^{FF}(b) = \int \frac{d^2\vec{Q}}{(2\pi)^2} e^{i\vec{Q}\cdot\vec{b}} \left(\frac{\nu^2}{Q^2 + \nu^2}\right)^4 = \frac{b\nu^2\sqrt{\nu^2}}{96\pi} \mathcal{K}_3(b\sqrt{\nu^2}) \quad \nu^2 = 0.71 \text{ GeV}^2 \quad (8)$$

as proposed by L.Durand et al. [5] in the first eikonal mini-jet model for proton-proton collisions, for γp collision would become

$$A_{\gamma p}^{FF} = \frac{1}{4\pi} \frac{\nu^2 k_o^2}{k_o^2 - \nu^2} \left[\nu b K_1(\nu b) - \frac{2\nu^2}{k_o^2 - \nu^2} [K_0(\nu b) - K_0(k_o b)] \right] \quad (9)$$

and for $\gamma\gamma$ collisions

$$A_{\gamma\gamma}^{FF}(b) = \frac{1}{4\pi} k_o^3 b K_1(bk_o) \quad (10)$$

As for P_γ^{had} , this is clearly expected to be $\mathcal{O}(\alpha_{em})$ and from VMD one would expect $1/250$. It should be noticed that the eikonalised minijet cross-sections do not depend on $A_{\gamma\gamma}$ and $P_{\gamma\gamma}^{had}$ separately, but depend only on the ratio of the two [21, 22]; which for our ansätze for $A_{\gamma\gamma}$ means ratio of k_0 and $P_{\gamma\gamma}^{had}$. From phenomenological considerations [18] and fits to HERA data, fixing $k_o = 0.735 \text{ GeV}$ one finds a value $P_{had}^\gamma \approx 1/200$, which indicates at these energies a non-VMD component of $\approx 20\%$.

As mentioned, the QCD description requires the definition of p_{tmin} . From HERA data, one notices that while lower values of p_{tmin} , i.e. 1.4 GeV , can be invoked to describe the beginning of the rise, a higher value, i.e. 2.0 or even 2.5 GeV , is better suited to describe the rise at higher energy. In Fig.1a we show the fit to HERA data obtained with the above parameters, using a purely phenomenological fit to determine the non-perturbative part of the cross-section.

Having thus established the range of variability of the quantities involved in the calculation of total photonic cross sections, we now proceed to calculate and compare with existing data the eikonalized minijet cross-section for $\gamma\gamma$ collisions. For photon photon collisions, we use the central value $p_{tmin} = 2.0 \text{ GeV}$. We also use $P_\gamma^{had} = 1/204$, and $A(b)$ from eq.(10) with 3 different values of k_o which correspond to values within two standard deviations from the ZEUS [20] collaboration value. Our predictions for $\gamma\gamma$ collisions are shown in Fig. (1b). A comparison with existing data shows that data points are better fitted by a higher larger value of k_0 , and we choose $k_0 = 1 \text{ GeV}$.

As stressed, the theoretical description is rather unsatisfactory and we now move to present a model for the overlap function which, in principle, should allow for a clearer predictability and to provide an expression for $A(b)$ which could be applied to various cases of interest. We shall use Bloch-Nordsieck techniques to sum soft gluon transverse momentum distributions to all orders and compare our results with both the intrinsic transverse momentum approach as well as the form factor approach. In what follows, we shall first illustrate the Bloch-Nordsieck result and show that it gives a gaussian fall-off

Figure 1: Total inelastic photon-proton and photon-photon cross-section as described in the text.

with an intrinsic transverse size consistent with MonteCarlo models [12]. We then calculate the relevant distributions and discuss their phenomenological application.

In ref.([23]) it has been proposed that in hadron-hadron collisions, the b-distribution of partons in the colliding hadrons is the Fourier transform of the transverse momentum distribution resulting from soft gluon radiation emitted by quarks as the hadron breaks up because of the collision. This distribution is obtained by summing soft gluons to all orders, with a technique amply discussed in the literature [24, 25]. The resulting expression[26, 27] is

$$\mathcal{F}_{\mathcal{BN}}(K_{\perp}) = \frac{1}{2\pi} \int bdb J_0(bK_{\perp}) e^{-h(b;M,\Lambda)} \quad (11)$$

with

$$h(b; M, \Lambda) = \frac{2c_F}{\pi} \int_0^M \frac{dk_{\perp}}{k_{\perp}} \alpha_s\left(\frac{k_{\perp}^2}{\Lambda^2}\right) \ln \frac{M + \sqrt{M^2 - k_{\perp}^2}}{M - \sqrt{M^2 - k_{\perp}^2}} [1 - J_0(k_{\perp}b)] \quad (12)$$

where $c_F = 4/3$ and the hadronic scale M accounts for the maximum energy allowed in a

single ($k^2 = 0$) gluon emission.

The definition given in eqs.(2,3), requires for its consistency a normalized b-distribution, i.e.

$$\int d^2\vec{b}A(b) = 1. \quad (13)$$

so that the proposed Bloch-Nordsieck expression for the overlap function $A(b)$, satisfying the above normalization, reads

$$A_{BN} = \frac{e^{-h(b;M,\Lambda)}}{2\pi \int b db e^{-h(b;M,\Lambda)}} \quad (14)$$

An inspection of eq.(12), immediately poses the problem of extending the known asymptotic freedom expression for α_s to the very small k_\perp region. To avoid the small k_\perp divergence in eq.(12), it has been customary to introduce a lower cut-off in k_\perp and freeze α_s at $k_\perp = 0$, i.e. to put

$$\alpha_s(k_\perp^2) = \frac{12\pi}{33 - 2N_f} \frac{1}{\ln[(k_\perp^2 + a^2\Lambda^2)/\Lambda^2]} \quad (15)$$

with $a = 2$ in ref. [28]. For applications where the scale M is large (e.g., W-transverse momentum distribution calculations) eq.(12) is dominated by the (asymptotic) logarithmic behaviour and the small k_\perp -limit, albeit theoretically crucial, is not very relevant phenomenologically. However, this is not case in the present context, where we are dealing with soft gluon emission in low- p_t physics (responsible for large cross-sections). The typical scale of such peripheral interactions, is that of the hadronic masses, i.e. we expect $M \sim \mathcal{O}(1 \div 2 \text{ GeV})$ and the small k_\perp limit plays a basic role. This can be appreciated on a qualitative basis, by considering the limit $bM \ll 1$ of eq. (12). In this region, one can approximate $1 - J_0(kb) \approx b^2k^2/4$, to obtain

$$h(b; M, \Lambda) \approx b^2 A \quad (16)$$

with

$$A = \frac{c_F}{4\pi} \int dk^2 \alpha_s\left(\frac{k^2}{\Lambda^2}\right) \ln \frac{4M^2}{k^2} \quad (17)$$

One obtains a function $h(b; M, \Lambda)$ with a gaussian fall-off as in models where $A(b)$ is the Fourier transform of an intrinsic transverse momentum distribution of partons, i.e. $\exp(-k_\perp^2/4A^2)$. Note that the relevance of an integral similar to the one in eq.(17) has been recently discussed in connection to hadronic event shapes [29].

Our choice for the infrared behaviour of α_s for a quantitative description of the distribution in eq.(12), does not follow eq.(15), but is inspired by the Richardson potential for quarkonium bound states [30]. In a number of related applications [31, 32], it has been proposed to calculate the above integral using the following expression for α_s :

$$\alpha_s(k_\perp) = \frac{12\pi}{(33 - 2N_f)} \frac{p}{\ln[1 + p(\frac{k_\perp}{\Lambda})^{2p}]} \quad (18)$$

which coincides with the usual one-loop expression for large (relative to Λ) values of k_\perp , while going to a singular limit for small k_\perp . For the special case $p = 1$ such an α_s coincides with one used in the Richardson potential [30], and which incorporates - in a compact expression - the high-momentum limit demanded by asymptotic freedom as well as linear quark confinement in the static limit. In [31] we have generalized Richardson's ansatz to values of $p \leq 1$. For $1/2 < p \leq 1$, this corresponds to a confining potential rising less than linearly with the interquark distance r . The range $p \neq 1$ has an important advantage, i.e., it allows the integration in eq.(12) to converge for all values of k_\perp . For the motivations given in [31] the value $p = 5/6$ was chosen in previous calculations of the transverse momentum distribution of Drell Yan pairs [31, 33].

Having set up our formalism, we shall now examine its implications. The distribution $A(b)$ depends upon the hadronic scale M in the function $h(b)$. This scale depends upon the energy of the specific subprocess and, through this, upon the hadron scattering energy. It plays a crucial role, just as it did for the Drell-Yan process, where the expression of eq.(11) has been successfully [34, 35, 33, 28] used to describe the transverse momentum distribution of the time-like virtual photon or W-boson. In these cases [34, 33], the scale M was found to be energy dependent and to vary between $\sqrt{Q^2}/4$ and $\sqrt{Q^2}/2$. In the calculation of the transverse momentum distribution of a lepton pair produced in quark-antiquark annihilation [34], the scale M was obtained as the maximum transverse momentum allowed by kinematics to a single gluon emitted by the initial $q\bar{q}$ pair of c.m.energy $\sqrt{\hat{s}}$ in the process

$$q\bar{q} \rightarrow g + \gamma(Q^2) \quad (19)$$

In the Drell-Yan case, one needed $h(b)$ to calculate the transverse momentum distribution of the lepton pair, here we use it to evaluate the average number of partons in the overlap region of two colliding hadrons. In this case $e^{-h(b)}$ is the F-transform of the transverse momentum distribution induced by initial state radiation in the process

$$q\bar{q} \rightarrow jet\ jet + X \quad (20)$$

where the jet pair in process (20) is the one produced through gluon-gluon or other parton-parton scattering with total jet-cross-section σ_{jet} and X can also include the quark-antiquark pair which continues undetected after emission of the gluon pair which stimulated the initial state bremsstrahlung. We work in a no-recoil approximation, where the transverse momentum of the jet pair is balanced by the emitted soft gluons. Then the maximum transverse momentum allowed to a single gluon is given by

$$q_{max}(\hat{s}) = \frac{\sqrt{\hat{s}}}{2} \left(1 - \frac{\hat{s}_{jet}}{\hat{s}}\right) \quad (21)$$

with $\sqrt{\hat{s}_{jet}}$ being the jet-jet invariant mass over which one needs to perform further integrations. An improved eq.(3) now reads

$$n(b, s) = n_{soft}(b, s) + \sum_{i,j} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} f_i(x_1) f_j(x_2) \int dz \int dp_t^2 A_{BN}(b, q_{max}) \frac{d\sigma}{dp_t^2 dz} \quad (22)$$

where f_i are the quark densities in the colliding hadrons, $z = \hat{s}_{jet}/(sx_1x_2)$, and $\frac{d\sigma}{dp_t^2 dz}$ is the differential cross-section for process (20) for a given p_t of the produced jets.

Unlike the usual expressions for $n(b, s)$, eq.(22) does not exhibit factorization between the longitudinal and transverse degrees of freedom since the distribution A_{BN} depends upon the quark subenergies. Factorization can be obtained however, through an averaging process whereupon one can factorize the b-distribution in eq.(22), by evaluating A_{BN} with q_{max} at its mean value, i.e. write

$$n(b, s) = n_{soft}(b, s) + A_{BN}(b, \langle q_{max}(s) \rangle) \sigma_{jet} \quad (23)$$

with

$$\sigma_{jet} = \sum_{i,j} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} f_i(x_1) f_j(x_2) \int dz \int dp_t^2 \frac{d\sigma}{dz dp_t^2} \quad (24)$$

and

$$\langle q_{max}(s) \rangle = \frac{\sqrt{s} \sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \sqrt{x_1 x_2} \int dz (1-z)}{2 \sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \int (dz)} \quad (25)$$

with the lower limit of integration in the variable z given by $z_{min} = 4p_{tmin}^2/(sx_1x_2)$. To grasp the energy dependence of this scale, one can use a simple toy model, in which the valence quark densities are approximated by $1/\sqrt{x}$ and thus obtain

$$\langle q_{max}(s) \rangle \sim \frac{3}{8} p_{tmin} \ln \frac{\sqrt{s}}{2p_{tmin}} \quad (26)$$

for $2p_{tmin} \ll \sqrt{s}$. For $p_{tmin} = 1.4 \text{ GeV}$, as in typical eikonal mini-jet models for proton-proton scattering [6], one obtains values of $\langle q_{max}(s) \rangle$ which range from 0.5 to 5 GeV for \sqrt{s} between 10 GeV and 14 TeV respectively. A more precise evaluation of the above quantities depends upon the type of parton densities one uses, and will be discussed in a forthcoming paper.

From the discussion about the large b-behaviour of the function $h(b)$, we then expect $A_{BN}(b, s)$ to fall at large b more rapidly as the energy increases from $\sqrt{s} = 10 \text{ GeV}$ into the TeV region. In Fig. 2, we compare this behaviour for the function $A(b)$ with the one obtained through the Fourier transform of the squared e.m. form factor of the proton, eq.(8). The function $A(b)$ from the Bloch-Nordsieck model is calculated for $\Lambda = 0.1 \text{ GeV}$ and values of $\langle q_{max} \rangle$ which include those obtainable from eq.(26) in the energy range $\sqrt{s} \approx 10 \text{ GeV} \div 14 \text{ TeV}$.

We notice that, as the energy increases, $A(b)$ from the form factor model remains substantially higher at large b than in the Bloch-Nordsieck case. As a result, for the same σ_{jet} the Bloch-Nordsieck model will give smaller $n(b, s)$ at large b than the form factor model and a softening effect of the total eikonal mini-jet cross-sections can be expected.

In conclusion, we have studied the parameter dependence of the eikonalized mini-jet model for photonic and hadronic total inelastic cross-section and found that a large uncertainty arises through the description of parton distributions in impact parameter space.

Figure 2: Comparison between the $A(b)$ distribution function from the Bloch-Nordsieck model (full) and the form factor model (dots).

A model, derived from soft gluon summation techniques, is described and compared with expectations from the currently used form factor models for such distribution. Such comparison indicates a distinctly different behaviour in this large b -region suggesting a softening of the rise of the total cross-section in mini-jet models relative to the ones with the hadron form-factors.

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