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Recent advances in lattice gauge theories

R V GAVAI
Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, India

Abstract. Recent progress in the field of lattice gauge theories is briefly reviewed for a non-specialist audience. While the emphasis is on the latest and more definitive results that have emerged prior to this symposium, an effort has been made to provide them with minimal technicalities.

Keywords. Lattice gauge theory; QCD; hadron spectra; chiral; finite temperature.

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1. Introduction

There are many quantitative, and some even qualitative, aspects of the physics of the Standard Model (SM) for which the beloved tool of theoretical physicists, namely, perturbation theory, is grossly inadequate. Thus the widely accepted theory of strong interactions, quantum chromodynamics (QCD), has mainly been confronted with the experimental world in rather restricted kinematic ranges such as those of the deep inelastic scattering or e^+e^- collisions. It must also explain the phenomenon of quark confinement and spontaneous breaking of chiral symmetries (or why pion is so light) and provide us with a quantitative understanding of the masses of all hadrons and their other properties. Indeed, this is, in fact, essential if we are to accept QCD as the correct theory of strong interactions; any demonstration of its failure to do so will be tantamount to one of the best experimental evidence for physics beyond the standard model. On a more practical level, several quantities such as the quark masses or the matrix elements of weak interaction operators in hadronic states usually become additional parameters whereas QCD should be able to constrain them severely, if not determine them.

One needs new tools, preferably a non-perturbative regularization of field theories, to deal with these problems, as they necessarily involve large values of the strong coupling, α_s. Lattice field theories [1], defined on a discrete space-time lattice, provide a gauge invariant regularization which is suitable to address them. The lattice spacing a between two adjacent sites on the lattice provides a momentum cut-off: $-(\pi/a) < p \leq (\pi/a)$. Interestingly, it turns out to be relatively easy to handle the very strong coupling region of the theory. One can demonstrate [2] quark confinement and also compute various hadron masses analytically in this region. However, as in other quantum field theories, the regulator must be removed to obtain physically relevant answers. The lattice scaffolding has to be eliminated by making the lattice progressively finer, i.e., $a \to 0$. One needs to take recourse to numerical Monte Carlo techniques in order to take this limit. With the advent of substantial and affordable computing power, the lattice approach is finally yielding results where a meaningful comparison with the experimental results is increasingly becoming feasible, as we will see below. In the case of heavy mesons, there are now *predictions* available from lattice QCD for decay constants, such as f_B and f_D. Availability of a non-perturbative tool has also permitted exploration of the Standard Model in unusual environments such as high temperature (relevant to the physics of early universe and heavy ion collisions) and to seek its novel predictions such as those for glueballs and other exotic mesons. The current best understanding of the quark-hadron phase transition and the subsequent quark-gluon plasma phase has come from lattice QCD. An experimental confirmation of these in the upcoming heavy ion collisions in Brookhaven National Laboratory, USA and CERN, Geneva will be a dramatic test of QCD in the uncharted region of strong coupling.

The plan of the rest of my talk is as follows. A lightning short review of how the lattice computations are made will be presented in the next section. Its primary purpose is to highlight the approximations involved so that the reliability of the results can be assessed by the reader. Recent impressive results from CP-PACS on hadron spectroscopy will be covered in §3 while §4 will comprise of finite temperature physics. Section 5 is devoted to the recent developments in the area of exact chiral symmetry on the lattice and the summary is presented in §6. Since I will have to be necessarily very selective, due to space constraints, let me mention that proceedings of the annual lattice field theory symposia [3] provide the best place to obtain further details on the topics covered here as well as those left out.

2. Formalism

The quark fields, $\psi(x)$, and the antiquark fields $\bar{\psi}(x)$ are associated with a site $x = (x_1, x_2, x_3, x_4)$ of a 4-dimensional hypercubic lattice. Similarly scalar fields $\Phi(x)$, which are needed for the electroweak interactions (or any theory with Higgs mechanism, in general) are defined on sites too. As in the case of the continuum field theory, one obtains a lattice gauge theory by demanding invariance of the Lagrangian for free quark–antiquarks (e.g. obtained by a straight forward discretization of the usual Dirac Lagrangian) under any *local* phase rotation of these fields. This can be accomplished by introducing lattice gauge fields $U_x^\mu \equiv U_\mu(x)$ which are associated with a directed link from the site x to $x + \hat{\mu}a$. A simple gauge invariant quark action thus is

$$S_F = \frac{1}{2} \sum_{x,\mu=1}^{4} \bar{\psi}(x) \gamma_\mu \left[U_x^\mu \psi(x + \hat{\mu}a) - U_{x-\hat{\mu}a}^\mu \psi(x - \hat{\mu}a) \right] + ma \sum_x \bar{\psi}(x) \psi(x), \tag{1}$$

where the gauge transformations are defined by

$$\psi'(x) = V(x)\psi(x),\ \bar{\psi}'(x) = \bar{\psi}(x)V^\dagger(x),\ U_x^{\mu'} = V(x)U_x^\mu V^\dagger(x + \hat{\mu}a) \tag{2}$$

and $U_x^\mu \in \mathrm{SU}(3)$, $V(x) \in \mathrm{SU}(3)$ in case of QCD (SU(2) for electroweak). From the gauge transformations of the U_x^μ-fields, one can see that the simplest gauge invariant actions for these fields is given by

$$S_G = \frac{6}{g^2} \sum_P \left[1 - \frac{1}{3} \mathrm{Re\ tr}\ U_p \right]. \tag{3}$$

Here $U_p = U_x^\mu U_{x+\hat{\mu}a}^\nu U_{x+\hat{\nu}a}^{\mu\dagger} U_x^{\nu\dagger}$, called plaquette, is the smallest closed loop of the directed gauge links in the (μ, ν) plane at site x. The sum over P runs over all possible plaquettes P on the lattice. Defining $U_x^\mu = \exp[iga \sum_{b=1}^8 A_\mu^b(x + a\hat{\mu}/2) T^b]$, where $A_\mu^b(x)$ is the continuum gauge field in bth colour direction and μth space direction, and T^b is the corresponding adjoint matrix for bth colour, one can easily show that in the limit of $a \to 0$, eqs (1) and (3) reduce to the usual continuum quark and gauge actions respectively [2].

Defining a partition function Z for these fields, which are complicated versions of the familiar Ising-spins,

$$\begin{aligned} Z &= \int \prod_{x,\hat{\mu}} \mathrm{d}U_\mu(x) \prod_x \mathrm{d}\psi(x)\, \mathrm{d}\bar{\psi}(x) \mathrm{e}^{-S_G - S_F} \\ &= \int \prod_{x,\hat{\mu}} \prod_f \det(D + ma_f) \mathrm{e}^{-S_G}, \end{aligned} \tag{4}$$

one can compute quantum expectation values of any physical observable as averages with respect to the Z above. Thus, e.g., masses of physical particles are obtained from the exponential decays of appropriate correlation functions. Taking $\Theta(x) = \bar{\psi}(x)\Gamma\psi(x)$, where the choice of the matrix Γ in the spin and flavour space decides whether it is a π-meson correlator or a ρ-meson correlator, one can compute the correlation function $C(t)$,

$$C(t) = Z^{-1} \int \prod_{x,\hat{\mu}} \mathrm{d}U_x^\mu \prod_x \mathrm{d}\psi(x)\, \mathrm{d}\bar{\psi}(x) \prod_f \det{}_f \mathrm{e}^{-S_G} \Theta(t)\Theta(0), \tag{5}$$

with $\Theta(t) = \sum_{\vec{x}} \Theta(\vec{x}, t)$. As $t \to \infty$, $C(t) \simeq A \exp(-mt)$, yielding thus the lowest mass with quantum numbers of Θ. Its decay constant can be obtained from the coefficient A.

The Monte Carlo technique to evaluate $C(t)$, or the expectation value of any other observable, consists of (1) generating as large a set of links $\{U_x^\mu\}$ for the whole lattice as possible, such that each set of $\{U_x^\mu\}$ occurs with a probability proportional to $\prod_f \det_f \cdot \exp[-S_G(\{U_x^\mu\})]$ and (2) evaluating $C(t)$ for each configuration $\{U_x^\mu\}$ and taking its average over all the configurations in the set.

A first hurdle in carrying out this program comes in the form of the fermion doubling problem [4]. It turns out that a single flavour of quark on the lattice becomes equivalent to $16 (= 2^d)$ flavours in the continuum limit, $a \to 0$, if one insists on having reasonable properties for the S_F in eq. (1), such as (i) locality (discretizing the derivative using only few terms like two in eq. (1)), (ii) chiral symmetry (γ_5 anticommutes with the lattice Dirac operator) and (iii) real Hamiltonian. The two popular solutions [2], which are used in the results presented below, are (i) Wilson fermions, which break *all* chiral symmetries on the lattice but have a one-to-one correspondence of flavours on the lattice and in the continuum and (ii) staggered or Kogut–Susskind fermions, which have an exact chiral symmetry on the lattice but broken flavour symmetry. Furthermore, even the restored flavour symmetry in the $a \to 0$ limit for the latter is well defined for 4 light flavours only.

The second hurdle relates to the enormity of the computational task if one wishes to generate the set of $\{U_x^\mu\}$ for full QCD, i.e., for a theory with all virtual quark loops included.

Let us assume that the lattice has N_s sites in each of the space directions and N_t in time direction. Usually $N_t \gg N_s$ for a zero temperature propagator calculation, while $N_t \ll N_s$ for a finite temperature calculation. The temperature $T = (N_t a)^{-1}$ and the spatial volume of the lattice is $V = L^3 = N_s^3 a^3$. The continuum limit corresponds to making the lattice finer by reducing a, and thus correspondingly making the lattice size $N_s^3 \times N_t$ bigger so that T and L are held constant in physical units. The currently best algorithm [5] to generate a configuration of $\{U_x^\mu\}$ for full QCD needs computer time which scales as $V^{5/4}$ and $(m_q^{\rm sea} \cdot a)^{-5/2}$. Thus it increases rapidly as the sea quark mass is lowered and/or continuum limit is approached. The possible solutions, in increasing order of approximations but decreasing order of computer time, are (i) full QCD simulations on smaller lattices, (ii) partially quenched QCD simulations with $am^{\rm sea}$ large and greater than $am^{\rm valence}$ and (iii) quenched QCD simulations with $am^{\rm sea} = \infty$ (i.e. no dynamical quarks). The early lattice results and today's best results are obtained in the quenched approximation. Indeed, one has now begun to answer quantitatively the question as to how good the approximation itself is. One is constrained by the available computer resources to perform these calculations for small but nonzero a and large $am^{\rm valence}$ and has to then extrapolate the results to $a = 0$ and small quark masses (for u, d and s quarks). It is *a priori* not clear how small am_q and a have to be to obtain reliable results. The recent simulations seem to provide some indications for them though.

3. Hadron spectroscopy

The Japanese CP-PACS collaboration recently presented their results [6] on hadron spectroscopy which take a significant step forward in answering the questions raised above. Table 1 gives the parameters of their simulations performed in quenched approximation. Note that as a decreases, N_s and N_t increase so that $V = L^3$ (and T) is almost constant in physical units. They employed Wilson fermions and made computations at 5 values of $am^{\rm valence}$ for each a above. Combining the quark propagators, they looked for the mass spectrum of hadrons for $m_u = m_d$ but a larger m_s. At each value of a in table 1 (or at each $6/g^2$), and for each quark mass m_u, they obtained various non-strange hadronic masses am_h from the correlation functions of the operators with appropriate quantum numbers of the hadron h, as explained in §2. These were used to fix the lattice spacing a and the quark mass m_u. Forming ratios of am_π/am_ρ, and studying the mass am_ρ as a function of this ratio, which is equivalent to studying it as a function of m_u, am_ρ was extrapolated to experimental value for $m_\pi/m_\rho = 140/770$, while the ratio from simulations itself spanned a range of 0.4–0.75. Using $m_\rho = 770$ MeV and the extrapolated value of am_ρ, value of a in fermi was fixed. Note that this procedure amounts to using the ratio m_π/m_ρ from experiments to fix the quark mass $m_u = m_d$. The lattice values of the proton or the delta (Δ) mass can now be converted using this value of a to MeV and are *predictions* of QCD (albeit for a fixed cut-off a). Combining light quark and heavy (anti)quark for getting a K-meson or a Λ-baryon propagator, or taking the heavy quark–antiquark pair for the ϕ-meson, one can similarly obtain m_K, m_Λ, m_ϕ and other strange mesons and baryons as a function of the strange quark mass m_s. By interpolating the set of masses of *either* m_K or m_ϕ to its experimental value, the strange quark mass m_s was fixed, yielding again QCD *predictions* for the rest at the fixed cut-off a and the right strange quark mass. Using finally the 4-sets of spectra at each a of table 1, and making an ansatz $m_h(a) = m_h(0)(1+\alpha a)$, one obtains

Table 1. Parameters of CP-PACS quenched simulations.

$6/g^2$	a (fm)	Lattice size	L (fm)	# sets of U_x^μ
5.9	0.1	$32^3 \times 56$	3.3	800
6.1	0.078	$40^3 \times 70$	3.1	600
6.25	0.064	$48^3 \times 84$	3.1	420
6.47	0.05	$64^3 \times 112$	3.2	150

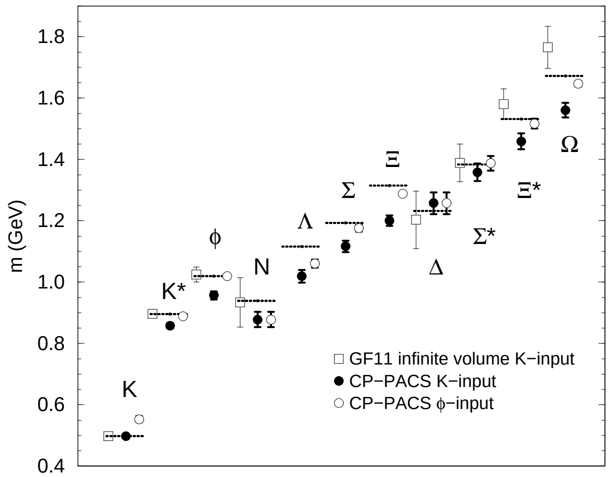

Figure 1. Light hadron spectrum for quenched QCD obtained by CP-PACS [6].

the continuum results for the hadronic spectra, displayed in figure 1. Compared to the earlier results of the GF11 collaboration these results are much more precise, enabling one to see differences due to the different input m_ϕ or m_K to fix the strange quark mass. While one sees an impressive agreement with the experimental data, strongly supporting QCD in its non-pertubative form as well, one also clearly sees the inadequacies of the quenched approximation: the nucleon mass seems lower than the experimental value by about $1.5\,\sigma$ whereas m_Δ is marginally higher. Using m_ϕ as input to fix m_s seems to give better results for strange baryons but m_K turns out to be higher by about 50 MeV in that case.

The same collaboration has also obtained results for dynamical Wilson fermions on coarser (larger a) and smaller ($L \simeq 2$–2.5 fm) lattices in a partially quenched simulation which still had m_s infinitely heavy. Their results for m_ϕ and m_{K^*} using m_K as input are shown in figure 2. Comparing them with figure 1 and m_K as input, one sees that the masses of m_{K^*} and m_ϕ shift in the right direction due to the inclusion of the effects of dynamical u-quarks. In fact, about 60% of the difference in figure 1 is explained this way. It is interesting to speculate that the rest will be accounted for by the inclusion of dynamical strange quarks. These impressive results have set a formidable benchmark for future high precision tests of non-perturbative QCD and seem certain to establish it firmly as the correct theory of strong interactions.

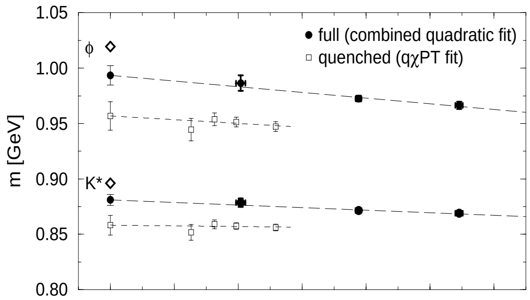

Figure 2. K^* and ϕ masses for dynamical 2 light flavour simulation. From ref. [6].

Table 2. World averages from ref. [7].

f_B (Mev)	f_{B_S} (MeV)	f_D (MeV)	f_{D_S} (MeV)	R_D (MeV)
165 ± 20	185^{+25}_{-20}	200 ± 20	220^{+25}_{-25}	190 ± 25
$B_{B_d}(m_b)$	B_{B_d}/B_{B_s}	R_D/R_s		
$0.86 \pm 0.04 \pm 0.08$	$1.00 \pm 0.01 \pm 0.02$	1.14 ± 0.06		

There are many interesting results which too follow from these simulations. For a lack of space, let me just mention them without getting into the details of how they are obtained. The information on bare masses of u, d and s quarks can be used to obtain renormalized quark masses. CP-PACS found [6] $(m_u + m_d)/2 = 4.6 \pm 0.2$ MeV and $m_s = 115 \pm 2$ MeV (or 143 ± 6 MeV, if m_ϕ is used as input) in the MS-bar scheme at a scale of $\mu = 2$ GeV. An interesting puzzle is whether and when the string between a pair of heavy quark and antiquark snaps as r, the distance between them, is increased. One expects this string breaking to occur only in full QCD simulations which are currently constrained to employ somewhat heavy dynamical quarks due to computational problems. Nevertheless, it is a bit puzzling to note that one has so far found no string breaking for $r \leq 2$ fm.

The lattice QCD approach has made a variety of predictions in the area of heavy quark physics which may be soon tested in the upcoming B-factories. One cannot employ the same methodology as in §2 in this case since $m_{\rm bottom} \simeq 4.5$ GeV and $m_{\rm charm} \simeq 1.5$ GeV(or ~ 0.045 fm^{-1} and 0.133 fm^{-1} respectively). If the cut-off effects are to be small, one wishes $m^{-1} \gg a$ (or $1 \gg ma$), whereas from table 1 one sees that the best values of a used even in the quenched approximations are still unacceptably large for heavy quark physics. Many approaches have been developed to deal with this situation, such as integrating out the heavy quarks (NRQCD approach) or static approximation. One can also compute for small ma and extrapolate up. The results obtained by these different approaches are in broad agreement. Table 2, taken from ref. [7] where additional details can be found, lists the world averages for various decay constants and the B-factors.

4. Finite temperature

According to the widely accepted Big Bang theory of the universe, our universe must have existed at very high temperatures soon after the Big Bang. As it cooled down, it may have undergone many phase transitions. Depending upon the nature of the high temperature phases and the phase transitions, one may be able to explain the baryon asymmetry of the universe or predict the dark matter to be predominantly baryonic. Very dense neutron stars may have a new form of matter at their core. Reliable information about the physics of the Standard Model for such situations can be obtained using lattice field theory techniques at finite temperature or density. As mentioned in §2, all one needs to do is to employ asymmetric, $N_s^3 \times N_t$, lattices with $N_s \gg N_t$. The phase transitions can be investigated with the help of several order parameters, such as the chiral condensate $\langle \bar\psi\psi\rangle$ for a chiral symmetry restoring phase transition or $\langle L\rangle \sim \exp(-F_q(T)/T)$ for studying the deconfinement phase transition, where $F_q(T)$ is the free energy of a static quark. The nature of the high T phase just beyond the transition can be explored by various hadronic screening lengths (obtained from correlators like those discussed in §2 but in a spatial direction), susceptibilities, energy density, pressure, etc. I will here restrict myself to updates of the results already available in the literature [8] and to new results of interest.

The chiral symmetry restoring phase transition in full QCD is one of the most striking non-perturbative prediction of lattice QCD. It is also theoretically very interesting because of the significant difference the quenched QCD has with full QCD in this case: the nature of the chiral phase transition and the transition temperature depend very critically on the number of light dynamical flavours. For two light flavours, $T_{\rm ch}/m_\rho$ has been estimated for various quark masses (or equivalently m_π/m_ρ values) and lattice spacings (or equivalent N_t). For the Wilson fermions, these ranges are $0.5 \lesssim m_\pi^2/m_\rho^2 \lesssim 1$ and 0.17 fm $\lesssim a \lesssim$ 0.33 fm, while for the staggered fermions they are slightly better: $0.2 \lesssim m_\pi^2/m_\rho^2 \lesssim 0.7$, 0.11 fm $\lesssim a \lesssim$ 0.33 fm. One finds [9] very small cut-off dependence and $T_{\rm ch}/m_\rho \simeq 0.2$ after extrapolation to zero quark mass for *both* types of fermions. This is a crucial check of universality. Since for a given continuum field theory, infinitely many lattice actions can be written down, by e.g., simply changing the approximation to the derivatives in eq. (1) or taking the trace in a different representation in eq. (3), one wonders whether they all have the same quantum continuum limit. By definition classical continuum limit yields the same continuum Lagrangian. The quantum continuum limit, on the other hand, is defined by holding a physical quantity, say, mass of the proton, constant in usual physical units, say, MeV. A non-trivial limit exists only at those couplings where the corresponding correlation length in lattice units diverges. Thus the lattice theory has to be at its critical point for a continuum limit to be nontrivial. As in statistical mechanics, one therefore expects a universal behaviour irrespective of the details of the lattice action (provided no terms in additional relevant direction in the sense of renormalization group are added). The universal result for $T_{\rm ch} \simeq 150$ MeV for two different lattice actions for the quarks is, therefore, very encouraging.

Unfortunately, the current results for the order of the chiral symmetry restoring phase transition seem to be at odds with universality. The Wilson fermions yield a second order phase transition with $O(4)$ exponents which is in disagreement with the staggered fermion results. For the latter, small lattices do suggest a second order phase transition but with different exponents. Moreover, the situation changes on larger lattices and even the order itself becomes, and still is, unclear.

A similar but somewhat stronger violation of universality was observed in the case of SU(2) gauge theory [10], which is a toy model, to examine critically various theoretical concepts. Quenched SU(2) theory with an action given by eq. (3) (but with $6/g^2 \rightarrow 4/g^2$, $1/3 \rightarrow 1/2$ and $U_x^\mu \in \mathrm{SU(2)}$) has a second order deconfinement phase transition whose critical exponents match with those of the three dimensional Ising model. What ref. [10] showed is that an addition of a term which is similar to that in eq. (3) but with the trace taken in adjoint representation leads to a first order deconfinement phase transition. The added term has no extra relevant direction and was expected to leave at least the qualitative results unaffected. Similarly SO(3) lattice gauge theory, expected to be similar to the SU(2) theory in the continuum limit, also yielded a first order deconfinement phase transition [11]. While this puzzling lack of universality could perhaps be explored further on bigger and bigger lattices with a hope that the universality of the continuum limit will eventually be restored, how so ever implausible it may seem in current day simulations, a novel attempt was made [12] recently to look for universal results. The lattice SO(3) theory has Z_2 monopoles, which are topologically nontrivial objects but are purely lattice artifacts. As $a \rightarrow 0$, they disappear or become very rare. Suppressing them with the help of a large chemical potential, ref. [12] investigated the specific heat of the SO(3) gauge theory, shown in figure 3. The increase in its peak height as the spatial volume is increased is consistent with that expected for a three dimensional Ising model-like second order transition. This agreement with universality is remarkable especially considering that the SO(3) lattice gauge theory has no order parameter for deconfinement phase transition.

Another way to confirm or rule out a first order deconfinement phase transition for the SU(2) theory with the extended action above is to check whether the physical properties of the high T phase are the same or not. From the determination of glueball screening lengths in both SU(2) and SU(3) gauge theory with the usual action (in eq. (3)), it was shown [13] that already at $T \simeq 2T_c$ the spectrum exhibits dimensional reduction, i.e., a symmetry expected to be valid as $T \rightarrow \infty$. A similar calculation in the extended theory will shed further light on the issue of universality. If the first order phase transition is *not* a deconfining one then the spectrum of glueballs should not exhibit dimensional reduction.

Our observed universe seems to have only baryons although it was presumably born with a net baryon number zero in the Big Bang. Several ideas have been proposed to generate the

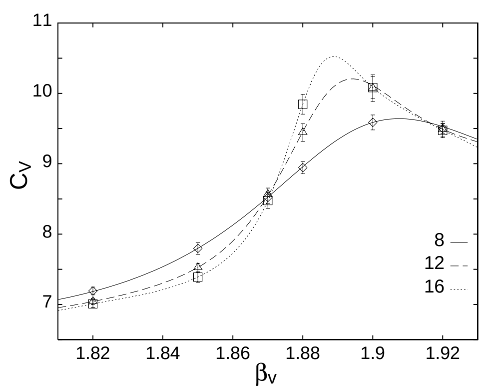

Figure 3. Specfic heat for SO(3) lattice gauge theory with monopole suppression. From ref. [12].

baryon asymmetry. If the electroweak phase transition (EWPT) restoring the SU(2) $\times$ U(1) symmetry is strongly first order, with the vacuum expectation value of the Higgs field dropping to zero discontinuously at T_c and with $v_H(T_c)/T_c > 1$, then EWPT can provide the necessary departure from equilibrium for baryogenesis to occur [14]. EWPT has been investigated on the lattice using 2 different approaches: (1) SU(2) gauge theory coupled to scalar Higgs fields in $3+1$ dimensions (ignoring the fermions which are chiral and hence difficult to handle on the lattice although perhaps not so important due to their weak interactions) and (2) dimensional reduction, assuming T to be large enough to integrate modes with masses proportional to T (including all modes of all the fermions) to obtain a 3-dimensional SU(2) gauge theory coupled to scalars. Both approaches [16] find that *no* first order EWPT is possible if $m_{\rm Higgs} > 72$ GeV which is already ruled out by the recent LEP results.

5. Exact chiral symmetry

The Standard Model is a chiral gauge theory: the left-handed quarks and leptons transform as a doublet under SU(2) but the right-handed ones are singlets. Such theories have anomalies at quantum level which are cancelled in the perturbation theory for SM by clever assignments of quantum numbers. Various extensions of SM continue to be chiral theories as well. In view of this, it is important to ask whether such theories can be defined non-perturbatively as well. The difficulties of defining fermions on the lattice without doubling and with chiral symmetries seemed to suggest a negative answer. Various attempts have been made to define chiral gauge theories on the lattice, including the overlap formalism [17] many of which seemed to fail. Developments in the past year now give hopes that it may be possible to define a chiral gauge theory [18].

A lattice Dirac operator, D_O, for the overlap fermions [17] was proposed by Neuberger [20] which, like the almost simultaneously but independently proposed Dirac operator using the perfect action approach [21], obeyed Ginsparg–Wilson relation [22]

$$\gamma_5 D + D\gamma_5 = a\, D\gamma_5 D. \tag{6}$$

Although this relation has been known for a long time, it has recently been shown that any D satisfying it, (1) has exact but nonlocal chiral symmetry, and (2) satisfies the index theorem (has chiral zero modes). Chiral fermions could then be introduced using appropriate projectors. There are many interesting applications of these ideas even for a vector theory like QCD since one can now define exact chiral symmetry for any given number of flavours. How to include these fermions in simulations and what impact they have on, e.g., the chiral phase transition, is just beginning to be explored.

6. Summary

Lattice gauge theories, defined on a discrete space time lattice, provide us with the best tool yet to obtain qualitative and quantitative consequences of the standard model in regions of its coupling for which weak coupling–perturbative–expansion fails. Extracting results valid in the continuum limit of vanishing lattice spacing is currently possible only numerically, by using Monte Carlo techniques. Tremendous progress has been made in the calculations of hadronic masses, decay constants, quark masses etc. in the quenched approximation which corresponds to setting the sea quark masses to infinity. Hadronic spectrum results from CP-PACS are close enough to the experimental results to strengthen our belief in QCD as the theory of strong interactions but are precise enough to show clear disagreements. Inclusion of dynamical quarks, by tuning the up and down sea quark masses to be finite but still large, reduces the disagreement, bringing the theoretical results closer to the experiments. These simulations also seem to yield smaller values for light quark masses, favouring a smaller m_s ($\sim$ 100 MeV) which in turn suggests a larger ϵ'/ϵ for the K-mesons.

Different formulations of fermions (Wilson and Kogut–Susskind) lead to a similar value for the chiral symmetry restoring temperature, $T_{\rm ch} \sim$ 150 MeV, for two massless flavours in the continuum limit. However, there is an apparent violation of universality in their predictions of the order of the chiral phase transition. Bigger lattices or simulations with the recently proposed Dirac–Neuberger operator for fermions may clarify the situation in future. Violations of universality in the SU(2) gauge theory, whose second order deconfinement phase transition turned first order as a result of increase in an irrelevant coupling (in the sense of renormalization group), were shown to be related to the presence of lattice topological objects, such as monopoles, which die away in the $a \to 0$ limit. Suppressing the monopoles led to a rising specific heat in the SO(3) lattice gauge theory with universal critical exponent of the SU(2) theory and the Ising model in three dimensions. Nevertheless, it still remains unclear as to how rapidly these objects become rare in the unsuppressed theory, leaving the almost zero rate of approach to universal results in the case unexplained.

Exact chiral symmetry on the lattice seems to have arrived on the horizon and is being intensely pursued. One looks forward to more stringent checks of universality using the new formulations of lattice fermions.

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