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$\sigma^{tot}_{ee\gamma\gamma}$ at e^+e^- colliders [1]

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Abstract

In this talk I briefly summarize different models for $\sigma^{tot}_{2\gamma}$ ($e^+e^- \rightarrow \gamma\gamma \rightarrow$ hadrons) and contrast model predictions with the data. I will then discuss the capability of the future e^+e^- and $\gamma\gamma$ colliders to distinguish between various models and end with an outlook for future work.

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$\sigma^{tot}_{ee\gamma\gamma}$ at e^+e^- colliders

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INTRODUCTION

The subject of this discussion is total hadronic cross-section in e^+e^- collisions. At high energies this is essentially given by $\sigma^{tot}_{ee\gamma\gamma} \equiv \sigma^{tot}$ ($e^+e^- \to e^+e^-\gamma\gamma \to e^+e^-$ hadrons). Further it is also established that the major contribution to the hadron production in 2γ processes at high energies, comes from the hadronic structure of the photon [1]. Experimentally, recent data on $\sigma^{tot}_{\gamma\gamma\gamma}$ has shown [2,3] that the cross-section rises with $\sqrt{s}$ just like the γp [4,5] and pp/p̄p [6] case. $\sigma^{tot}_{ee\gamma\gamma}$ is given by

$$\sigma^{tot}_{ee\gamma\gamma} = \int dx_1 \int dx_2 \;\; f_{\gamma_1/e}(x_1) \;\; f_{\gamma_2/e}(x_2) \;\; \sigma^{tot}_{\gamma\gamma}(\hat{s} = s x_1 x_2) \tag{1}$$

where $\sigma^{tot}_{\gamma\gamma}$ is the total hadronic cross-section σ^{tot} ($\gamma\gamma \to$ hadrons) and $f_{\gamma_i/e}(x_i)$ are the flux factors for γ in e^-/e^+. Hence it is clear that the $\sqrt{s_{\gamma\gamma}}$ dependence of $\sigma^{tot}_{\gamma\gamma}$ controls the rate of the rise of $\sigma^{tot}_{ee2\gamma}$ with $\sqrt{s_{e^+e^-}}$ and this knowledge is necessary to estimate the hadronic backgrounds due to the 2γ processes at the future linear colliders. It has been pointed out that these can threaten to spoil the clean environment of an LC [7,8]; particularly at high energy e^+e^- colliders like CLIC as well as the $\gamma\gamma$ colliders [9] that are being discussed. Apart from this pragmatic need for a good model to extrapolate the $\sigma^{tot}_{2\gamma}$ at high energies, the 2γ system also provides an additional theoretical laboratory to test our models of calculating σ^{tot}_{AB}. Understanding the observed rise with energy of all the hadronic cross-sections in a QCD based picture is a theoretical challenge. Since the cross-sections of photon induced processes [10,9] show some special features, such studies increase our understanding of the photon as well. The dramatic improvement in the state of the data on $\sigma^{tot}_{2\gamma}$ [2,3] from the study of 2γ processes at LEP has already helped provide discrimination among predictions of theoretical models [11–13].

THEORETICAL MODELS

There exist two types of theoretical models [14] for calculation of $\sigma^{tot}_{\gamma\gamma}$; what we can call loosely as (i) 'Photon is like a proton' models [15–19] and (ii) QCD based models [20–23]. In the first class of models, the energy dependence of the $\gamma\gamma$ cross-sections is essentially similar to that for pp/p̄p. In Ref. [15] the total $\gamma\gamma$ cross-section is assumed to be described in the form

$$\sigma^{tot}_{\gamma\gamma} = Y_{\gamma\gamma} s^{-\eta} + X_{\gamma\gamma} s^{\epsilon} \tag{2}$$

The powers η and ϵ are assumed to be universal and hence the same as those for pp/p̄p; ϵ=0.079 and η=0.467. $X_{\gamma\gamma}$ is determined by assuming factorization, *i.e.* $X_{\gamma\gamma} X_{pp} = X^2_{\gamma p}$ and similarly for $Y_{\gamma\gamma}$. The values $X_{\gamma p}$, X_{pp} are taken from fits to (pp) p̄p and γp data in a form similar to that given by equation (2). Ref. [16] has a more elaborate treatment, but their final predictions for $\sigma_{\gamma\gamma}$ follow a pattern similar to equation (2). BSW [17] predictions just assume $\sigma_{\gamma\gamma}$ = A σ_{pp} and try to estimate A. Aspen model [18] and GLMN model [19] actually are a mixture of QCD based models, to be described later, and treating the photon like a proton. It is assumed in these models that the rise of total $\gamma\gamma$ cross-section is caused by increased number of parton collisions in photons. However, all the parameters of the model for photons are obtained from those for protons using the ideas of quark model. Thus, their predictions of $\sqrt{s_{\gamma\gamma}}$ dependence of $\sigma_{\gamma\gamma}$ are similar to those of Refs. [15,16].

The models which are based on QCD use the information on the photon structure obtained experimentally as crucial inputs. In BKKS model [20] $\sigma_{\gamma\gamma}$ is related to F_2^γ. In the eikonalised minijet model [21], the total eikonalized cross-section for σ^{tot}_{AB}(A+B → hadrons) is written as

$$\sigma^{tot}_{AB} = 2P^{had}_{AB} \int d^2\vec{b}[1 - e^{\chi^{AB}_I} cos\chi^{AB}_R] \tag{3}$$

where χ^{AB}_R can be taken to be ≈ 0 and the imaginary part of the eikonal, χ^{AB}_I given by

$$2\chi^{AB}_I = A_{AB}(b)[\sigma^{soft}_{AB}(s) + \frac{1}{P^{had}_{AB}} \sigma^{jet}_{AB}(s, p^{min}_T)] \tag{4}$$

In equation (4) above, σ^{soft}_{AB} (s) is the nonperturbative, soft cross-section of hadronic size which is fitted, A_{AB}(b) is the overlap function of the partons in the two hadrons

A and B in the transverse space, P_{AB}^{had} is the product of the probabilities that the projectiles A and B hadronize, $P_{A/B}^{had}$ being unity if either A or B is a hadron and is $\sim$ O(α_{em}) for a photon. The QCD input is in the quantity σ_{AB}^{jet} which can be symbolically written as

$$\sigma_{AB}^{jet}(p_T^{min}, s) \equiv \int_{P_T^{min}}^{s/2} \frac{d\sigma}{dp_T}(A + B \rightarrow j_1 + j_2) \tag{5}$$

$$= \sum_{l,m,p,q} \int_{p_T^{min}}^{s/2} \int dx_1 \int dx_2 \; f_{l/A}(x_1) \; f_{m/B}(x_2) \; \frac{d\hat{\sigma}}{dp_T}(l + m \rightarrow p + q)$$

$f_{l/A}(x_1), f_{m/B}(x_2), d\hat{\sigma}/dp_T$ are the QCD inputs. The very steep rise of σ^{jet} wih s is tempered by the eikonal function, such that unitarity bound is satisfied. The modelling aspect is in the choice of P^{had} and ansatz for A_{AB}(b). We take

$$P_{\gamma p}^{had} = P_{\gamma}^{had} \equiv P^{had} = \sum_{V=\rho,\omega,\phi} \frac{4\pi\alpha}{f_V^2} \simeq \frac{1}{240} \tag{6}$$

and $P_{\gamma\gamma}^{had} = (P_{\gamma}^{had})^2$. A_{AB}(b) is normally taken to be Fourier Transform (F.T.) of the product of the e.m. form factors of the colliding hadrons. For a photon, instead of modelling it through the F.T. of the pion form factor, as done previously [24], we take it to be the F.T. of the internal k_T distribution of the partons in the photon as measured by ZEUS [25]. In our model [21,22] we determined the soft parameter for $\gamma\gamma$ through a Quark Model ansatz and used

$$\sigma_{\gamma\gamma}^{soft} = \frac{2}{3}\sigma_{\gamma p}^{soft} = \frac{2}{3}[\sigma_0 + \frac{\mathcal{A}}{\sqrt{s}} + \frac{\mathcal{B}}{s}] \tag{7}$$

where $\mathcal{A}$ and $\mathcal{B}$ are fitted to the γp data.

In Aspen model [18] the formulation is the same as in equation (3). However, χ_I^{AB} is completely decided by using that for protons and quark model ideas. Other model which uses the EMM formulation [23] actually tries to calculate A_{AB}(b) from QCD resummation and is even more close to QCD than the formulation discussed earlier [21,22].

PREDICTIONS OF THE MODELS

Left panel of Fig. 1 shows the γp data [4,5,26,27] along with a band of EMM model predictions [21,22,13]. The figure includes the old photoproduction data before and from HERA experiments, as well as the BPC extrapolation of the DIS data from HERA [26], along with the latest, preliminary data [27] from ZEUS. The parameter k_0 controls the b dependence of A_{AB}(b) and A in the legend in the figure corresponds to $\mathcal{A}$ of equation (7). Note here that the experimentally measured value of k_0 is $k_0 = 0.66 \pm 0.22$ GeV [25]. We then use $\sigma_{\gamma\gamma}^{soft}$ determined

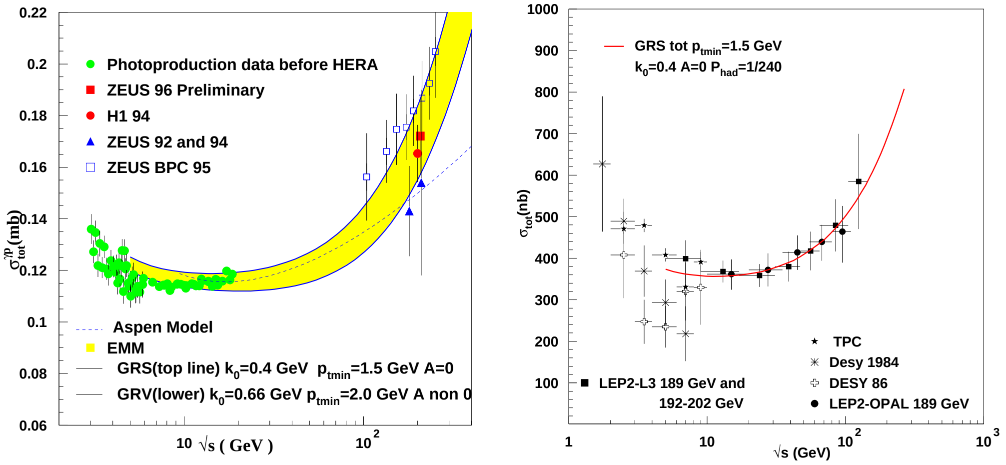

FIGURE 1. Comparison between the eikonal minijet model predictions and data for total $\gamma\ p$ cross-section as well as BPC data extrapolated from DIS(left panel) and prediction for the $\gamma\gamma$ case (right panel) correpsonding to the parameters for the topmost curve for the γp case in the left panel.

from $\sigma_{\gamma p}^{soft}$ as in equation (7) and calculate $\sigma_{\gamma\gamma}$ for the choice of parameters which correspond to the upper edge of the band in the left panel of Fig. 1. The right panel in Fig. 1 shows the prediction along with the latest compilation of the 2γ data on $\sigma_{\gamma\gamma}^{tot}$ [2,3]. One sees from the figure that the values of the parameters which give a good fit to the $\gamma\gamma$ data actually predict a normalisation for γp data higher by 10%. The situation should clarify once the newer photoproduction data from HERA firm up. Of course, variations of the parameters within the limits allowed by the γp data give a band of predictions for the EMM model for $\gamma\gamma$ case. This band of predictions is shown in Fig. 2 where alongwith the EMM model predictions [21,12,22,13] the predictions of various other models [15,16,19,17,18,20] are shown too. We observe that in general the data on $\sigma_{\gamma\gamma}^{tot}$ seem to rise faster than the predictions of most of the 'photon like a proton' models. The data certainly seems to rise faster than the $\sigma_{pp}/\sigma_{\bar{p}p}$ with $\sqrt{s}$. Predictions of different QCD based models [20,22] reproduce the data to a similar degree of satisfaction [2]. The question to ask now is how can the future LC help us distinguish between the various models in the e^+e^- mode and in the Compton mode.

DISCRIMINATION BETWEEN THEORETICAL MODELS AT FUTURE COLLIDERS

In view of the inherent experimental uncertainties in unfolding $\gamma\gamma$ cross-sections $\sigma_{\gamma\gamma}^{tot}$ from the measured hadronic cross-sections in e^+e^- collisions $\sigma_{ee\gamma\gamma}^{tot}$, of course

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[2] BKKS predictions have a lattitude in overall normalisation which can bring these predictions down at lower $\sqrt{s_{\gamma\gamma}}$

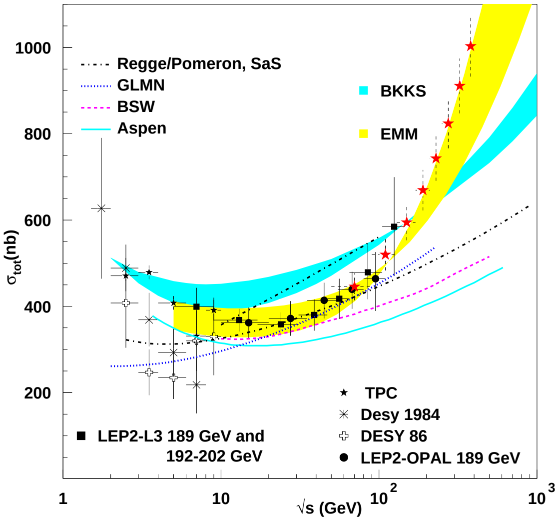

FIGURE 2. The predictions from factorization models, Regge-Pomeron exchange and QCD structure function models together with those from the EMM and a comparison with present data. ‘Pseudo’ data points with errors expected at a future Compton collider are indicated by stars.

the Compton colliders will offer the best discriminatory power. Tables 1 and 2 show [9] the precision required to distinguish at 1 σ level between different models based on factorisation and various predictions of QCD based models respectively. The ‘pseudo’ datapoints with error bars [28] expected at a Compton collider with an e^+e^- collider of TESLA design, are plotted in Fig. 2. This clearly shows that a Compton collider with a parent e^+e^- collider of $\sqrt{s} = 500$ GeV, can certainly distinguish between the different theoretical models and provide an opportunity to learn about the interactions of high energy photons.

However, the discriminatory power is not lost even if one considers only the e^+e^- option. This can be seen by calculating $\sigma_{ee\gamma\gamma}^{tot}$. Recall that $\sigma_{ee\gamma\gamma}^{tot}$ is given by equation

TABLE 1. Predictions for different ‘proton-like’ models. GRV,GRS correspond to the parametrisations of the photonic parton densities given in Refs. [29,30] respectively.

$\sqrt{s_{\gamma\gamma}}(GeV)$	Aspen	BSW	DL	1σ
20	309 nb	330 nb	379 nb	7%
50	330 nb	368 nb	430 nb	11%
100	362 nb	401 nb	477 nb	10%
200	404 nb	441 nb	531 nb	9%
500	474 nb	515 nb	612 nb	8%
700	503 nb	543 nb	645 nb	8%

TABLE 2. Predictions for different QCD based models.

$\sqrt{s_{\gamma\gamma}}$	EMM,Inel,GRS (p_{tmin}=1.5 GeV)	EMM,Tot,GRV (p_{tmin}=2 GeV)	BKKS GRV	1σ
20	399 nb	331 nb	408 nb	2 %
50	429 nb	374 nb	471 nb	9%
100	486 nb	472 nb	543 nb	11%
200	596 nb	676 nb	635 nb	6%
500	850 nb	1165 nb	792 nb	7 %
700	978 nb	1407 nb	860 nb	13 %

(1). The photon spectra $f_{\gamma/e}(x)$ receive contributions from both bremstrahlung (Weizäcker-Williams - WW) photons and beamstrahlung. The WW spectra with which one folds $\sigma_{\gamma\gamma}$ have to take into account the (anti) tagging conditions at e^+e^- colliders as well as inclusion of the effect of virtuality of tageed photon on the cross-section [31]. Major uncertainties in the unfolding of $\sigma_{\gamma\gamma}^{tot}$ from $\sigma_{ee\gamma\gamma}^{tot}$ come from modelling the behaviour of the hadronic system that is boosted in the beam direction and lost to the detectors. Hence one way of making comparisons with data free of this modelling is to make predictions for $\sigma_{ee2\gamma}^{tot}$ by restricting the integration region in equation (1) to regions of $\sqrt{s_{\gamma\gamma}}$ where these uncertainties are least. Fig. 3 shows $\sigma_{ee2\gamma}^{tot}$ as a function of $(\sqrt{s})_{e^+e^-}$, where the bands show the range of predictions by using $\sigma_{2\gamma}^{tot}$ $(s_{\gamma\gamma})$ from different theoretical models. The lower edge corresponds to models which treat 'photon like a proton' and the upper edge to the QCD based models. The upper band corresponds to the predictions when no (anti) tagging requirement has been imposed on the γ spectra. The lower band represents the more realistic predictions by assuming for the NLC, $\theta_{tag} = 0.025$ rad and $E_e^{min} = 0.20$

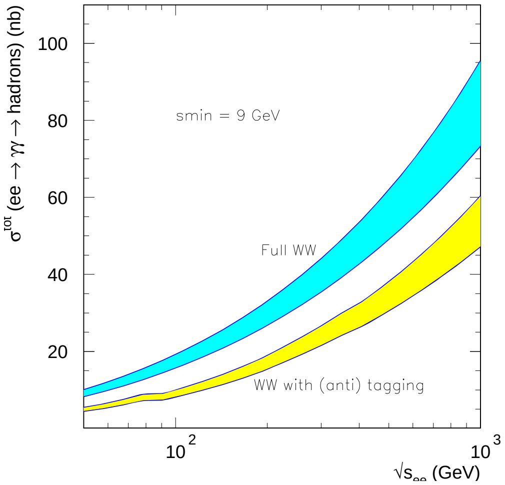

FIGURE 3. Predictions for $\sigma_{ee2\gamma}^{tot}$ as a function of $\sqrt{s_{\gamma\gamma}}$.

E_{beam}. This causes $\sim$ 40% reduction in the rates for $(\sqrt{s_{\gamma\gamma}})_{min} = 9$ GeV. Note that, the differences in $\sigma^{tot}_{\gamma\gamma}$ of factor $\sim$ 2-3 for different models is reduced to $\sim$ 30% for $\sigma^{had}_{ee\gamma\gamma}$. However, the demands on precision required to discriminate between different theoretical models are still very much within the reach of the LC measurements even for the e^+e^- mode. In the calculation I present here **only** the contribution of bremsstrahlung photons is included. The inclusion of the beamstrahlung photons might increase the discriminatory power, but that needs to be investigated.

CONCLUSIONS AND OUTLOOK

We can summarise our discussions as follows:

1. Models which treat photon like a proton tend to predict a rise of cross-sections $\sigma^{tot}_{\gamma\gamma}$ with energy slower than shown by $\gamma\gamma$ data. QCD based models predict a faster rise.

2. γp data seems also to show tendency of needing a value of ϵ ($\sim s^{\epsilon}$) higher than that for pp/$\bar{\text{p}}$p.

3. Extraction of $\sigma_{\gamma\gamma}$ ($\sigma_{\gamma p}$) from data is no mean task.

4. Accurate measurements of $\sigma_{\gamma\gamma}$ at a $\gamma\gamma$ collider will be capable of distinguishing between these different models. A precision of $\sim$ 20% is required for that.

5. When folded with bremstrahlung spectra the difference of 200 - 300 % at high $\sqrt{s}$ in $\sigma^{tot}_{\gamma\gamma}$ in different models reduces to 30%.

6. The issue needs to be investigated for high energy e^+e^- colliders including the effects of beamstrahlung.

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