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Eikonalised minijet model predictions for cross-sections of photon induced processes

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Abstract

In this talk I present the results of an analysis of total and inelastic hadronic cross-sections for photon induced processes in the framework of an eikonalised minijet model (EMM). We fix the various input parameters to the EMM calculations by using the data on photoproduction cross-sections and then make predictions for $\sigma(\gamma\gamma \rightarrow \text{hadrons})$ using *same* values of the parameters. We then compare our predictions with the recent measurements of $\sigma(\gamma\gamma \rightarrow \text{hadrons})$ from LEP. We also show that in the framework of the EMM the rise with \sqrt{s} of the total/inelastic cross-sections will be faster for photon induced processes than for the processes induced by hadrons like protons.

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1 Introduction

The rise of hadronic total cross-sections $\sigma(A + B \rightarrow \text{hadrons})$ with energy, has been now observed for a set of comparable values of \sqrt{s} where both A,B are hadrons [1, 2], when one of them is a photon [3, 4] and when both of them are photons [5, 6]. It is well known that interactions of a photon with another hadron or photon receive contributions from the ‘structure’ of a photon which the photon develops due to its fluctuation into a virtual $q\bar{q}$ pair. The recent measurements [5, 6] of $\sigma(\gamma\gamma \rightarrow \text{hadrons})$ at higher energies of upto and beyond $\sim \mathcal{O}(100)$ GeV, have confirmed that the hadronic cross-sections rise with \sqrt{s} and preliminary claims [7, 8] are that they rise faster than the $p\bar{p}$ cross-sections. A measurement of γ^*p cross-sections by ZEUS collaboration, extrapolated to $Q^2 = 0$ [9, 10], lies above the photoproduction measurements [3, 4]. These extrapolated γ^*p data and the new $\gamma\gamma$ data seem to indicate that the rise of cross-sections with \sqrt{s} gets faster as one replaces hadrons with photons successively. In the Pomeron-Regge picture [11] the total cross-section is given by

$$\sigma_{ab}^{\text{tot}} = Y_{ab}s^{-\eta} + X_{ab}s^{\epsilon} \quad (1)$$

where η and ϵ are related to the intercept at zero of the leading Regge trajectory and of the Pomeron, respectively. The value of the Pomeron intercept indicated by the unpublished results of ZEUS [9, 10] is $0.157 \pm 0.019 \pm 0.04$ whereas the corresponding value for the $\gamma\gamma$ data obtained by the L3 collaboration [5] is $0.158 \pm 0.006 \pm 0.028$ which is to be compared with the value of ~ 0.08 [11] for pure hadronic cross-sections.

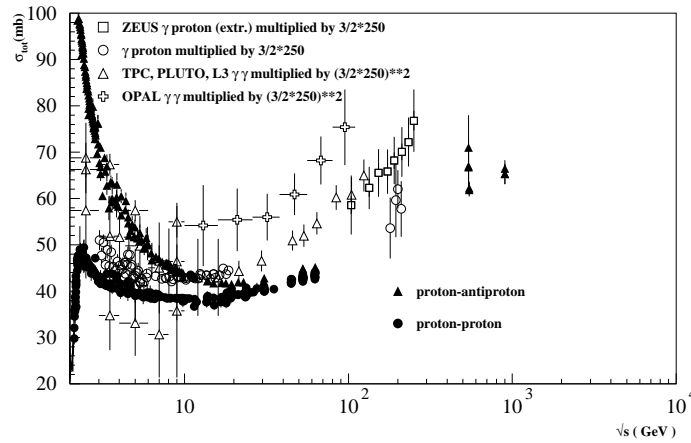


Figure 1: Energy dependence of σ_{ab}^{tot} .

Fig. 1 shows the energy dependence of the hadronic cross-sections as well as those for the photon induced processes. The latter are multiplied by a quark model motivated factor of $3/2$ and the inverse of the probability for a photon to fluctuate into a $q\bar{q}$ pair: P_γ^{had} . The value chosen for P_γ^{had} is $1/250$ which is close to a value motivated by VMD picture. i.e.

$$P_\gamma^{\text{had}} = P_{VMD} = \sum_{V=\rho,\omega,\phi} \frac{4\pi\alpha_{\text{em}}}{f_V^2} \approx \frac{1}{250}. \quad (2)$$

Fig. 1 illustrates that *all* total cross-sections show a similar rise in energy when the difference between photon and hadron is taken into account, albeit with indications of somewhat steeper energy dependence for photon induced processes¹. The similarity in the energy dependence makes it interesting to attempt to give a description of all three data sets in the same theoretical framework[12]. We have analysed the cross-sections for photon-induced processes alone [13, 14, 15] and we find that the EMM calculations generically predict faster rise with \sqrt{s} for $\gamma\gamma$ case than would be expected by an universal pomeron hypothesis. I will further present arguments why in the framework of EMM the Pomeron intercept is expected to increase as the number of colliding photons in the process increase.

2 Eikonalised Minijet Model

There are some basic differences between the purely hadronic cross-section measurements and those of the total cross-sections in photon induced processes. In the purely hadronic case the measurement of the total cross-section comes from the combined methods of extrapolation of the elastic diffraction peak and total event count, whereas in the case of photon induced processes, this has to be extracted from the data using a Monte-carlo; for example the total $\gamma\gamma$ cross-sections are extracted from a measurement of hadron production in the untagged e^+e^- interactions. Experimentally this gives rise to different types of uncertainties in the measurement². Theoretically, the concept of the ‘elastic’ cross-section can not be well defined for the photons. In models satisfying unitarity like those which use the eikonal formulation, it is important to understand what is the definition of the elastic cross-section and if the data include all of this elastic cross-section or part or a small fraction. To that end let us summarise some of the basic features of the Eikonal formulation.

¹It should be noted here that although the rate of rise with \sqrt{s} is similar for both OPAL and L3 [7], on this plot the OPAL data seems to stand a bit apart, which may be due to the difference in the normalisation of the two data sets.

²This is clear from the discussions, for example, in Ref. [7].

Let us start from the very beginning. Consider the eikonal formulation for the elastic scattering amplitude

$$f(\theta) = \frac{ik}{2\pi} \int d^2\vec{b} e^{i\vec{q}\cdot\vec{b}} [1 - e^{i\chi(b,s)/2}] \quad (3)$$

which, together with the optical theorem leads to the expression for the total cross-sections

$$\sigma^{\text{el}} = \int d^2\vec{b} |1 - e^{-i\chi(b,s)/2}|^2 \quad (4)$$

$$\sigma^{\text{tot}} = 2 \int d^2\vec{b} [1 - e^{-\chi_I(b,s)/2} \cos(\chi_R)] \quad (5)$$

$$\sigma^{\text{inel}} = \sigma^{\text{tot}} - \sigma^{\text{el}} = \int d^2\vec{b} [1 - e^{-\chi_I(b,s)}] \quad (6)$$

According to the minijet model [16, 17, 18, 19] the rise of total cross-sections can be calculated from QCD. In the model, there is an *ad hoc* sharp division between a soft component, which is of non-perturbative origin and for which the model is not able to make theoretical predictions, and a hard component, which receives input from perturbative QCD. The minijet model assumes that the rise with energy of total cross-sections is driven by the rise with energy of the number of low-x partons (gluons) responsible for hadron collisions and in its simplest formulation reads

$$\sigma_{ab}^{\text{inel},u} = \sigma_0 + \int_{p_{tmin}} d^2\vec{p}_t \frac{d\sigma_{ab}^{\text{jet}}}{d^2\vec{p}_t} = \sigma_0 + \sigma_{ab}^{\text{jet}}(s, p_{tmin}), \quad (7)$$

the superscript *u* indicating that this is the ununitarised cross-section. This concept can be embodied in a unitary formulation as in 5-6, by writing [20, 21]

$$\sigma_{ab}^{\text{inel}} = P_{ab}^{\text{had}} \int d^2\vec{b} [1 - e^{-n(b,s)}] \quad (8)$$

with

$$n(b, s) = A_{ab}(b) \left[\sigma_{h/a,h/b}^{\text{soft}} + \frac{\sigma_{ab}^{\text{jet}}(s, p_{tmin})}{P_{ab}^{\text{had}}} \right] \quad (9)$$

In eq.8, we have inserted, to include the generalization to photon processes, a factor P_{ab}^{had} defined as the probability that particles *a* and *b* behave like hadrons in the collision. This parameter is unity for hadron-hadron processes, but of order α_{em} or α_{em}^2 for processes with respectively one or two photons in the initial state. The definition of $\sigma_{h/a,h/b}^{\text{soft}}$ in eq.9 is such that, even in the photonic case, it is of hadronic size, just like $\sigma_{ab}^{\text{jet}}(s, p_{tmin})/P_{ab}^{\text{had}}$. A simple way to understand the need for this factor [22] is to realise that the unitarisation in this formalism is achieved by multiple parton interactions in a given scatter of hadrons and once the photon has ‘hadronised’ itself, one should not be paying the price of P_{γ}^{had} for further multiparton scatters.

At high energies, the dominant term in the eikonal is the *jet* cross-section which is calculable in QCD and depends on the parton densities in the colliding particles and p_{tmin} ,

which admittedly is an ad hoc parameter separating the perturbative and nonperturbative contributions to the eikonal. The basic assumption in arriving at eq. 8 is that the multiple parton scatters responsible for the unitarisation are independent of each other at a given value of b . In this model $n(b, s)$ in eq. 9, is identified as the average number of collisions at any given energy \sqrt{s} and impact parameter b . The b dependence is assumed to be given by the function $A_{ab}(b)$ which is modelled in different ways. This function measures the overlap of the partons in the two hadrons a, b in the transverse plane.

Before going to the discussion of different models of $A_{ab}(b)$, we note that the mini-jet model is particularly well suited for generalisation to the photon-induced processes where the concept of ‘elastic’ cross-section is not very well defined. Whereas for the hadronic case one starts from the elastic amplitude followed by the optical theorem as done in eqs. 3 – 6, in this case the starting point is actually the eq. 8 and then one defines σ_{ab}^{tot} using eq. 6 with $\chi_R = 0$ and using χ_I as given by 8. The above discussion specifies the total cross-section formulation of the EMM for photon-induced processes. While our earlier analyses [13, 14, 15] assumed that the $\gamma\gamma$ cross-sections presented were the inelastic cross-sections, the analysis of [12] had used the total cross-section formulation but with a different ansatz for the eikonal. Our analysis uses the total cross-section formulation of the EMM with the perturbative part of the Eikonal as given by QCD *a-la* eq. 8.

3 Overlap function and jet cross-sections.

The overlap function $A_{ab}(b)$ is normally calculated in terms of the convolution of the matter distributions $\rho_{a,b}(\vec{b})$ of the partons in the colliding hadrons in the transverse plane

$$A_{ab}(b) = \int d^2\vec{b}' \rho_a(\vec{b}') \rho_b(\vec{b} - \vec{b}'). \quad (10)$$

If we assume that the $\rho(\vec{b})$ is given by Fourier Transform of the form factor of the hadron, then $A_{ab}(b)$ is given by,

$$A_{ab}(b) = \frac{1}{(2\pi)^2} \int d^2\vec{q} \mathcal{F}_a(q) \mathcal{F}_b(q) e^{i\vec{q}\vec{b}}, \quad (11)$$

where $\mathcal{F}_{a,b}$ are the electromagnetic form factors of the colliding hadrons. For protons this is given by the dipole expression

$$\mathcal{F}_{prot}(q) = \left[\frac{\nu^2}{q^2 + \nu^2} \right]^2, \quad (12)$$

with $\nu^2 = 0.71 \text{ GeV}^2$. For photons a number of authors [20, 21], on the basis of Vector Meson Dominance, have assumed the same functional form as for pion, i.e. the pole

expression

$$\mathcal{F}_{pion}(q) = \frac{k_0^2}{q^2 + k_0^2}, \quad (13)$$

with $k_0 = 0.735$ GeV from the measured pion form factors, changing the value of the scale parameter k_0 , if necessary in order to fit the data.

Yet another philosophy would be to assume that the b-space distribution of partons is the Fourier transform of the transverse momentum distribution of the colliding system [23]. To leading order, this transverse momentum distribution can be entirely due to an intrinsic transverse momentum of partons in the parent hadron. While the intrinsic transverse momentum (k_T) distribution of partons in a proton is normally taken to be Gaussian, a choice which can be justified in QCD based models [24], in the case of photon the origin of all partons can, in principle, be traced back to the hard vertex $\gamma q\bar{q}$. Therefore, also in the intrinsic transverse momentum philosophy, one can expect the k_T distribution of photonic partons to be different from that of the partons in the proton. The expected functional dependence can be deduced using the origin of photonic partons from the $\gamma \rightarrow q\bar{q}$ splitting. For the photon one can argue that the intrinsic transverse momentum ansätze would imply the use of a different value of the parameter k_0 [15, 25], which is extracted from data involving ‘resolved’ [26] photon interactions [27], in the pole expression for the form factor. By varying k_0 one can then explore various possibilities, i.e. the VMD/ π hypothesis if $k_0 = 0.735$ GeV, or the intrinsic transverse distribution case for other values of k_0 [27].

The ansatz of eqs. 5– 6 and 9, requires that the overlap function be normalised to unity, i.e.,

$$\int d^2\vec{b} A_{ab}(b) = 1. \quad (14)$$

Taking the form factor ansatz for the proton we then have

$$A_{pp}(b) = \frac{\nu^2}{96\pi} (\nu b)^3 \mathcal{K}_3(\nu b) \quad A_{\gamma\gamma} = \frac{k_0^3 b}{4\pi} \mathcal{K}_1(k_0 b) \quad (15)$$

and

$$A_{\gamma p}(b) = \frac{k_0^2 \nu^4}{2\pi(\nu^2 - k_0^2)^2} [\mathcal{K}_0(k_0 b) - \mathcal{K}_0(\nu b)] + \frac{k_0^2 \nu^2}{4\pi(k_0^2 - \nu^2)} \nu b \mathcal{K}_1(\nu b) \quad (16)$$

where ν and k_0 are the scale factors mentioned earlier and \mathcal{K}_i are the modified Bessel functions.

If we look at eqs. 8- 9 it is easy to see that $A_{ab}(b)$ and P_{ab}^{had} always appear in the combination $A_{ab}(b)/P_{ab}^{had}$ [28, 26]. Hence only one of them can be varied independently. Note also that σ^{soft} can always be renormalised since it is a function fitted to the low energy data. By looking at eq. 9 we can see that if the s -dependence of the *jet* cross-sections were similar for *all* the colliding particles, then the difference in the s -dependence

of the total/inelastic cross-sections can be estimated by looking at the behaviour of A_{ab} . It is also clear then that changing the scale parameter k_0 in $A_{ab}(b)$ is equivalent to changing P_{ab}^{had} . Note also that

$$P_{\gamma p}^{had} = P_{\gamma}^{had}, \quad P_{\gamma\gamma}^{had} = (P_{\gamma}^{had})^2. \quad (17)$$

Hence, in analysing the photon-induced reactions, i.e, the γp and $\gamma\gamma$ cross-sections, the only hadronisation probability that is an independent parameter is $P^{had} \equiv P_{\gamma}^{had}$.

Thus now we are ready to list the total number of inputs on which the EMM predictions depend:

- The soft cross-sections σ_{ab}^{soft} ,
- p_{tmin} and the parton densities in the colliding hadrons,
- P^{had} and the ansatz as well as the scale parameters for the $A_{ab}(b)$,

Out of these the protonic and photonic parton densities are known from eP and $e\gamma$ DIS. The nonperturbative part $\sigma_{\gamma p}^{soft}, \sigma_{\gamma\gamma}^{soft}$ has to be determined from some fits. We outline the procedure used by us below. It is true that the jet cross-sections of eq. 7 depend very strongly on the value of p_{tmin} . Hence it would be useful to have an independent information of this parameter, which as said before separates the perturbative and nonperturbative contribution in an ad hoc manner. Luckily, there is more direct evidence that the ansatz of eq. 8 can describe some features of hadronic interactions. Event generators which have built in multiple parton interactions in a given ab interaction, for the case of a, b being p, \bar{p} , were shown [29] to explain many features of the hadronic interactions such as multiplicity distributions with a p_{tmin} around 1.5 GeV. Recent analyses of the γp interactions seem to show [30] again that a consistent description of many features such as energy flow, multiplicity distributions is possible with a p_{tmin} value between 1.5 – 2.0 GeV.

The energy dependence of σ_{ab}^{jet} as defined in eq. 7 will of course get reflected in the energy rise of the eikonalised total or inelastic cross-section. It is therefore instructive to see how this depends on the type of the colliding particles. We compare this for $p\bar{p}$, γp and $\gamma\gamma$ case, where we have multiplied the γp and $\gamma\gamma$ jet cross-sections by factor of α and α^2 respectively. In the comparison of Fig. 2 we have used the GRV, LO parametrisations for both the proton [31] and the photon [32]. We note here that at high energies, the rise with energy of the jet cross-sections is very similar in all the three cases, when difference between a photon and a proton is accounted for. A study of the b -dependence of the respective $A_{ab}(b)$ given by eqs. 15, 16, shows that the photon is much ‘smaller’ as compared to the proton in the transverse space, which is also understandable as the photon after all owes its ‘structure’ to the hard $\gamma q\bar{q}$ vertex. Hence, we expect that the

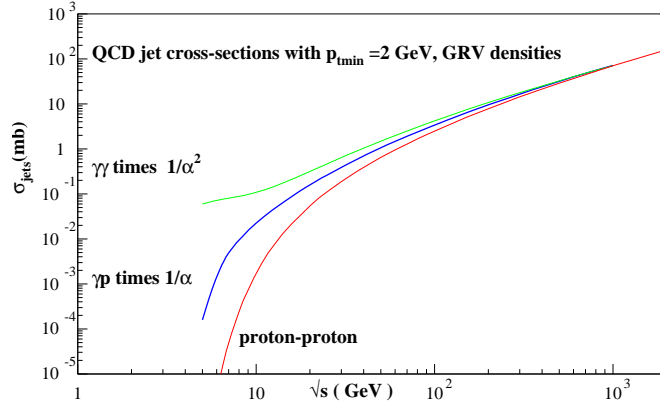


Figure 2: Energy dependence of σ_{ab}^{jet} for $p_{tmin} = 2\text{GeV}$.

damping of the cross-section rise due to multiple scattering for photons will be less than for a proton. This, coupled with the above observation of the *jet* cross-section, implies that in the EMM, total/inelastic cross-sections are expected to rise faster with energy as we replace a proton by a photon. I.e., an increase in the pomeron intercept as we go from $\bar{p}p$ to γp and $\gamma\gamma$ as indicated by the data, is expected in the EMM framework.

4 Results

Now in this section let us spell out our strategy of fixing the various inputs to the EMM. We restrict our analysis only to the photon-induced processes, i.e., γp and $\gamma\gamma$ cross-section. We follow the same procedure as we had adopted in [15], i.e., we fix all the inputs to the EMM by a fit to the data on the available photoproduction data on $\sigma_{\gamma p}$. Here we do not include the data [9, 10], shown in Fig. 1, which has been obtained by an extrapolation of the low Q^2 data to 0. We determine $\sigma_{\gamma p}^{\text{soft}}$ by a fit to the photoproduction data using a form suggested in [21],

$$\sigma_{\gamma p}^{\text{soft}} = \sigma_{\gamma p}^0 + \frac{\mathcal{A}_{\gamma p}}{\sqrt{s}} + \frac{\mathcal{B}_{\gamma p}}{s}. \quad (18)$$

We then determine $\mathcal{A}_{\gamma p}$, $\mathcal{B}_{\gamma p}$ and $\sigma_{\gamma p}^0$ from the best fit to the low-energy photoproduction data, starting from the quark-model motivated ansatz $\sigma_{\gamma p}^0 = 2/3\sigma_{\bar{p}p}$. In earlier work [15] we had used the inelastic formulation. Now we have repeated the same exercise with the total cross-section formulation of the EMM, which we believe is the more appropriate to use [12]. The results of our fit, using the total cross-section formulation of the EMM, are

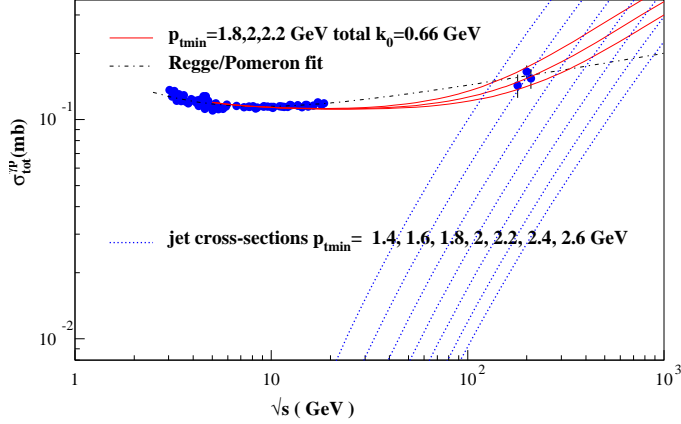


Figure 3: Comparison of the photoproduction data with the EMM fits with total cross-section formulation, and jet cross-sections as a function of p_{tmin} .

shown in Fig. 3. The fit values of the parameters are

$$\sigma_{\gamma p}^0 = 31.2 \text{ mb} \quad ; \quad \mathcal{A}_{\gamma p} = 10 \text{ mb GeV} \quad ; \quad \mathcal{B}_{\gamma p} = 37.9 \text{ mb GeV}^2. \quad (19)$$

As compared with the similar exercise done in [15], we find that the rise of the eikonalised cross-sections with \sqrt{s} is faster in this case than in the inelastic formulation. However, $p_{tmin} = 2 \text{ GeV}$ is still the best value to use, as seen from Fig. 3. We use here the form factor ansatz for the proton and the intrinsic k_T ansatz for the photon with a value of parameter $k_0 = 0.66 \text{ GeV}$, which corresponds to the central value from the measurement [27] of the intrinsic k_T distribution. We have used GRV distributions for both the proton and photon and $P^{\text{had}} = 1/240$. We also find, similar to the analysis in the inelastic formulation by us [15] and others [21, 33], that the description of the photoproduction data in terms of a single eikonal leaves leeway for improvement. We restrict ourselves to the use of a single eikonal, so as to minimize our parameters but note that this can perhaps be cured by using an energy dependent P^{had} or alternatively an energy dependent k_0 .

Now, having fixed all the inputs for the γp case, we determine the corresponding parameters for the $\gamma\gamma$ case again by appealing to the Quark Model considerations and we use,

$$\sigma_{\gamma\gamma}^{\text{soft}} = \frac{2}{3} \sigma_{\gamma p}^{\text{soft}}. \quad (20)$$

All the other inputs are exactly the same as in the γp case. In this manner, we have really *extrapolated* our results from γp case to the $\gamma\gamma$ case. The results of our extrapolation are shown in Fig. 4.

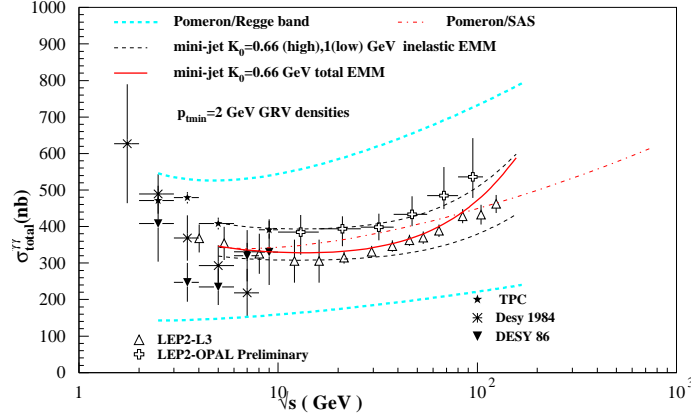


Figure 4: EMM predictions for $\gamma\gamma$ cross-sections for total and inelastic cross-section formulations along with data and Regge -Pomeron prediction.

Notice that the overall normalization of the photonic cross-section depends upon P^{had} . When extrapolating from photoproduction to photon-photon using the inelastic assumption, we had used $P^{had} = 1/200$, which can be thought of as corresponding to a 20% non-VMD component. Using the total cross-section formulation, the low energy production data suggest rather to use $P^{had} = 1/240$, a value which implies that the photon is practically purely a vector meson. Then, Fig.4 shows that in the total cross-section formulation, the extrapolation from γp to $\gamma\gamma$ leads to cross-section which lies lower at low energies, but rises faster, than in the inelastic fits, for the same values of the parameters.

Both the inelastic and the total cross-section are seen to rise faster than is expected in an universal pomeron picture. This feature is same both for γp and $\gamma\gamma$ cases. We show the dependence of our results on the scale parameter k_0 . The band in the figure corresponds to using the Regge-Pomeron hypothesis of eq. 1, measured values of X_{ab}, Y_{ab} for $\bar{p}p/pp, \gamma p$ case and the factorisation idea [34]

$$X_{\gamma\gamma} = X_{\gamma p}^2/X_{pp}; \quad Y_{\gamma\gamma} = Y_{\gamma p}^2/Y_{pp}.$$

Here $X(Y)_{pp}$ stand for an average for pp and $\bar{p}p$ case.

We see that while our analysis using inelastic formulation and the default value of $k_0 = 0.66$ GeV [27] gave predictions closer to the OPAL data the total cross-section formulation, *for the same value of k_0* , gives results closer to the L3 data, as already pointed out in [12]. The sensitivity of the predictions to the difference between different parametrisations for the photonic partons increases with energy. At higher energies one is sensitive to the low- x region about which not much is known. Our earlier analysis [

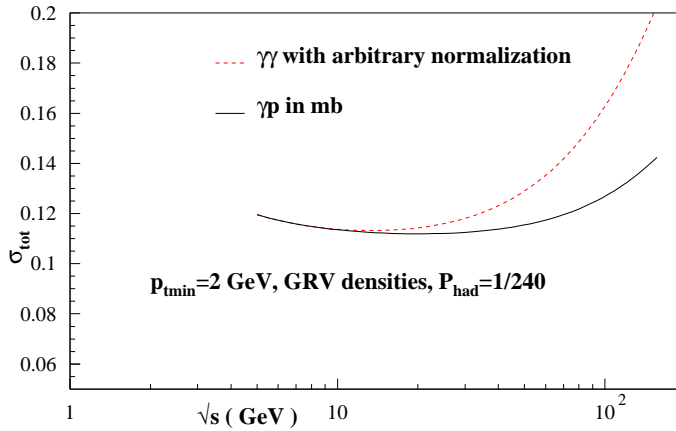


Figure 5: Comparison of the energy dependence of the EMM predictions for the total $\gamma\gamma$ and γp cross-sections with $p_{tmin} = 2$ GeV, $P^{\text{had}} = 1/240$.

15] in the inelastic formulation had shown that the $\gamma\gamma$ cross-sections rise more slowly for the SAS [35] parametrisation of the photonic parton densities. The dependence of our results in this analysis on the parton densities in the photon will be presented elsewhere [36].

Fig. 5 shows a comparison of the eikonalised γp and $\gamma\gamma$ cross-sections, in the total formulation, with the different parameters for γp and $\gamma\gamma$ case related as described before. We see indeed that in the EMM the $\gamma\gamma$ cross-sections rise faster with \sqrt{s} than γp case, as was expected from the results shown in Fig. 3 and arguments following that. However, the dependence of this observation on P^{had} and/or the scale parameter needs to be still explored.

5 Conclusions

In conclusion we discuss the results of an analysis of total and inelastic hadronic cross-sections for photon induced processes in the framework of an eikonalised minijet model (EMM). We have fixed various input parameters to the EMM calculations by using the data on photoproduction cross-sections and then made predictions for $\sigma(\gamma\gamma \rightarrow \text{hadrons})$ using *same* values of the parameters. We then compare our predictions with the recent measurements of $\sigma(\gamma\gamma \rightarrow \text{hadrons})$ from LEP. We find that the total cross-section formulation of the EMM predicts faster rise with \sqrt{s} as compared to the inelastic one, for the *same* value of p_{tmin} and scale parameter k_0 . In the former case our extrapolations yield results closer to the L3 data whereas in the latter case they are closer to the OPAL results.

We also find that in the framework of EMM it is natural to expect a faster rise with \sqrt{s} for the $\gamma\gamma$ case as compared to the γp case.

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