Filtering of two-photon quantum correlations by optical cavities: Cancellation of dispersive effects

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The spectral filtering of a field with strong quantum correlations is studied. The changes in the correlation characteristics of a field, produced by a down converter, passing through a strongly dispersive element like a Fabry-Pérot cavity, are calculated. In the special case when the central frequency of the idler and signal photons coincides with the resonance frequency of the cavity, there is a *cancellation* of the dispersive effects of the cavity. Detailed numerical results for the two-photon joint counting rate are presented. The effects of the absorption in the medium on the two-photon correlations are also discussed.

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I. INTRODUCTION

In recent years one has discovered many interesting features of spectral filtering. For example, the spectrum of light passing through a filter can change in a number of ways depending on the spectral characteristics of the filter itself as can be seen from the following simple consideration. Let $T(\omega)$ be the spectral response of the filter. Then the spectrum $S_{\text{out}}(\omega)$ of the output is related to the spectrum $S_{\text{in}}(\omega)$ of the input via

$$S_{\text{out}}(\omega)=T(\omega)S_{\text{in}}(\omega) . \tag{1}$$

If both $T(\omega)$ and $S_{\text{in}}(\omega)$ are Gaussians centered at different frequencies, then $S_{\text{out}}(\omega)$ is also Gaussian but shifted in frequency [1]. The shift depends on the two central frequencies and the two widths. The output spectrum can also be a multiline spectrum [2] even if the input is a single line. This depends on the resonant character of $T(\omega)$. Note that the spectrum represents a second-order statistical property of the field and a relation like (1) sheds no light on the higher-order statistical properties of the output field. The situation is rather simple if the input field is described by a Gaussian stochastic process. In such a case the output field is also described by a Gaussian stochastic process. Then this fact coupled with Eq. (1) is enough to derive all the statistical properties of the output field. Woerdman and co-workers found a very interesting result [3] for the case of an input field described by the phase diffusion model. They showed that if the filter bandwidth is very narrow compared to the width of the input beam, then the intensity-intensity correlation of the output beam behaves the same way as the intensity-intensity correlation of a chaotic light beam. Thus the higher-order statistical characteristics of the output beam are very sensitive to the statistics of the input beam. While all this is based on the classical treatment of light beams, which is quite adequate for many situations, there are however situations which require a quantized field treatment. This is particularly the case if the input light is produced by down converting [4,5] a pump beam of frequency 2Ω into two photons of frequencies $(\Omega+\nu)$ and $(\Omega-\nu)$ with the allowed values of ν determined by the phase-matching considerations. The quantum correlations between two photons of the pair $(\Omega+\nu,\Omega-\nu)$ are extremely large and produce a number of important effects. For example, Franson [6] has discovered an unusual property of the two-photon correlated source with a wave function of the form

$$|\psi\rangle \sim \int \phi(\nu)|\Omega+\nu,\Omega-\nu\rangle d\nu . \tag{2}$$

He found that if the two photons of the pair travel through a dispersive medium, then under certain conditions there is cancellation of the dispersive effects in the two-photon detection probability. Franson assumed weak dispersion, i.e., a dispersion relation of the form

$$k(\Omega+\nu)\cong k_0+\nu k_1+\nu^2 k_2 \tag{3}$$

was used. Clearly the issue of the effects of absorption and dispersion on the two-photon correlations is wide open.

In this paper we examine the changes in the photon correlations if the signal and idler beams are filtered by a *resonant* structure like a Fabry-Pérot interferometer. The correlated nature of the signal and idler beams lead to the rather remarkable result—the two-photon detection probability is independent of the presence of the Fabry-Pérot interferometer provided the Fabry-Pérot interferometer is tuned so that its resonance frequency coincides with Ω. Thus, under the condition, the resonant dispersive effects of the filter (cavity) cancel as far as the two-photon detection probability is concerned though the transmission of each beam itself is affected by the dispersive characteristics of the filter.

II. CALCULATION OF THE TWO-PHOTON COINCIDENCE PROBABILITY

Consider the filtering of the two-photon correlations as shown schematically in Fig. 1. Let the idler and the signal beams produced in down conversion be incident on the two sides of the Fabry-Pérot interferometer. The Fabry-Pérot interferometer may in addition be filled by a

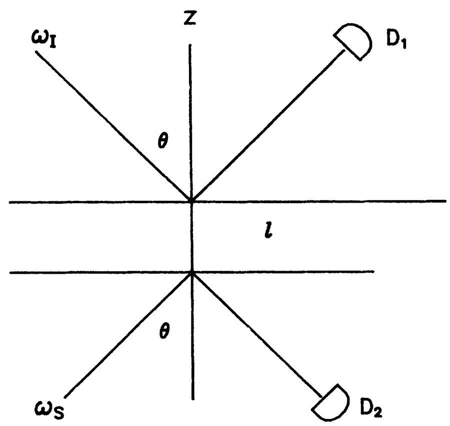

FIG. 1. Schematic illustration of the interference arrangement.

dispersive or absorptive medium. Let the photons at the detectors D_1 and D_2 be detected in coincidence. Let $\mathcal{E}_I^{(+)}$ and $\mathcal{E}_S^{(+)}$ be the amplitudes associated with the idler and the signal fields. In quantum theory these represent the positive frequency components of the electric-field operators. The fields at the detectors D_1 and D_2 are then given by

$$\mathcal{E}_1^{(+)}(t)=\int d\omega\, e^{-i\omega t+ik_\omega(z_1)}[r_I(\omega)\mathcal{E}_I^{(+)}(\omega)+t_S(\omega)\mathcal{E}_S^{(+)}] , \quad (4)$$

$$\mathcal{E}_2^{(+)}(t)=\int d\omega\, e^{-i\omega t-ik_\omega z_2}[r_S(\omega)\mathcal{E}_S^{(+)}(\omega)+t_I(\omega)\mathcal{E}_I^{(+)}(\omega)] , \quad (5)$$

where z_1 and z_2 give the position of the detectors and

$$k_\omega=\frac{\omega}{c}\cos\theta . \quad (6)$$

In Eq. (4), $r(\omega)$ and $t(\omega)$ represent, respectively, the reflection and transmission amplitudes of the Fabry-Pérot interferometer. The subscript $S(I)$ refers to the signal (idler wave). For a lossless Fabry-Pérot interferometer the matrix

$$U=\begin{bmatrix} t_S(\omega) & r_I(\omega) \\ r_S(\omega) & t_I(\omega) \end{bmatrix} \quad (7)$$

should be unitary. Let $\chi(\omega)$ be the susceptibility of the medium inside the Fabry-Pérot interferometer. Then the theory [7] of the Fabry-Pérot (FP) interferometer leads to

$$r_S(\omega)=r_I(\omega)=\sqrt{R}\,\frac{(1-e^{2il\tilde{k}_z})}{(1-Re^{2il\tilde{k}_z})} , \quad (8)$$

$$t_S(\omega)=t_I(\omega)=\frac{Te^{il\tilde{k}_z}}{1-Re^{2il\tilde{k}_z}} , \quad (9)$$

$$\tilde{k}_z^2=\frac{\omega^2}{c^2}\{[1+4\pi\chi(\omega)]-\sin^2\theta\}, \quad R+T=1 . \quad (10)$$

where R (T) are the reflectivity (transmissivity) of the FP mirrors. Using (8)–(10) the unitarity of U can be proved for the case when $\chi(\omega)$ is real.

The input state of the field is given by the relation (2) with the function $\phi(\nu)$ having the form

$$\phi(\nu)=\frac{\phi_0}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{\nu^2}{2\sigma^2}\right\} . \quad (11)$$

The joint probability of detecting a photon at the detector 1 at time t and a photon at the detector 2 at time $t+\tau$ is given by [4]

$$P_2(\tau)=\langle\psi|\mathcal{E}_1^{(-)}(t)\mathcal{E}_2^{(-)}(t+\tau)\mathcal{E}_2^{(+)}(t+\tau)\mathcal{E}_1^{(+)}(t)|\psi\rangle , \quad (12)$$

where we assume that the two detectors are located such that $z_1=z_2$. This probability P_2 can be calculated using (2), (4), and (5). One should remember that the structure of the wave function is such that both signal and idler photons are present simultaneously. The calculations show that

$$P_2(\tau)=\left|\int d\nu\, e^{-i\nu\tau}\Phi(\nu)[r_S(\Omega+\nu)r_I(\Omega-\nu)+t_S(\Omega-\nu)t_I(\Omega+\nu)]\right|^2 . \quad (13)$$

The measured coincidence probability will be obtained by integrating over the resolving time T_R of the detector [4]

$$P_2=\frac{1}{T_R}\int_{-T_R/2}^{T_R/2}P_2(\tau)d\tau . \quad (14)$$

The Eq. (14) gives the final results for the two-photon joint detection probability. On using Eqs. (2) and (4) the probability of detecting a photon at the detector 1 will be

$$P_1=\langle\psi|\mathcal{E}_1^{(-)}(t)\mathcal{E}_1^{(+)}(t)|\psi\rangle=\int d\omega|\phi(\omega)|^2[|r_I(\omega)|^2+|t_S(\omega)|^2] \quad (15)$$

which for a lossless medium reduces to

$$P_1=\int d\omega|\phi(\omega)|^2=|\phi_0|^2 . \quad (16)$$

The result for the detection of the *single events is independent* of the Fabry-Pérot interferometer. This is in contrast to the result (1). The very special nature of the two-photon source leads to (16).

III. EFFECTS OF FABRY-PÉROT DISPERSION ON TWO-PHOTON COINCIDENCE DETECTION

In this section we present explicit results for P_2. We show its dependence on the Fabry-Péroy parameters. We assume that there is no medium between the two plates of the Fabry-Pérot (FP) interferometer. Let ω_c be the resonance frequency of the FP interferometer for a given angle of incidence θ. We define

$$\Delta\equiv\Omega-\omega_c;\quad \delta=\frac{\Delta l}{c}\cos\theta,\quad \delta_\nu=\frac{\nu l}{c}\cos\theta . \quad (17)$$

Then on using (8)–(10) and (17) we can prove that

$$r_S(\Omega+\nu)r_I(\Omega-\nu)+t_S(\Omega-\nu)t_I(\Omega+\nu)=1+\frac{T[T+(Re^{2i\delta}-e^{-2i\delta})]}{(e^{-i\delta}-Re^{i\delta})^2+4R\sin^2\delta_\nu} . \quad (18)$$

From Eq. (18) we see that

$$r_S(\Omega+\nu)r_I(\Omega-\nu)+t_S(\Omega-\nu)t_I(\Omega+\nu)=1 \text{ if } \delta=0 \quad (19)$$

and hence

$$P_2(\tau)=\left|\int d\nu\, e^{-i\nu\tau}\Phi(\nu)\right|^2 \text{ if } \Omega=\omega_c \ . \quad (20)$$

We have thus shown that the Fabry-Pérot dispersion does not affect the two-photon coincidence probability if the central frequency of the idler and signal photons matches the resonance frequency ω_c of the Fabry-Pérot cavity. There is a cancellation of the effects of the FP dispersion. This is so in spite of the fact that the FP transmission and reflection characteristics exhibit resonant behavior. Above result holds irrespective of the width of the FP relative to the width of the signal or idler photons.

We next discuss the behavior of P_2 as a function of the parameter Δ, i.e., P_2 as the Fabry-Pérot interferometer is scanned across its resonance frequency. We show the numerical results in Fig. 2 where P_2 is plotted as a function of the parameter $(q\Delta/\sigma)$ for a detector resolution time $\sigma T_R=50$ and for different values of the FP reflectivity parameter R. The parameter q $[=\sigma\cos(\theta)l/c]$ was taken to be equal to 0.15. The peak height is independent of the FP reflectivity as expected from our general result (19). For $q\Delta/\sigma$ of the order of the FP linewidth, the joint probability developes minima. The minimum grows deeper as the reflectivity R goes down. For Δ greater than the width of the FP Airy resonances, the joint detection probability grows, since in that case it is almost like having no FP dispersion. In order to understand the minima in the joint probability P_2 we have investigated $P_2(\tau)$ as a function of dimensionless time $\sigma\tau$ (see Fig. 3) for various values of $q\Delta/\sigma$. We have calculated $P_2(\tau)$ for values of $q\Delta/\sigma$ corresponding to the minimum (i.e., $q\Delta/\sigma\cong 0.1$) and for two other values, namely, $q\Delta/\sigma=0$ and 0.3. It is clear that at resonance (i.e., $\Omega=\omega_c$), where dispersion gets nullified, the area under the curve $P_2(\tau)$ gets maximized. Due to destructive interference the area under P_2 (τ) reduces to a minimum for $q\Delta/\sigma\cong\pm 0.1$, whereas, for larger values of $q\Delta/\sigma$, the FP signature is erased and there is a growth in the joint counting probability.

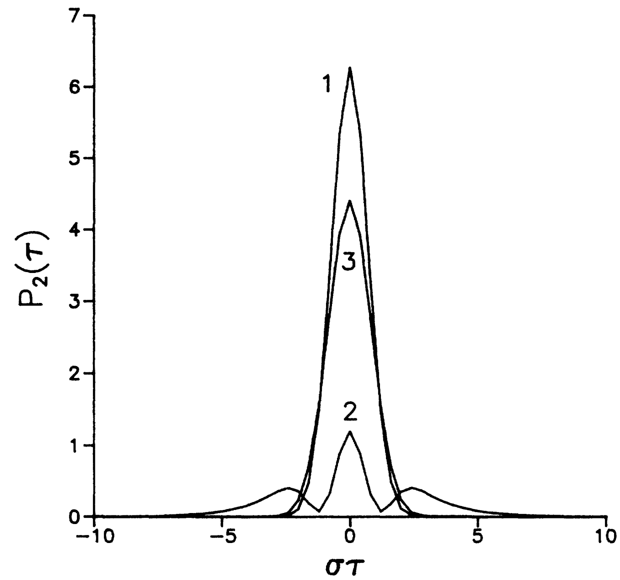

FIG. 3. The time dependence of the joint detection probability for an empty cavity for $q=0.15$, $R=0.9$ and for different values of $q\Delta/\sigma$, namely, 0.0 (curve 1), 0.1 (curve 2) and 0.3 (curve 3).

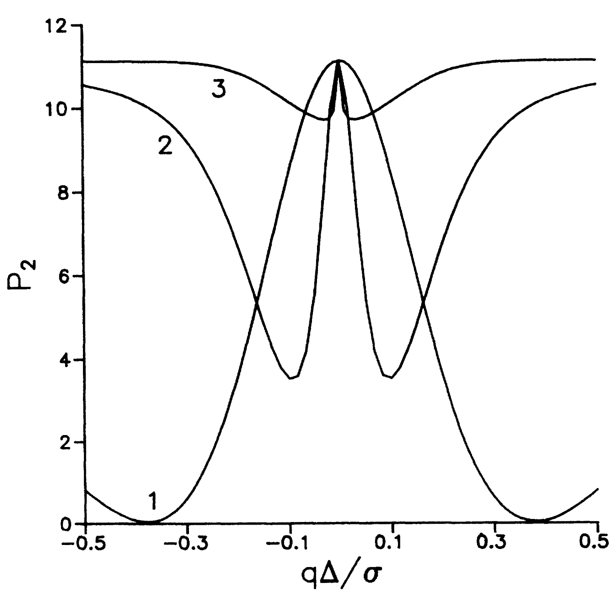

FIG. 2. Joint detection probability P_2 as a function of $q\Delta/\sigma$. Curves labeled by 1, 2, 3 are for $R=0.5$, 0.9 and 0.99. Other parameters are as follows: $q=0.15$, $\sigma T_R=50$.

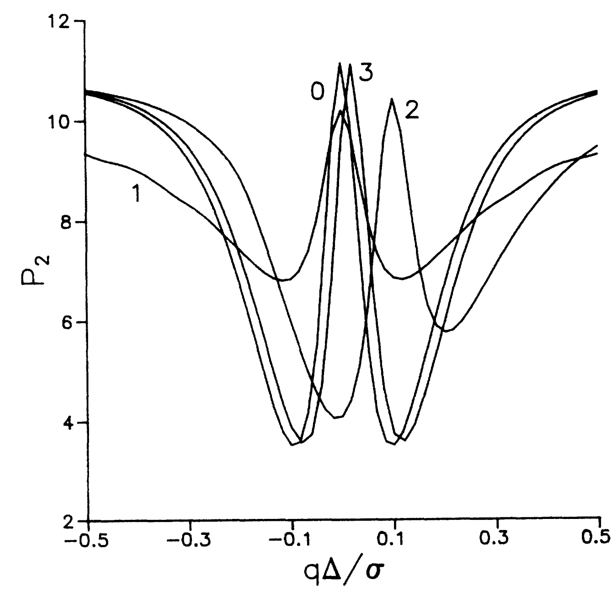

FIG. 4. Joint detection probability P_2 as a function of $q\Delta/\sigma$. The curve labelled by 0 is for the empty cavity, whereas, 1, 2, 3 are for a FP cavity with atoms with $q\Delta_0/\sigma$ given by 0.02, 0.2, and 1.0, respectively. Other parameters are as follows, $q=0.15$, $\sigma T_R=50$, $\gamma/\omega_0=10^{-4}$, $(\omega_p/\omega_0)^2=10^{-5}$, $R=0.9$.

IV. COMBINED EFFECTS OF THE FABRY-PEROT DISPERSION AND ATOMIC MEDIUM DISPERSION ON JOINT COUNTING RATE

Recently the work of Franson to a frequency region where the absorption of the medium is important, has been generalized [8]. In particular, the absorption of the medium affects the joint counting rate significantly. In this section we examine the combined effects of absorption and dispersion. This is done by including a resonant medium within the FP interferometer. We write the susceptibility of the resonant medium as

$$4\pi\chi(\omega)=\frac{(\omega_p^2/\omega_0^2)}{\left[1-\left[\frac{\omega^2}{\omega_0^2}\right]-i\frac{\omega}{\omega_0}\frac{\gamma}{\omega_0}\right]}, \tag{21}$$

where ω_0 is the resonant frequency of the medium and γ gives the absorption in the medium. The computed numerical results using Eq. (13) are shown in Fig. 4. We have plotted P_2 as a function of $q\Delta/\sigma$ for various values of $q\Delta_0/\sigma$ where $\Delta_0=\Omega-\omega_0$. We have shown the curves for only positive values of Δ_0 since for negative values of Δ_0, they are symmetric with respect to $\Delta=0$. For comparison we have also shown the case of an empty FP cavity. It is clear from Fig. 4 that absorption and dispersion introduced by the atoms lead to a suppression and a shift of the peaks and dips of the joint counting probability. The effect of absorption is dominant for zero detuning. Positive Δ_0 leads to a pulling of the features to the right. However, for large Δ_0, when the atoms are far detuned their effect becomes negligible and we recover the case for the empty FP cavity.

In summary, we have examined the filtering of the radiation produced by a two-photon correlated source. The filter is a dispersive element like a Fabry-Pérot cavity. We show that under certain conditions, the two-photon detection probability becomes independent of the presence of the optical cavity. We have also examined the effect of an absorbing medium inside the cavity.

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