A SURVEY OF PRESENT-DAY APPROACHES TO STRONG INTERACTIONS*

by A. N. MITRA, *Department of Physics, University of Delhi, Delhi 6*

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Till about the middle of the present decade (which incidentally ends today), conventional field theory with specific interaction Lagrangians used to provide the main basis for the treatment of all interactions, weak and strong. Along with a few general invariance principles, coupled with the ideas of renormalization, this basis had seemed conceptually to be quite satisfactory to a large section among modern physicists, as is eloquently testified by the two volumes of Bethe's treatise on the subject (Bethe *et al.* 1955). The spectacular success of the theory applied to specifically electromagnetic interactions had appeared to provide enough justification for such an attitude, so much so as to allow its extrapolation to strong interactions where the coupling constant is 15 instead of 1/137. However, the lack of reliable mathematical tools for effective treatment of strong interactions had consistently defied attempts to check on the premises of a Lagrangian-type field theory against experimental background. Among the various methods that had been devised up to the middle of this decade for the treatment of strong interactions the following may be mentioned:

(*a*) The strong coupling method of Pauli (1947), which was largely resurrected later at the hands of Chew (1954) and Chew and Low (1956).

(*b*) The Tamm-Dancoff, Bethe-Salpeter and Tomonaga Intermediate Coupling Theories.

(*c*) Soluble models in field theory like the Lee (1954) and Thirring (1958) models.

(*d*) The method of functional integrations due of Peierls and Edwards (1954).

While some of these have had limited successes (e.g. (*a*) and (*b*) for low energy pion-nucleon scattering and photoproduction), none of them can really be taken to have produced results of a standard anywhere near the electromagnetic case. This, in turn, started leading to serious misgivings in many minds as to the correctness of the very premises of field theory as applied

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to strong interactions with Yukawa-type coupling of pseudoscalar mesons to nucleons.

During the second half of this decade, certain general aspects of field theory (without specific reference to any particular model of interaction) have, however, received a good deal of attention. Thus the Heisenberg representation techniques developed by Lehmann and collaborators (*see* e.g. Lehmann 1959) have proved particularly fruitful for the derivation of certain very general results on arbitrary scattering and production amplitudes. In particular these techniques have greatly helped towards concrete realization of the so-called dispersion relations in the framework of field theory in a model-independent manner. Thus dispersion relations in both forward and non-forward directions have been rigorously derived within the premises of field theory (*see* e.g. Bogoluibov and Shirkov 1959).

It is well known that dispersion relations have proved extremely useful for correlating important experimental results with theoretical parameters. Thus the manifold of ambiguities in the various phase shift determinations in π-N scattering from the corresponding cross-sections have been resolved almost entirely through dispersion relations for forward scattering (*see* e.g. Puppi 1959). However, important as such results are, it had no doubt to be recognized that these dispersion relations, being independent of specific interaction models, could not by themselves be expected to provide any answer to the question of 'how to handle strong interactions in practice'. In other words, it was clear that one could not go very far in the theoretical understanding of strong interactions without additional input information not contained in dispersion relations.

Another phase of progress in field theory during the same period, which seems to have had a more positive bearing on the theoretical understanding of strong interactions, consists in the application of the so-called invariance principle to Lagrangian-type field theory. Of course some of the 'orthodox' invariance principles based on continuous transformation had never been questioned, e.g. translational and rotational invariance. On the other hand, our faith in the corresponding principles for *discrete* transformations (e.g. charge conjugation, parity, time-reversal) received a set-back through the Lee-Yang discovery of parity violation in weak interactions. This naturally led to a good deal of heart-searching and, in the process, many more symmetries (some purely speculative) were discovered. Indeed, though the Lee-Yang discovery had set the stage for rethinking on weak interactions, concrete progress was achieved only after the discovery of another symmetry, viz. the gauge or γ_5-invariance for neutrino fields.

While the applications of symmetry principles have largely helped in the quantitative understanding of weak interactions, they have served at most to provide some qualitative understanding of strong interaction phenomena, in spite of the fact that, in principle, strong interactions are supposed to obey much stronger symmetries than weak ones. The essential reason for this apparent anomaly is that for weak interactions one can start with definite interaction Lagrangians, apply various symmetry requirements on them and test the effects of the latter through the calculation of physically measurable entities by straightforward perturbation techniques. And, of course, this is precisely what cannot be done for strong interaction phenomena in the absence of reliable calculational tools. Thus there is no practical way of seeing if the conjectured symmetries in strong interactions have any experimental significance.

Symmetries have still been conjectured for strong interactions, the earliest and most important of them being of course charge independence, viz. rotational invariance in the space of isobaric spin. Fortunately, this symmetry has had many experimental manifestations even without explicit reference to an interaction Lagrangian (e.g. existence of isobaric multiplets in nuclei, equality of n-p and p-p forces, resonances in π-N scattering and photoproduction, etc.). Charge independence is therefore a concept whose physical validity for strong interaction is taken almost for granted, so much so that it has come to be regarded as almost synonymous with strong interactions. Therefore, with the discovery of the so-called strange particles with numerous manifestations of strong interactions, the concept of charge independence was readily extended to them. The assignments of isospin to these strange particles were of course made in such a way as to be in conformity in the so-called Gell-Mann-Nishijima selection rules, with the result that (with the exception of Ξ-particles) the isobaric space transformation properties of 'strange' particles (Λ, Σ, K) happen to be just the opposite of those for 'normal' particles (N, π). Thus we know that Λ, Σ are isobosons (with $T = 0, 1$ respectively) and K's (and also Ξ) are isofermions ($T = \frac{1}{2}$). Once these assignments were known, it was immediately possible to write down Yukawa-type Lagrangians for the pionic and kaonic interactions of all particles as straightforward generalizations of the charge independent pion-nucleon interaction (for notation *see* Roman 1960):

$$L_\pi = iG_1\bar{\psi}_N\gamma_5\vec{\tau}\cdot\psi_N\vec{\phi}_\pi + G_2\left(\bar{\psi}_\Lambda\gamma_5\vec{\psi}_\Sigma\cdot\vec{\phi}_\pi + h.c.\right)$$
$$+ iG_3\left(\vec{\bar{\psi}}_\Sigma\gamma_5\times\vec{\psi}_\Sigma\right)\cdot\vec{\phi}_\pi + iG_4\bar{\psi}_\Xi\gamma_5\vec{\tau}\cdot\psi_\Xi\vec{\phi}_\pi; \quad \ldots \quad \ldots \quad (1)$$

$$L_K = G_5\left(\bar{\psi}_N\gamma_5\psi_\Lambda\phi_K + h.c.\right) + G_6\left(\bar{\psi}_N\vec{\tau}\phi_K\gamma_5\cdot\vec{\psi}_\Sigma + h.c.\right)$$
$$+ G_7\left(\bar{\psi}_\Xi\tau_2\overset{*}{\phi}_K\gamma_5\psi_\Lambda + h.c.\right) + G_8\left(\bar{\psi}_\Xi\vec{\tau}\tau_2\overset{*}{\phi}_K\gamma_5\cdot\vec{\psi}_\Sigma + h.c.\right). \quad \ldots \quad (2)$$

These forms imply of course that the kaons are pseudoscalar and the hyperons are all Dirac particles (of spin $\frac{1}{2}$)—results which are now taken to be fairly well established experimentally (with the exception of Ξ).

It may once again be emphasized that, unlike the corresponding (V-A) weak interaction Lagrangian with universal Fermi coupling, the expressions (1) and (2) are not experimentally established for the same reason, as given earlier in this article, viz. the failure of standard field theoretical techniques for large coupling. Still it was tempting for many authors to look for further (higher) symmetries than implied by charge independence in (1) and (2). Thus it was suggested by Gell-Mann (1957) and more emphatically by Schwinger (1957) that, to avoid the appearance of as many as eight independent coupling constants, each class of interaction (pionic and kaonic) must be specified by additional symmetry requirements. Thus, making all the G's equal in (1) has the effect of rendering L_π rotationally invariant in a 4-dimensional isobaric spin space, where the nucleons and hyperons are regarded as different states of a single entity—the 'baryon'. This symmetry Gell-Mann named 'global symmetry' to distinguish it from mere charge independence. Gell-Mann tried to make the concept plausible on the ground that the (experimentally stronger) pionic interaction should exhibit a higher degree of symmetry than the (experimentally weaker) kaonic interaction which is only charge independent, in much the same way as the π-N interaction shows stronger symmetry (charge independence) than the e.m. interaction of the nucleon (which conserves only T_3). In Gell-Mann's picture the mass splitting of the various hyperon groups comes from the assumption of different constants G_i in (2). In Schwinger's picture, on the other hand, the number of independent constants in (2) is still *one*, but their *phases* are such as to make (2) exhibit only 3-dimensional isospin invariance, hence essentially responsible for the mass splittings between the hyperon groups.

While such ideas are extremely attractive in principle, their practical content does not seem to be very profound. For, in practice, the kaonic interaction is strong enough to distort global symmetry so violently as to reduce the latter to no more than an academic concept. In other words, it is very difficult to test the above hypothesis experimentally simply because it is not possible in practice to 'switch off' one kind of interaction in favour of the other. Such symmetry principles, even if they are true, are therefore of a highly approximate nature in their practical manifestations. Moreover the practical manifestations of such principles based on specifically strong interactions must necessarily be very indirect because of the limitations of calculational tools. Thus, e.g., resonances in pion-hyperon scattering should in principle be able to throw some light on the validity of global symmetry by comparison with corresponding resonances in π-N scattering provided of course their spin, parity and energy dependencies tally. From this limited point of view, of course, the recently observed π-Λ, Σ resonances have tended to favour the concept of global symmetry for π-interactions. Yet the concept cannot be taken for granted in the absence of more quantitative evidences as obtained e.g. for the corresponding case of the weak interaction. Thus Pais (1958) found from the analysis of K-N scattering that experimental data are clearly incompatible with simultaneous universality of π- and K-interactions. On the other hand, Salam *et al.* (1961) concluded from the data on non-leptonic decays of hyperons that the results are not very far removed from the predictions of universality. None of these conclusions can however be taken as final, in so far as these are not based on reliable computational techniques which have concrete quantitative content.

It thus appears that applications of abstract symmetry principles to strong Lagrangian-type interactions in conventional field theory have not by themselves helped much in the quantitative understanding of strong interaction phenomena. In recent times there has emerged a strong group of opinion according to which the malady lies with the starting point itself, viz. the very premises of field theory. People have therefore been trying to advocate other approaches outside the scope of the so-called Lagrangian formalism, and it may be worth while to describe some briefly.

A rather popular approach at present is one based on the idea of analyticity of various amplitudes (scattering, production, etc.) in their appropriate arguments regarded as complex variables. This is not a new idea, for it had its seeds in the original S-matrix theory of Heisenberg and was subsequently developed by Møller (1945). In more recent times Chew and Low (1956) successfully exploited these ideas of analyticity in a simple and convincing manner to develop a satisfactory theory of π-N scattering in the fixed-source model. A more important fact that has emerged in recent times (from this point of view) is the recognition that, for all its limitations, perturbation theory has one important virtue, viz. a perturbation series exhibits analytic behaviour term by term. This fact was first recognized by Mandelstam (1958) for two-body scattering amplitudes and by Karplus *et al.* (1958) for vertex functions. These and similar results, derived within the premises of field theory, have led to the view that the S-matrix is presumably the boundary value of an analytic function of certain invariant combinations of 4-momenta (regarded as complex variables), when these variables take on specific values corresponding to the occurrence of the physical process under study. Further, the 'strengths' of the singularities of the complex function (viz. residues at the poles and discontinuities across branch-cuts) are severely restricted by unitarity.

Specifically, the idea is the following: In a two-particle scattering problem, the scattering amplitude is a function of the form $f(E^2, \Delta^2)$ where E is the total c.m. energy and Δ^2 is the squared 4-momentum transfer. Physical values of these two variables do *not* overlap. This makes it possible to define a function $f(z_1, z_2)$ whose boundary value appropriate to the scattering problem at hand is the amplitude $f(E^2, \Delta^2)$. Once such a function $f(z_1, z_2)$ is visualized, it becomes possible to define two other sets of non-overlapping boundary values of z_1, z_2 (in which the roles of physical energy and momentum transfer are interchanged) corresponding to two other processes, without extra charge. Graphically this can be understood by looking at the Feynman diagram for a scattering process in *three* distinct ways corresponding to the possible pairings of the four external lines involved. This is the famous 'Substitution Law', well known in perturbation theory, by virtue of which, e.g., the matrix elements of (1) bremsstrahlung and pair production; (2) compton scattering and two quanta annihilation of a fermion pair; and like pairs of processes have identical algebraic structures. In the present context this law was rediscovered by Mandelstam (1958) who took it as an exact statement of facts applicable to all orders in perturbation theory. However, to give this statement a concrete content without having to invoke the premises of field theory, he was led to a study of the analytical structure of the function $f(z_1, z_2)$ in z_1 and z_2.

Mandelstam's (1959) investigations based again on perturbation theory showed that $f(z_1, z_2)$ satisfies a dispersion representation in *both* z_1 and z_2 simultaneously, with the spectral functions vanishing unless these variables have values corresponding to their physical magnitudes for any one of the three processes mentioned above. In addition, the singularities of $f(z_1, z_2)$ in either variable are confined to poles and branch-cuts lying on the *real* axes in z_1 or z_2. The mass of a 'particle' in this picture happens to be directly related to the location of the pole. The 'particle' of course may be elementary or composite, but it must have quantum numbers corresponding to one of the three channels described by $f(z_1, z_2)$. In the same way, the residue at the pole is interpreted as a product of the two coupling constants (renormalized) operative at the respective vertices connected by the propagator for the 'particle'. In this new approach, this is generally taken as the *definition* of the coupling constant operative at a particular vertex, and it reduces to the field-theoretical definition in the Born approximation. Finally, the discontinuities across the branch-cuts are interpreted, via unitarity, as proportional to the cross-sections for the various processes.

These ideas have received a thorough and systematic treatment at the hands of Chew (*see* Chew 1960) who, in collaboration with Mandelstam, has given a complete theory of pion-pion scattering (Chew and Mandelstam 1960).

It may be noted that the π-π problem plays a special role in this theory for two reasons: (1) for the π-π system, the 'substitution rule' gives rise only to π-π scattering amplitudes for all the 3 channels, so that their discontinuities can all be related to *physical* π-π cross-sections alone, (2) the π-π system being the state of lowest mass for a scattering problem, the branch-cut corresponding to their physical amplitude is the first to appear as we go along the real axis in z_1 or z_2. In so far as therefore the discontinuities across the line singularities are restricted by unitarity, the higher mass branch-cuts may be taken to give smaller contributions at moderate energies. This gives rise to a scheme of successive approximations, by virtue of which one first solves the π-π problem entirely in terms of itself (neglecting the higher mass singularities) and then goes on towards the solution of more 'massive' system like π-N, N-N which successively need higher and higher mass states. In this spirit, only the first stage of the work has really been carried out so far. The second stage (π-N problem) is still in the offing (*see* e.g. Frazer and Fulco 1960).

While these techniques of Chew and Mandelstam have proved quite useful for two-particle system, they are not extendable to amplitudes involving 3 or more particles (e.g. production amplitudes). A major step in this direction was provided by the work of Landau (1960) whose techniques are based essentially on Feynman graphs in a manner strongly reminiscent of perturbation theory. Landau's idea is that whether or not field theory is valid, at least the 'near' singularities of the various amplitudes are given correctly by the Feynman denominators of their perturbation theoretic expressions calculated from field theory. As for the 'strengths' of these singularities Cutkosky (1960) has found a prescription for relating the discontinuities across the branch-cuts to physically observable quantities (e.g. production cross-sections) so that in principle the integrand of a dispersion integral is expressible in terms of the latter and at most some coupling constants. Although there are still many loop-holes in the formalism as it stands now, this 'graphical calculus', as Landau puts it, seems capable of providing some answer to the long-standing question of 'how to treat interactions in practice'.

This new approach towards physical problems, using the analyticity properties of the S-matrix, is appealing in a number of ways. The ideas contained in the formalism were no doubt visualized through (approximately derived) results in field theory. Yet it is quite likely that there is a good deal of truth in these concepts, though we do not as yet know how to derive them from more fundamental premises, which according to persons like Landau should form a more satisfactory alternative to field theory. It may be noted that some of the general concepts on which local field theory is based, viz. Lorentz isotopic invariance, causality and unitarity, are also incorporated in this new approach. The additional assumption needed to make the theory more specific is the requirement of analyticity. The most pressing question in this context is therefore whether this additional requirement of analyticity really gives the theory a fundamental dynamical content on par with, say, quantum electrodynamics. There is a certain group of opinion (*see* e.g. Chew 1960) which believes that the principle of *maximal analyticity* should suffice to give the theory a complete dynamical content. Of course in so far as such a principle has not yet been satisfactorily formulated, the theory in its present form has less input information than one with a specific field theoretic Lagrangian. There are, on the other hand, people (Gell-Mann, Sakurai) who believe that such a theory can never hope to substitute the information contained in a concrete Lagrangian formalism.

In very recent times some notable attempts have been made toward understanding strong interactions within the premises of local field theory. One of these is due to Sakurai (1960) who suggests that instead of looking for higher symmetries like global symmetry (described earlier in this article), the existing symmetries should be taken as exact results and the theory so formulated as to bring these about explicitly. Now the quantities known to be exactly conserved under strong interactions are (1) the so-called baryonic charge B (number of baryons minus the number of antibaryons), (2) isospin T, and (3) the hypercharge $Y \equiv S+B$. Sakurai then uses a generalization of a result due to Yang and Mills (1954) for the conservation of T (and of course a much earlier one known in quantum electrodynamics for conservation of electric charge), viz. whenever a gauge group of the first kind exists there must exist a *vector field*, coupled uniquely to the particles obeying the above gauge invariance, and that the original gauge group (of first kind) must be enlarged to incorporate the gauge transformation of the second kind. He, therefore, associates with the above 'internal' conservation laws, appropriate gauge transformations of the first kind, and requires that these should be consistent with the local field concept. In this way he is led to the introduction of 3 distinct vector fields which are respectively the carriers of B, T and Y, in much the same way as the vector e.m. field is the carrier of electric charge. The carriers of B and Y are isoscalars and that of T an isovector. As a result, the interaction Lagrangian postulated by Sakurai has the form

$$L_{\text{Int}} = L_T + L_Y + L_B ; \quad \quad .. \quad .. \quad .. \quad .. \quad (3)$$

$$L_T = -f_T \sum_{\alpha=1}^{3} B_{\mu\alpha}^{(T)} J_{\mu\alpha}^{(T)} ; \quad \quad .. \quad .. \quad .. \quad .. \quad (4)$$

$$L_Y = -f_Y B_{\mu}^{(Y)} J_{\mu}^{(Y)} ; \quad \quad .. \quad .. \quad .. \quad .. \quad (5)$$

$$L_B = -f_B B_{\mu}^{(B)} J_{\mu}^{(B)} . \quad \quad .. \quad .. \quad .. \quad .. \quad (6)$$

Here, the J_μ's are the currents to which the various B-fields are coupled.

The essential point of this theory is the recognition that the carriers of such fundamental attributes like B, T and Y must be vector fields. Thus the (pseudoscalar) pion field should not be expected to serve as the carrier of 4-dimensional isospin or hypercharge (*see* e.g. Schwinger 1957); for, if such were the case, the corresponding (global) symmetry would be broken violently by other (kaonic) interactions. In Sakurai's 'vector' picture, on the other hand, the internal attributes are designed to be exactly conserved no matter how strong the interactions with other (unknown) fields may be.

The theory is extremely simple in applications and gives very sensible results even in perturbation theory. A good number of its predictions already seem to be in fair accord with some experimental results hitherto difficult to understand on the 'older' premises. Thus the theory can account, in a rather natural way, for (1) the isospin anomaly in S-wave π-N scattering phase shifts, (2) the short-range spin-orbit potential in N-N scattering, (3) the pion multiplicity in N-$\overline{N}$ annihilation, to mention only a few. The only snag in the vector theory so far is that it is difficult to reconcile finite masses ($> m_\pi$) for these vector mesons (which is required by experiment) with the theoretical requirement of zero mass fields as carriers of internal attributes. However it may be quite possible to overcome such conceptual difficulties in the near future.

Sakurai's theory has put vector fields hitherto 'disliked', because of unrenormalizability, in an altogether different light. The theory brings out rather clearly the fact that the positive quality of a vector field (viz. it represents the only possible way for the dynamical manifestation of an internal attribute) more than offsets its disadvantages due to unrenormalizability. The possibility of a deeper connection between strong and weak interactions (hitherto conjectured by various people) also seems capable of realization in this approach, since the theory describes the effective phenomenological interaction between any two fields connected by any one of the 'carrier' vector fields, as $\vec{j}_\mu(1) \cdot \vec{j}_\mu(2)$, reminiscent of the famous V-A theory of weak interactions. In short, Sakurai's theory serves to dispel the widespread impression that conventional field theory can no longer 'deliver the goods', and that one must necessarily look to other approaches. Rather, what appears to have been wrong is not so much with the general premises of field theory, but with the view that π- and K-fields are the basic carriers of strong interactions through a Yukawa-type γ_5 coupling. It may therefore be hoped that local field theory need not necessarily be given up in favour of other (unknown) premises, before a fair trial has been given to a more satisfactory formulation within the jurisdiction of the former.

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