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Trends and the pole tide in Indian tidegauge records

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Abstract. This paper studies tidegauge records of stations on the Indian coastline. An analysis of trends did not reveal a monotonic trend. Trends were seen for limited periods at only five of the eight stations on the Indian coast. A spectral analysis of annual records produced evidence of long period cycles with shorter cycles riding on them. The shorter cycles had a period of 5·0 years. The spectra of monthly records revealed evidence of a pole tide and an annual cycle. The amplitude of the pole tide was estimated to be around 7·5 mm. This was larger than the equilibrium tide. A spectral analysis of monthly rainfall at Bombay, a station on the Indian west coast, also showed a 13·9 month cycle and a (3, 1, 0) autoregressive model. But the coherence between monthly rainfall and relative sealevel fluctuations was low.

Keywords. Trends; sealevel; spectral analysis; pole tide.

1. Introduction

There is considerable interest on relative sealevel fluctuations because of the recent concern about global warming. Tidegauge records have revealed another phenomenon of meteorological interest. This is an approximately 14 month oscillation of the earth's pole round its axis of the largest moment of inertia. It is referred to as the pole tide. This is caused by a wobble in the earth's axis of rotation. The wobble is named the Chandler Wobble after its discoverer. Many attempts have been made in the past to find its source of excitation, but Indian tidegauges do not appear to have been considered in detail. Consequently, our purpose will be to (a) identify trends in Indian records and (b) to see if there is evidence of a pole tide.

2. Data

In an earlier paper (Das and Radhakrishna 1991) we have examined the records of four Indian stations. The data were provided by the Survey of India with suitable corrections for changes in location. Subsequently, we obtained data which were reduced to a common datum. The monthly and annual mean observations adjusted in this manner form the "Revised Local Reference (RLR)" data base of the Permanent Service for Mean Sea Level (PSMSL) at Bidston observatory in the United Kingdom. This data set will be used in the present paper to minimise inconsistencies due to changes in location. Consequently, the results presented here will be a little different from those obtained earlier.

3. Trends

Different methods have been used to determine trend. By fitting linear regression lines, Emery and Aubrey (1989) found trends ranging from 1·3 to −2·1 mm/yr, with an average of −0·5 mm/yr, for five Indian stations. This suggests a monotonic trend for the entire span of each record which need not be the case. Douglas (1991) used a multi-year sliding filter of 5 years on successive parts of the record. We have used a different approach in this paper.

We used a Mann-Kendall test (Kendall and Stuart 1961) to find out if a trend could be found that was statistically significant. This could be only for a part of the total record.

Mann-Kendall's test finds out whether the n-observations of a time series are independent of each other by counting inversions.

Let $U_1, U_2, \ldots U_n$ be individual observations. For any observation U_i, the number of inversions in the sequence preceding it is represented by the number of occasions when $U_j > U_i$ for $j < i$.

If n_i is the number of inversions then the test statistic

$$S_k = \sum_1^k n_i \tag{3.1}$$

is a measure of trend. A null hypothesis of all observations being independent implies a normal distribution of S_k. Its mean and variance are expressed by

$$\mu = k(k-1)/4 \tag{3.2}$$

and

$$\sigma^2 = k(k-1)(2k+5)/72 \tag{3.3}$$

To test departures from a normal distribution let

$$Z = (S_k - \mu)/\sigma \tag{3.4}$$

The 95% confidence limits of Z are then $-1{\cdot}96 < Z < 1{\cdot}96$. If the computed values of Z exceeded these limits then a trend was indicated. When the values of Z are significant an increasing or decreasing trend is indicated by $Z > 0$ or $Z < 0$. The advantage of this test is that when a time series shows a significant trend then it is possible to locate the period for which a trend exists. Thus, there might be a trend for only a limited period of the entire time span and this will not be revealed by fitting a linear trend over the entire span of a record.

The data were smoothed by a Gaussian ordinate filter (WMO 1966; Sneyers 1990). Weights were applied to successive terms of a series which were proportional to the ordinates of a Gaussian curve. The filter captured over 95% of the variance over a ten year period round the central frequency. Finally, we computed the standard error of the trend. Tushingham and Peltier (1991) computed approximate values of the Post-Glacial Rise (PGR). These are vertical displacements of sea level due to the collision of the Indo-Australian and Eurasian plates computed by a numerical model (ICE-3G). Values of differences between the trend (T) and the PGR were also computed to compare our results with those of Douglas (1991).

There were 8 Indian stations where the length of the PSMSL records exceed 20 years. They are shown in table 1. The last column of the table shows the period for

Table 1. Stations with records exceeding 20 years.

(1) Station	(2) Latitude (°N)	(3) Longitude (°E)	(4) Time span (years)	(5) Trend period (years)
1. Bombay	18·6	72·5	1878–1987 (109)	1940–1987 (47)
2. Cochin	9·6	76·2	1939–1977 (38)	1951–1977 (26)
3. Diamond Harbour	22·1	88·1	1954–1987 (39)	1965–1987 (22)
4. Garden Reach	22·3	88·1	1954–1987 (33)	1974–1987 (13)
5. Kandla	23·1	70·1	1950–1987 (37)	1957–1987 (30)
6. Madras	13·6	80·2	1953–1976 (23)	0
7. Saugor	21·4	88·3	1945–1987 (32)	1965–1987 (22)
8. Vizag	17·4	83·1	1937–1986 (49)	0

Table 2. Trends and standard errors.

(1) Station	(2) Trend (mm/yr)	(3) Standard error (mm)	(4) Trends (Douglas 1991) (mm/yr)	(5) PGR (mm/yr)	(6) T-PGR (mm/yr)
1. Bombay	−0·3	0·1	−0·3	−0·2	−0·1
2. Cochin	2·2	0·8	2·2	−0·5	2·7
3. Diamond Harbour	7·8	0·7	5·9	−0·4	8·2
4. Kandla	4·8	0·9	—	−0·2	5·0
5. Saugor	−1·9	1·1	−4·4	−0·3	−1·6
Mean	2·5	0·7	0·9	−0·3	2·8

which a significant trend was found by Mann-Kendall's test. This will be referred to as the trend period.

The figures in parenthesis indicate the number of years. We find from the last column of table 1 that a trend period exceeding 20 years exists in only five of the eight stations. The trends in these five stations are shown in table 2.

Table 2 brings out a few interesting features. First, we note the negative trends at Bombay and Saugor. Despite the fact that the trend values of Douglas (1991) were for the periods 1930 to 1980, there is a general agreement between his values and ours. The large values of trend at Diamond Harbour are, we feel, due to sedimentation because the station is located in a region which has a high frequency of tropical cyclones and storm surges. From column (5) of table 1 we find that significant trends generally begin after 1950 with the exception of Bombay which starts around 1940 and this need not necessarily be an outcome of global warming because (a) only 5 out of the 8 stations in table 1 show a significant trend and (b) negative trends are observed at Bombay and Saugor. Douglas (1991) did not consider Kandla which, unlike Bombay and Saugor, has a large rising trend. Our overall conclusion is that there is still considerable uncertainty about a rise in sea level due to global warming over the Indian region. A few stations were also considered in the neighbourhood of India but the trend periods in their records were much less than 20 years which we feel is necessary to establish a significant trend.

4. The pole tide

4.1 *Annual means*

A spectral analysis was made by (a) Discrete Fourier Transforms (DFT) with a Parzen window and (b) by the Maximum Entropy Method (MEM) of Burg (1967). After removing high frequency noise by a 2 point moving average the data were analysed at levels without differencing. Most of the variance was observed to reside in long cycles with periods exceeding 50 years. But, unfortunately, these long cycles could not be studied in much detail because the longest record (Bombay) at our disposal was only of 100 year duration.

We removed the long period cycles by taking first differences. A diagnostic check was made to confirm the adequacy of first differences for detrending. Box and Pierce (1970) and later Ljung and Box (1979) showed that the statistic

$$\phi(k) = n(n+2) \sum_{k=1}^{K} (n-k)^{-1} r_k^2 \tag{4.1}$$

has a chi-square distribution with K degrees of freedom for a random process. The number of observations is n, k represents lag and r_k is the serial correlation at lag k. If $\phi(k)$ did not exceed the critical value of chi-square for the relevant degrees of freedom at 95% level of confidence then one assumes that the series was detrended by first differences.

The principal spectral peaks that we found are summarised in table 3.

A false alarm probability test (MacDonald 1989; Priestley 1981) was applied to confirm the reliability of the peaks. It ensures that the power in a spectral peak was larger than what would appear from noise. Under a null hypothesis one computes the probability of the peaks being independent. If independently distributed they follow a chi-square distribution with 2 degrees of freedom. The test measures the degree of independence with a specified level of confidence. We found that all the

Table 3. Spectral peaks in annual means.

(1)	(2) Spectral peaks (yr)	
	(i)	(ii)
Station	DFT	MEM
1. Bombay	5·2	4·5
*2. Madras (1)	5·7	4·5
†3. Madras (2)	4·5	3·9
4. Cochin	5·2	5·7
5. Vizag	3·7	3·7
6. Karachi	5·0	4·9
7. Rangoon	3·7	3·7
8. Aden	6·0	5·9
Mean	4·9	4·6

*Madras (1): 1881–1933
†Madras (2): 1953–1986

peaks in table 3 passed the false alarm test at 95% level of confidence except Cochin which was marginally insignificant.

The picture that emerges is one of short period cycles with periods around 5 years riding on the longer period cycles. It was considered whether this could be an outcome of aliasing. The Nyquist frequency (F_N) was 0·5 cpy with time intervals (Δt) of 1 year. The spectra were observed to lie in the frequency range of 0 to 0·5 cpy, with very little power at frequencies higher than F_N. It was thus unlikely that the 5 year cycle was an outcome of aliasing. The Parzen window which we used would also tend to inhibit aliasing.

On the other hand, Hameed and Currie (1989) suggest that the 5 year cycle was the outcome of an interaction between the pole tide and the annual cycle. The frequency of the pole tide is 0·0680 cpm and 0·0833 cpm for the annual cycle. The difference (0·0153 cpm) corresponds to a 5·5 year cycle. This is still a matter of uncertainty; consequently, we tried to see if the pole tide was visible in the monthly records.

4.2 *Monthly records*

In an earlier communication we have reported that the pole tide was observed by DFT analysis, if a 12 point filter was used to suppress the annual cycle. But, on subsequent analysis we found that the filter gave a higher bias to the power in the Chandler frequency or the pole tide.

To improve this we detrended the data and removed seasonal effects as far as possible by subtracting the mean of all Januaries from the observed January and so on for the other months. The residuals were then analysed by MEM which gave a better spectral resolution.

The spectral peaks are shown in table 4.

The length of the prediction error correction filter is shown in column (2) of the above table. Monthly data of Madras (2) were not available at present and the Rangoon monthly records had several missing segments. They were not considered.

The amplitudes in column (4) were determined by computing $(P_f \Delta f)^{1/2}$ where P_f is the power in a unit frequency interval and Δf is the frequency interval covered by the peak (Bolt and Currie 1975).

Table 4. Spectral peaks (MEM): Monthly means

(1)	(2)	(3) Periods (months)			(4) Amplitudes (mm)		
Station	PEC (%)	(i)	(ii)	(iii)	(i)	(ii)	(iii)
1. Bombay	20	14·3	12·7		6·8	6·4	
2. Madras(1)	20	14·3	11·4		8·4	8·7	
3. Cochin	20	16·2	14·0	12·6	7·5	5·1	8·6
4. Vizag	20	13·7			8·6		
5. Karachi	50	13·3			7·8		
6. Aden	50	14·0	12·8		6·0	8·4	
Mean		14·3	12·7		7·5	7·2	

An interesting feature of table 4 was the appearance of 2 peaks at all stations except Vizag. Cochin had 3 peaks. But, the mean periods of the first two peaks (I and II) were close to the frequency of the pole tide and the annual cycle. The amplitudes of (I), the pole tide, were of the right order of magnitude. At these latitudes, the amplitude of the equilibrium tide is 3 mm. The observed amplitudes of the pole tide were a little over 7 mm. This appears to us to be significant.

5. Rainfall and sealevel: Preliminary analysis

We examined the relationship between monthly rainfall and relative sealevel for Bombay, the coastal station with the longest record of relative sealevel. The monthly rainfall data for the period 1901–1986 were examined. This was done to find out if there was a steric rise or fall due to rainfall.

Interestingly, spectral analysis by MEM did reveal a prominent peak at 13·9 months in Bombay rainfall, after the seasonal component had been removed and the series was detrended by taking first differences. The spectrum is shown in figure 1. As before, the monthly records did not provide a meaningful spectrum when the DFT algorithm was used.

Another similarity between monthly rainfall and relative sealevel appeared when autoregressive (ARIMA) models were fitted to the data. We found that a (3, 1, 0) model fitted the data best. This was checked by applying the Kolmogorov-Smirnov test to the residuals after removing the model generated data. The test seeks to find out if the residuals satisfy the criterion for white noise, which they should if the model was good. As reported earlier (Das and Radhakrishna 1991) Bombay and Madras (2) were the only two Indian records which fitted a (3, 1, 0) autoregressive process. The other records did not show a similar feature. Data from the stations in table 2 were of too short a duration for designing a good model.

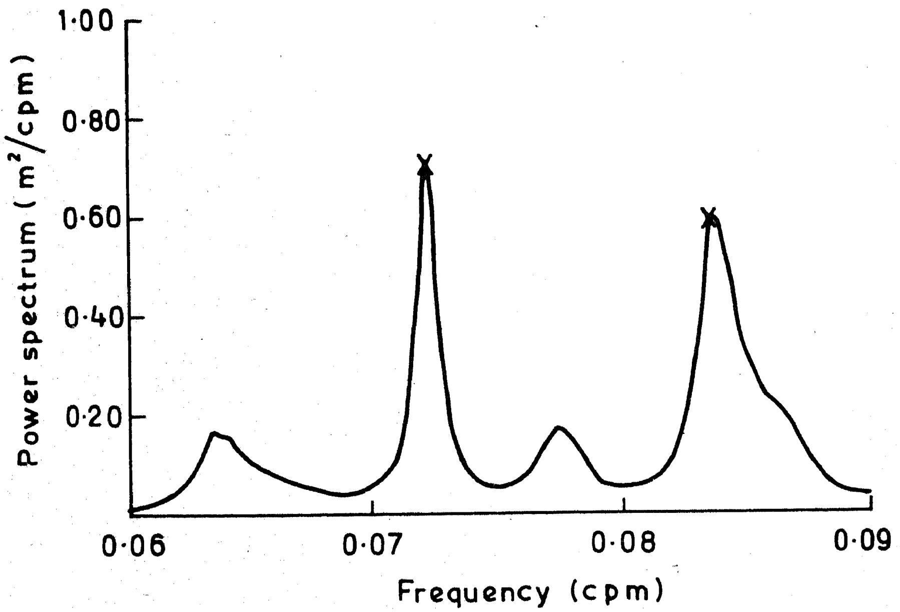

Figure 1. Power spectrum of Bombay rainfall. Crosses indicate the Chandler and the annual frequencies.

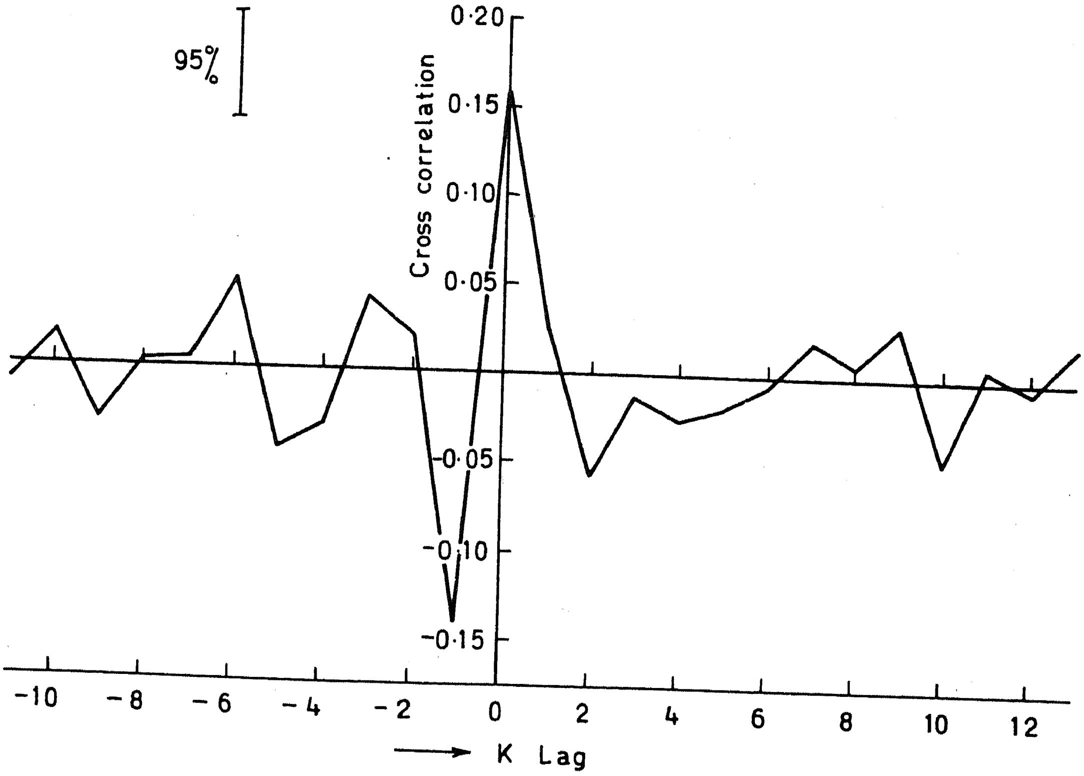

Figure 2. Cross spectrum of monthly rainfall and relative sea level at Bombay.

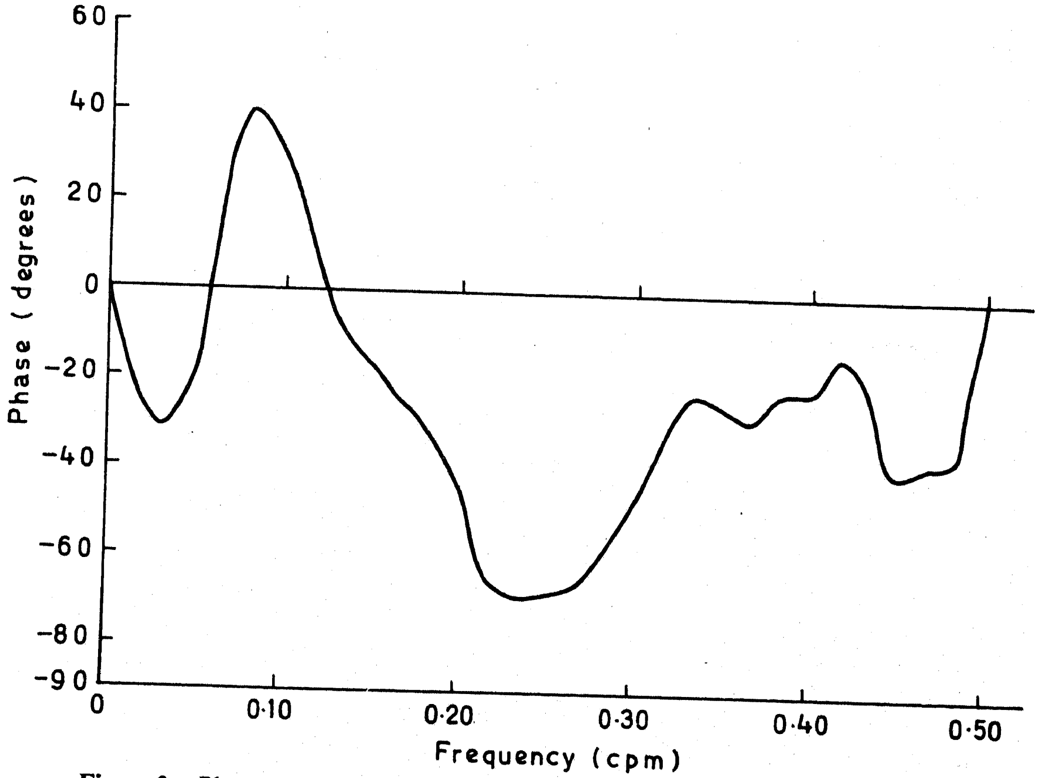

Figure 3. Phase spectrum of monthly rainfall and relative sea level. The Chandler and annual frequencies fall in a zone of positive phase.

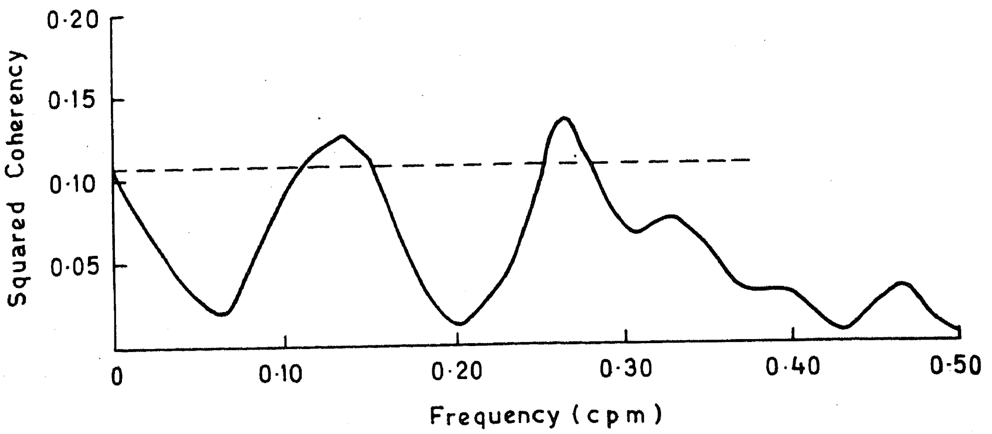

Figure 4. Squared coherency. Dotted line represents threshold of significant coherency.

Unfortunately, the relationship between rainfall and relative sealevel was not encouraging when the coherency between the two was examined. Figure 2 shows the cross correlation spectrum of monthly records. The correlation was small (0·15) even for small lag. The phase spectrum is shown in figure 3. It was interesting to note that both the annual and the frequency of the pole tide fall within a narrow band of positive phase. At other frequencies the phase was largely negative.

In figure 4 we show a plot of the coherency square against frequency. If we compute the degrees of freedom (ν) as by Munk and MacDonald (1960), that is, by twice the number of harmonics used to find the spectral density, then a value of coherency above $2\nu^{-1/2}$ indicates a useful phase relation between the two records. From figure 4 we found that the coherency was not significant at the frequency of the pole tide and the annual cycle. There could be several reasons for this. First, we examined the rainfall records of only one station. Secondly, the co-spectrum and the quadrature-spectrum were computed by using discrete Fourier transforms. As noted earlier, this did not provide us with a good spectrum for monthly records. Consequently, we need to extend MEM for this purpose. Lastly, as noted in an interesting study by Thompson (1980) the static pressure and local winds also need to be studied.

6. Summary

The main results of the present study may be summarised as follows:

- There was little evidence of a monotonic rising trend in annual tidegauge records. A rising trend was only visible for limited periods in the entire time span of three stations.
- Spectral analysis of annual records showed a large amount of variance residing in long cycles with periods exceeding 50 years. Shorter cycles with periods around 5 years were found riding on the larger cycles.
- The monthly records showed evidence of the pole tide and an annual cycle when analysed by the maximum entropy technique.
- Monthly rainfall records of Bombay also revealed a 13·9 month cycle. A (3, 1, 0) autoregressive model best fitted the monthly rainfall series. Relative sealevel fluctuations also fitted a (3, 1, 0) autoregressive model.

- There was little coherence between monthly rainfall at Bombay and sealevel fluctuations, although the two appeared to be in phase.

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