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Drift dissipative instability in a two temperature plasma

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Abstract. The presence of a small amount of relatively cold electrons in an otherwise hot plasma reduces the ion sound speed in the medium and hence reduces the growth rate of the drift dissipative ion acoustic mode in an inhomogeneous weakly ionized plasma. This is expected to improve the confinement time in certain magnetic confinement schemes. The propagation of a small but finite amplitude mode in the presence of ion viscosity is also investigated by using reductive perturbation method. It is shown that, when the damping due to ion viscosity is stronger than the growth due to collisions, there exists a stationary shock solution.

Keywords. Plasma; instabilities; ion viscosity.

1. Introduction

Magnetically confined plasmas are inherently inhomogeneous and drift instabilities are a cause of concern in such a plasma (Krall 1968) as they produce enhanced particle diffusion across the magnetic field and hence reduce the confinement time. In a collisional plasma it is the drift dissipative instabilities that are potentially dangerous in reducing the confinement time because they have large growth rates ($\text{Im}\omega \sim \text{Re}\omega$). These instabilities occur due to drift motion of the charged particles across the magnetic field and the diffusion along the field either in a weakly ionized plasma with a low neutral pressure (Timofeev 1963) or in a dense fully ionized plasma (Moiseev and Sagdeev 1963). In order to increase the confinement time it is extremely desirable to stabilize these instabilities. In this paper we present a linear mechanism by which a significant reduction in the growth rate of one of these instabilities (for which $\omega \gg \Omega_i$, ω, Ω_i being the characteristic wave frequency and ion gyrofrequency respectively) can be achieved.

Our proposition is based on the following considerations. The drift dissipative instability for which $\omega \gg \Omega_i$ (Kadomtsev 1965, henceforth, we shall call this mode as drift dissipative ion acoustic mode) has growth rate which is proportional to the ion acoustic speed in the medium. However, the ion acoustic speed in an otherwise hot plasma drastically decreases due to the presence of a small fraction of cold electrons. (Jones *et al* 1975, Goswami and Buti 1976). In section 2, we have shown that just by introducing a small amount of relatively cold electrons one can significantly reduce the growth rate of drift dissipative ion acoustic mode. This is expected to improve the confinement time in certain magnetic confinement schemes. Moreover, it is well known that (Hendel *et al* 1968) ion viscosity has a stabilizing effect on this instability. Now that the growth rate of the instability is reduced, a rather weak ion viscosity will be sufficient to quench the instability. In section 3 we derive a modified Korteweg-de Vries equation describing the propagation of a small but finite amplitude drift dissipative ion acoustic mode in such a system. It is shown that in the presence of an ion viscosity there exists a stationary shock solution when the viscous damping dominates over the collisional growth.

2. Linear analysis

Let us consider a plasma with cold ions (density N_0, $T_i=0$) embedded in a magnetic field in the z-direction where bulk of the electrons are hot (density N_h and temperature T_h) with a small fraction of cold electrons (density N_c and temperature T_c) such that $N_h/N_c \gg 1$ and $T_h/T_c \gg 1$. The density gradient will be considered to be in the x-direction while the wave propagation will be considered to be in the y-z plane. In equilibrium, the ions are at rest and the electrons have a drift in the y-direction. The charge neutrality condition, namely $N_o = N_h + N_c$ together with the conditions $N_h/N_c \gg 1$ and $T_h/T_c \gg 1$ demands that the density gradient scale lengths will be mostly governed by the hot electrons, i.e.

$$K = -\frac{1}{N_o}\frac{dN_o}{dx} = -\alpha\frac{1}{N_h}\frac{dN_h}{dx} - \beta\frac{1}{N_c}\frac{dN_c}{dx} = \alpha K_h + \beta K_c \tag{1}$$

where $\alpha = N_h/N_o$ and $\beta = N_c/N_o$. We shall assume that the main density gradient is produced by the hot electrons. Cold electrons are more or less uniformly distributed and hence $K_c \ll K_h$. Basic linearized set of equations consists of the continuity equations for ions and two types of electrons and the equations of motion for ions and the two types of electrons, namely

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0, \tag{2}$$

$$\frac{\partial n_{eh}}{\partial t} - \frac{C}{B_o}\frac{dN_h}{dx}\frac{\partial \phi}{\partial y} + N_h\frac{\partial v_{zh}}{\partial z} = 0, \tag{3}$$

$$\frac{\partial n_{ec}}{\partial t} - \frac{C}{B_o}\frac{dN_c}{dx}\frac{\partial \phi}{\partial y} + N_c\frac{\partial v_{zc}}{\partial z} = 0, \tag{4}$$

$$m_i\, \partial \mathbf{v}_i/\partial t = -e\nabla\phi, \tag{5}$$

$$-\frac{T_h}{N_h}\frac{\partial n_{eh}}{\partial z} + e\frac{\partial \phi}{\partial z} - m_e v_{zh}/\tau_h = 0 \tag{6}$$

and

$$-\frac{T_c}{N_c}\frac{\partial n_{ec}}{\partial z} + e\frac{\partial \phi}{\partial z} - m_e v_{zc}/\tau_c = 0, \tag{7}$$

where $\tau_{c(h)}$ is the collision time for the cold (hot) electrons respectively. In writing eqs (6) and (7) we have neglected the electron inertia. Moreover, use has been made of the perpendicular equations of motion in writing the continuity equations for electrons. The effect of the magnetic field on the ions is neglected as we shall restrict ourselves only to the mode for which $\omega \gg \Omega_i$. Assuming the perturbed quantities to go as exp $(i\,\mathbf{k}\cdot\mathbf{r} - i\omega t)$ we get (Kadomtsev 1965)

$$\frac{n_c}{N_c} = \left(k_z^2 - i\frac{k_y K_c}{\Omega_e \tau_c}\right)(D_c k_z^2 - i\omega)^{-1}\frac{e\phi\,\tau_c}{m_e} \tag{8}$$

$$\frac{n_h}{N_h} = \left(k_z^2 - i\frac{k_y K_h}{\Omega_e \tau_h}\right)(D_h k_z^2 - i\omega)^{-1}\frac{e\phi\,\tau_h}{m_e} \tag{9}$$

and

$$n_i/N_o = (k^2/\omega^2)\frac{e\phi}{m_i} \tag{10}$$

where $D_c = \tau_c T_c/m_e$ and $D_h = \tau_h T_h/m_e$.

Therefore, the linear dispersion relation can be written as

$$\frac{k^2}{m_i\omega^2} = \left(1-\frac{ik_y K_h}{\Omega_e k_z^2 \tau_h}\right)\frac{\alpha}{T_h} + \left(1-\frac{ik_y K_c}{\Omega_e k_z^2 \tau_c}\right)\frac{\beta}{T_c}$$

or

$$\omega^2/k^2 = (T_{\text{eff}}/m_i)\,[1 - ik_y(K\nu)_{\text{eff}}/\Omega_e k_z^2]^{-1} \tag{11}$$

where

$$T_{\text{eff}} = T_h T_c/(\alpha T_c + \beta T_h) \tag{12}$$

and

$$(K\nu)_{\text{eff}} = (\alpha T_c K_h \nu_h + \beta T_h K_c \nu_c)/(\alpha T_c + \beta T_h). \tag{13}$$

In deriving eq. (11) we have assumed that $D_c k_z^2$, $D_h k_z^2 \gg \omega$. It can be shown that eq. (11) is consistent with this assumption unless k_z is extremely small (Kadomtsev 1965). ν in the above equations is the collision frequency and the subscripts h and c refer to the hot and cold components respectively. Thus, from eq. (11), the frequency and growth rate of the wave is obtained as

$$\omega = kC_{\text{seff}}$$

and

$$\gamma = kC_{\text{seff}}\left[\frac{k_y(K\nu)_{\text{eff}}}{2\Omega_e k_z^2}\right] \tag{14}$$

where

$$C_{\text{seff}} = (T_{\text{eff}}/m_i)^{1/2}.$$

For large T_h/T_c, T_{eff} is closer to T_c (see eq. (12), also Jones *et al* 1975) and hence an appreciable reduction in C_{seff} takes place. Therefore, a significant reduction in the growth rate, which is given by eq. (14), also takes place. From eq. (13) it is seen that for T_h/T_c large $(k\nu)_{\text{eff}} < K_h \nu_h$ as $K_c \nu_c \ll K_h \nu_h$ and hence the decrease in the growth rate given by eq. (14) is genuine. Thus, just by introducing a small amount of relatively cold electrons, into the bulk of a hot plasma a significant reduction in the growth of the drift-dissipative ion acoustic mode and hence increase in the confinement time of certain magnetic confinement schemes can be obtained.

3. Nonlinear analysis

From the linear analysis we have seen that the presence of a small amount of cold electrons has a stabilizing effect on this instability. Therefore, a weaker ion viscosity will be sufficient to completely quench the instability. Following reductive perturbation method, in this section, we look for a steady state solution of the drift-dissipative ion acoustic mode in the presence of ion viscosity. The nonlinear set of equations is the following:

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial y}(n_i\, v_i)=0 \tag{15}$$

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial y}+\frac{\partial \phi}{\partial y}=\frac{\mu}{C_{\text{seff}}\ \lambda_{\text{deff}}\ m_i n_0 n_i}\frac{\partial^2 v_i}{\partial y^2} \tag{16}$$

$$\frac{m_e}{m_i}\frac{1}{\omega_{pe}\tau_h}\frac{\partial n_{eh}}{\partial t}+\frac{K_h}{\Omega_e\tau_h}\frac{\partial \phi}{\partial y}+\frac{\partial}{\partial z}\left(n_{eh}\frac{\partial \psi}{\partial z}\right)=0 \tag{17}$$

$$-\frac{1}{n_{eh}}\frac{\partial n_{eh}}{\partial z}+\frac{T_{\text{eff}}}{T_h}\frac{\partial \phi}{\partial z}-\frac{T_{\text{eff}}}{T_h}\frac{\partial \psi}{\partial z}=0 \tag{18}$$

$$n_{ec}=\beta\exp\left(\frac{T_{\text{eff}}}{T_c}\phi\right) \tag{19}$$

and

$$\frac{\partial^2 \phi}{\partial y^2}=n_{ec}+n_{eh}-n_i \tag{20}$$

with

$$\frac{\partial \psi}{\partial z}=\frac{m_e}{T_{\text{eff}}\,\tau_e}v_{ezh}. \tag{21}$$

In the above set of equations densities are normalized to n_0, velocities to C_{seff}, potential to T_{eff}/e, lengths to λ_{deff} ($\lambda^2_{\text{deff}}=T_{\text{eff}}/4\pi n_0 e^2$) and time to ion plasma period, $\omega_{P_i}^{-1}$ ($\omega_{P_i}^{2}=4\pi n_0 e^2/m_i$). Collisons for the cold electrons are ignored and density gradient exists only for the hot electrons. The quantity ψ appearing in the above set of equations is a velocity potential introduced through eq. (21). Since the mode under consideration is an electrostatic mode and the propagation is nearly perpendicular to the magnetic field, the motions of the ions along the direction of the magnetic field has been neglected. The term on the right hand side of eq. (16) represents the viscous force with $\mu=\eta_0/3+\eta_1$, $\eta_0=C_0 n_i T_i/\nu_{ii}$, $\eta_1=C_i n_i T_i \nu_{ii}/\Omega_i^2$, C_0, C_1 being constants and T_i and ν_{ii} being ion temperature and ion-ion collision frequency respectively. The constants as obtained by Braginskii (1965) are $C_0=0{\cdot}95$ and $C_1=0{\cdot}3$. In writing eq. (16) the ion pressure term is neglected because the ion temperature T_i is assumed to be much smaller than the electron temperature T_e and the strength of the ion pressure term compared to the ion viscosity term goes as $O(\Omega_i^2/\nu_{ii}k_y C_{\text{seff}})$. For $k_y C_{\text{seff}}\approx\omega\gg\Omega_i$, the strength of the ion pressure term is even smaller than that of the ion-viscosity term if $\nu_{ii}\gtrsim\Omega_i$.

Equation (18) can be integrated with respect to z to give $n_{eh} = \alpha \exp [T_{\text{eff}} (\phi - \psi)/T_h]$ and then the Poisson's equation (Eq. (20)) can be written as

$$\frac{\partial^2 \phi}{\partial y^2} = \frac{\alpha T_{\text{eff}}}{T_h}(\phi-\psi)+\tfrac{1}{2}\alpha \frac{T_{\text{eff}}^2}{T_h^2}(\phi-\psi)^2+\beta \frac{T_{\text{eff}}}{T_c}\phi+\tfrac{1}{2}\beta \frac{T_{\text{eff}}^2}{T_c^2}\phi^2-\tilde{n}_i \tag{22}$$

where $\tilde{n}_i$ is the perturbed part of the ion density.

Now, let us introduce stretched variables $\zeta=\epsilon^{1/2}(y-t)$ $\tau=\epsilon^{3/2}t$. The perturbed quantities can now be written, in terms of the smallness parameter ϵ as

$$\tilde{n}_i = \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \dots$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \dots$$

$$v_i = \epsilon v_i^{(1)} + \epsilon^2 v_i^{(2)} + \dots$$

$$\psi = \epsilon^2 \psi^{(1)} + \epsilon^3 \psi^{(2)} + \dots$$

The last expansion, namely that for ψ follows from eq. (18). In the absence of the collisions the electron density fluctuations are governed only by the potential fluctuations namely $n_e=\exp(\phi)$. The collisional term $\partial\psi/\partial z$ in eq. (18) is treated as a correction to the potential fluctuations $\partial\phi/\partial z$. Thus, $\partial\psi/\partial z$ is taken to be one order smaller than the term $\partial\phi/\partial z$. Hence, the above expansion for ψ.

The smallness parameter ϵ is chosen in such a way that to the lowest order eq. (17) gives

$$\frac{\partial^2 \psi^{(1)}}{\partial z^2} = -\frac{K}{\Omega_e \tau_e}\frac{\partial \phi^{(1)}}{\partial \zeta} \tag{23}$$

This requirement is satisfied if $(K/k_z^2\,\Omega_e\,\tau_e) \sim \epsilon^{1/2}$ and

$$\left(\frac{m_e}{m_i}\frac{1}{\omega_{P_i}\tau_e}\right)\left(\frac{K}{\Omega_e \tau_e}\right)^{-1} \sim \epsilon.$$

To the lowest order in ϵ eqs (15), (16), (22) give $n_i^{(1)} = v_i^{(1)} = \phi^{(1)}$. To the next higher order in ϵ these equations give

$$-\frac{\partial n_i^{(2)}}{\partial \zeta} + \frac{\partial v_i^{(2)}}{\partial \zeta} + \frac{\partial n_i^{(1)}}{\partial \tau} + \frac{\partial}{\partial \tau}(n_i^{(2)}\, v_i^{(1)})=0 \tag{24}$$

$$-\frac{\partial v_i^{(2)}}{\partial \zeta} + \frac{\partial v_i^{(1)}}{\partial \tau} + v_i^{(1)}\frac{\partial v_i^{(1)}}{\partial \zeta} + \frac{\partial \phi^{(2)}}{\partial \zeta} = \delta\,\frac{\partial^2 \phi^{(1)}}{\partial \zeta} \tag{25}$$

and

$$\frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} + \phi^{(2)} + \frac{\Delta}{2}[\phi^{(1)}]^2 - \alpha\frac{T_{\text{eff}}}{T_h}\psi^{(1)} - n_i^{(2)} \tag{26}$$

where

$$\delta = \frac{\mu}{C_{\text{seff}}\,\lambda_{\text{deff}}\,m_i n_0}$$

$$\Delta = \left(\frac{\alpha}{T_h^2}+\frac{\beta}{T_c^2}\right)T_{\text{eff}}^2 = \frac{\alpha\left(\dfrac{T_c}{T_h}\right)^2+\beta}{\left(\alpha\dfrac{T_c}{T_h}+\beta\right)^2} \tag{27}$$

Eliminating $n_i^{(2)}$, $\phi^{(2)}$ and $v_i^{(2)}$ and using the relation $n_i^{(1)}=v_i^{(1)}=\phi^{(1)}$ eqs (24)-(26) can be reduced to

$$\frac{\partial n_i^{(1)}}{\partial \tau}+\frac{(3-\triangle)}{2}n_i^{(1)}\frac{\partial n_i^{(1)}}{\partial \zeta}+\tfrac{1}{2}\frac{\partial^3 n_i^{(1)}}{\partial \zeta^3}+\tfrac{1}{2}\,a\frac{T_{\text{eff}}}{T_h}\frac{\partial \psi^{(1)}}{\partial \zeta}-\delta\frac{\partial^2 n_i^{(1)}}{\partial \zeta^2}=0 \quad (28)$$

Equations (23) and (28) constitute the coupled set of equations describing the propagation of a small but finite amplitude drift dissipative ion acoustic mode in a two temperature plasma. We note that the coefficient of the nonlinear term in eq. (28) is similar to one obtained by Goswami and Buti (1976). In order to obtain a stationary solution to the set of eqs (23) and (28) we follow a method similar to one used by Goswami and Buti (1977). Differentiating eq. (28) twice w.r.t. z and making use of eq. (23) we get,

$$\frac{\partial^3 n_i}{\partial \tau \partial z^2}+\frac{(3-\triangle)}{2}\frac{\partial^2}{\partial z^2}\left(n\frac{\partial n}{\partial \zeta}\right)+\tfrac{1}{2}\frac{\partial^5 n}{\partial \zeta^3 \partial z^2}-\gamma\frac{\partial^2 n}{\partial \zeta^2}-\delta\frac{\partial^4 n}{\partial \zeta^2 \partial z^2}=0 \quad (30)$$

where $(\gamma=aT_{\text{eff}}/2T_h)(K/\Omega_e\tau_e)$ and the subscript and superscript to n have been dropped for convenience. Now, let us consider propagation along a particular direction θ (θ being the angle between B_0 and $\mathbf{k}$). Therefore n would depend on ζ and z only through a single variable $\chi\equiv Z\cos\theta+\zeta\sin\theta$ and only parametrically in θ. Thus, integrating eq. (30) twice with respect to x under the boundary conditions such that n and its derivatives go to zero as $\chi\rightarrow\pm\infty$, we get

$$\frac{\partial n}{\partial \tau}+\frac{(3-\triangle)}{2}\sin\theta n\frac{\partial n}{\partial x}+\tfrac{1}{2}\sin^3\theta\frac{\partial^3 n}{\partial x^3}-\gamma\tan^2\theta\, n$$

$$-\delta\sin^2\theta\frac{\partial^2 n}{\partial x^2}=0. \quad (31)$$

Equation (31) is the modified *K-dV*-Burger's equation describing the propagation of a finite amplitude drift dissipative ion acoustic mode in presence of ion viscosity in a two temperature plasma. We would like to point out here that eq. (31) is valid only for $\theta>45°$, because, in deriving eq. (23) we assumed that $k_z \ll k_y$. Moreover, from linear theory we have seen that drift dissipative ion mode gets heavily Landau damped for $k_z>k_y$. Hence, the only interesting regime of propagation is that for which $\theta>45°$.

4. Solution of equation (31)

In this section we shall obtain some steady state solution of eq. (31) under different circumstances.

Case I: For $\gamma=\delta=0$, i.e. when both collisions and ion viscosity effects are absent eq. (31) reduces to

$$\frac{\partial n}{\partial \tau}+\frac{(3-\triangle)}{2}\sin\theta\, n\frac{\partial n}{\partial x}+\frac{1}{2}\sin^3\theta\frac{\partial^3 n}{\partial x^3}=0. \quad (32)$$

For exact perpendicular propagation eq. (32) reduces to eq. (14) of Goswami and Buti (1976). For oblique propagation ($45° < \theta < 90°$) eq. (32) has a solitary solution for $\triangle < 2$ which is given by

$$\eta = 3\, U\left(\frac{2}{3-\triangle}\right)\frac{1}{\sin\theta}\,\mathrm{sech}^2\left[\left(\frac{U}{2\sin^3\theta}\right)^{1/2}(\chi - U\tau)\right] \tag{33}$$

where U, the velocity of the moving frame, is rather arbitrary but of the order of C_{seff}. For $T_h = T_c$, $\triangle = 1$ the eq. (33) represents an obliquely propagating ion acoustic solitary wave. For $T_h > T_c$, $\triangle > 1$, the amplitude of the ion acoustic wave increases. The physical reason why this happens is given by Goswami and Buti (1976). For $\theta < 90°$, we notice that the amplitude of the solitary wave further increases whereas the width of the solitary wave further decreases. The most interesting thing to note, however, is that the width decreases much faster (proportional to $\sin^3\theta$) than the rate at which the amplitude increases (proportional to $1/\sin\theta$).

Case II: When $\gamma = 0$ and $\delta > 0$ eq. (31) becomes a *K-dV*-Burger's equation. Physically, this means either total absence of collisional growth or collisional growth is weaker than the viscous damping, δ representing the net damping. It is well known that this equation has a stationary shock solution (Johnson 1970). To see how the shock profile depends on the propagation angle let us do the following rather heuristic calculations. Let us say that n depend on τ and χ only through $\lambda \equiv \chi - U\tau$ then integrate eq. (31) w.r.t. λ under the boundary condition that n and its derivatives go to zero as $\lambda \to -\infty$ and we get,

$$\tfrac{1}{2}\sin^3\theta\,\frac{\partial^2 n}{\partial\lambda^2} - \delta\sin^2\theta\,\frac{\partial n}{\partial\lambda} + \frac{(3-\triangle)\sin\theta}{4}\,n^2 - Un = 0. \tag{34}$$

In the limit $\lambda = +\infty$ eq. (34) gives

$$n \approx \bar{n} = \frac{4U}{(3-\triangle)\sin\theta}.$$

In order to see the asymptotic behaviour of the solution we write $n = \bar{n} + \tilde{n}$ in eq. (34) and linearizing the equation we get

$$\tfrac{1}{2}\sin^3\theta\,\frac{d^2\tilde{n}}{d\lambda^2} - \delta\sin^2\theta\,\frac{d\tilde{n}}{d\lambda} + U\tilde{n} = 0. \tag{35}$$

Equation (35) has a solution $\tilde{n} = \exp(\nu\lambda)$ where ν is given by

$$\nu = \frac{\delta}{\sin^2\theta} \pm \left[\frac{\delta^2}{\sin^4\theta} - \frac{2U}{\sin^3\theta}\right]. \tag{36}$$

From eq. (36) it is clear that the shock profile has an oscillatory profile if

$$U > \frac{\delta^2}{2\sin\theta} \tag{37}$$

and it will have a monotonic profile if the inequality given by eq. (37) is not satisfied. Therefore, for a given strength of viscosity and a given Mach number for propagation there exists a critical angle θ at which the shock profile goes from monotonic to oscillatory.

Case III: When $\gamma > 0$, it is not possible to obtain a stationary solution of the eq. (31). Physically it is understood as follows. When the collisional growth dominates over the viscous damping, any initial perturbation keeps on growing and hence no stationary solution.

5. Conclusion

The presence of a small amount of relatively cold electrons in an otherwise hot plasma is shown to significantly reduce the growth rates of drift dissipative ion sound mode. Hence, an ion viscosity which is rather weak will be sufficient to quench this instability. Therefore, by simply introducing a small amount of cold electrons into a system, the confinement time of certain magnetic confinement schemes is expected to be improved significantly.

The propagation of a small but finite amplitude drift dissipative ion sound mode is also studied using reductive perturbation method. It is shown that in the presence of ion viscosity there exists a stationary shock solution. For a given strength of viscous damping and a given Mach number for propagation there exists a critical angle at which the shock profile changes from oscillatory to monotonic.

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