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Holograms of Branes in the Bulk and Acceleration Terms in SYM Effective Action

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If the AdS/CFT correspondence is valid in the Coulomb branch, the potential between waves on a pair of test branes in the bulk should be reproduced by the relevant Yang-Mills theory effective action on the boundary. Earlier work has provided evidence for this in the case of constant gauge field brane waves. In this paper we provide concrete evidence for an earlier proposal that the effects of exchange of supergravity modes with nonzero momentum in the brane directions are encoded in certain terms involving derivatives of the field strength in the gauge theory effective action. We explicitly calculate the force quadratic in the field strengths coming from the exchange of non-zero momentum two form fields between two 3-branes in $AdS_5 \times S^5$ to lowest nontrivial order in the momentum. We show that this is exactly the same as that between the branes living in flat space. The result is in agreement with the gauge theory effective action and consistent with the non-renormalization property of this term. We comment on the relationship of other "acceleration" terms in the SYM effective action with quantities in supergravity.

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1. Introduction.

The duality between Type IIB superstring theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in $3+1$ dimensions [1] [2] is most commonly studied at the conformally invariant point. Recently there has been some evidence that the duality is valid in the Coulomb branch [3] as well [3] [4] [5]. A simple setting for this is to study the force between waves on a pair of test 3-branes in the bulk of $AdS_5 \times S^5$ [3]. On the supergravity side this force is due to exchange of massless supergravity modes propagating in $AdS_5 \times S^5$, while on the Yang-Mills side this is read off from an effective action obtained by intergrating out fluctuations around the relevant expectation values of the scalars (representing the positions of the test branes) and other fields (representing the waves on the branes). Consider for simplicity brane waves made of gauge fields alone. Let y^a denote coordinates along the brane directions and Z^i denote the coordinates transverse to the branes. A pair of branes located at positions Z_1^i and Z_2^i is represented in the $U(N)$ Yang-Mills theory by the constant expectation value of the Higgs field ϕ^i

$$\phi^i = \begin{pmatrix} z_1^i & 0 & 0 \\ 0 & z_2^i & 0 \\ 0 & 0 & \mathbf{0}_{(N-2)\times(N-2)} \end{pmatrix} \tag{1.1}$$

where the expectation values z^i are related to the physical locations Z^i by

$$z^i = Z^i \; l_s^{-2} \tag{1.2}$$

where l_s is the string length. The gauge field excitations $(F_1)_{ab}$ and $(F_2)_{ab}$ on the two branes are represented by a $U(N)$ gauge field

$$F_{ab}(y) = \begin{pmatrix} (F_1)_{ab}(y) & 0 & 0 \\ 0 & (F_2)_{ab}(y) & 0 \\ 0 & 0 & \mathbf{0}_{(N-2)\times(N-2)} \end{pmatrix} \tag{1.3}$$

When $N = 2$ this represents branes in flat space, while for large N and in the Maldacena scaling limit these are branes in $AdS_5 \times S^5$.

When the gauge fields on the branes are constant, the terms in the one-loop effective action in the gauge theory involving cross-terms in F_1 and F_2 reads

$$\frac{g_{YM}^4}{\rho^4}[O_1^\phi O_2^\phi + O_1^\chi O_2^\chi + 2T_1^{ab}T_{2\;ab} + 2O_1^{ab}O_{2\;ab}|_4] \tag{1.4}$$

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[3] We will consider the gauge theory to live on $R^{(3,1)}$ rather than $S^3 \times R$ so that there is a Coulomb branch.

where g_{YM} is the Yang-Mills coupling constant and for each $i = 1, 2$

$$\begin{aligned}
O_i^\phi &= \frac{1}{4}(F_i)^{ab}(F_i)_{ab} \\
O_i^\chi &= \frac{1}{8}\epsilon_{abcd}(F_i)^{ab}(F_i)^{cd} \\
T_i^{ab} &= \frac{1}{2}[(F_i)^a_c(F_i)^{cb} - \frac{1}{2}\eta^{ab}(F_i)^{cd}(F_i)_{cd}] \\
O_i^{ab} &= \frac{1}{2}[(F_i)^{ab} + (F_i)^{ac}(F_i)_{cd}(F_i)^{db} - \frac{1}{4}(F_i)^{ab}(F_i)^{cd}(F_i)_{cd}]
\end{aligned} \tag{1.5}$$

while

$$\rho^2 = \sum_{i=1}^{6}(z_1^i - z_2^i)^2 \tag{1.6}$$

In the above expressions we have normalized the fields so that the kinetic term in the gauge theory action does not contain any power of g_{YM}. In (1.4) the subscript $|_4$ means the term with four factors of the gauge field.

Note that the one loop result is identical for $SU(2)$ and $SU(N)$ since the particles which run around the loop must be charged under both the $U(1)$'s in which z_1, z_2, F_1, F_2 lie. For $SU(2)$ this one loop result is known to be exact [6]. For $SU(N)$ this is also exact at a generic point on the Coulomb branch when one includes a sum over the various $U(1)$ factors [7]. For our situation, in which $SU(N) \to SU(N-2) \times [U(1)]^2$ these nonrenormalization theorems are still expected to be valid [3]. This means that the result (1.4) is valid at strong coupling where a comparison with supergravity can be made.

For $SU(2)$ the various terms in (1.4) have a straightforward supergravity interpretation in terms of exchange of supergravity modes with zero momentum in the brane directions. Note that ρ is related to the transverse distance ΔZ between the branes by

$$\Delta Z = \rho l_s^2 \tag{1.7}$$

while the 10d gravitational coupling constant κ is related to the gauge coupling by (upto a numerical constant)

$$\kappa = g_{YM}^2 l_s^4 \tag{1.8}$$

Thus

$$\frac{g_{YM}^4}{\rho^4} = \frac{\kappa^2}{(\Delta Z)^4} \tag{1.9}$$

However $1/(\Delta Z)^4$ is the static Coulomb propagator in the flat six dimensional transverse space, while the operators O^ϕ, O^χ, T^{ab} and O^{ab} are related to the operators $\tilde{O}$ in the brane action which couple to the dilaton, RR scalar, longitudinally polarized (ten dimensional) graviton and longitudinally polarized 2-form fields respectively by

$$\tilde{O}^\phi = \kappa O^\phi \qquad \tilde{O}^\chi = \kappa O^\chi \qquad \tilde{T}^{ab} = \kappa T^{ab}$$
$$(\tilde{O}_i^{ab})^{flat} = \frac{\sqrt{\kappa}}{2}(F_i)^{ab} + \frac{\kappa^{3/2}}{2}[(F_i)^{ac}(F_i)_{cd}(F_i)^{db} - \frac{1}{4}(F_i)^{ab}(F_i)^{cd}(F_i)_{cd}] \tag{1.10}$$

where the supergravity fields have been normalized such that there is no power of κ in their kinetic terms. Using the above formulae it may be checked that the gauge theory answer for the $O(F^4)$ terms is exactly of the same form as the supergravity answer. This is similar to the way velocity dependent forces between branes are reproduced from the relevant gauge theory and has played an important role in M(atrix) theory [8] In particular, the factors of g_{YM} and ρ in the former give the correct factors of κ and Δz in the latter.

For $SU(N)$ and in the scaling limit of [1], the result is due to exchange of supergravity modes propagating in $AdS_5 \times S^5$, but we expect that the final answer is the same as that in flat space ! In [3] it was shown that the zero (brane) momentum propagators for the dilaton, RR scalar and the longitudinal graviton [4] in $AdS_5 \times S^5$ are the same as those in flat space. Their couplings to the brane fields are also identical in $AdS_5 \times S^5$ and flat space, which is related to the fact that they couple to dimension four operators. This explains why the first three terms in (1.4) come from supergravity exchange in $AdS_5 \times S^5$.

In contrast, the propagators of the 2-form fields in $AdS_5 \times S^5$ are quite different from those in flat space [4], even at zero momentum. In particular the $NS - NS$ and $R - R$ 2-forms mix with each other in the $AdS_5 \times S^5$ background via the five form background field.

Let us use the standard form of the $AdS_5 \times S^5$ metric

$$ds^2 = (\frac{r}{R})^2[dy \cdot dy] + (\frac{R}{r})^2[dr^2 + r^2 \sum_{i=1}^{5} f_i(\theta_i)(d\theta_i)^2] \tag{1.11}$$

We will use the following conventions. The ten dimensional coordinates will be denoted by $y^\mu, \mu = 0, \cdots 9$. Out of these we denote the brane worldvolume directions by $y^a, a = 0, \cdots 3$. The remaining six transverse coordinates $y^5 \cdots y^9$ will be relabelled as $Z^i, i = 1, \cdots 6$. $r = \sqrt{\sum_{i=1}^{6}(Z^i)^2}$ is the radial coordinate in the transverse space and θ_i are angles on the S^5. (r, θ_i) are related to the cartesian coordinates Z^i in the transverse space by

[4] These are all minimally coupled scalars in the transverse space

the standard transformations and the metric coefficients $f_i(\theta_i)$ are determined from these transformations.

We combine the two 2-form fields into a complex field $B_{\mu\nu}$

$$B_{\mu\nu} = B_{\mu\nu}^{NSNS} + iB_{\mu\nu}^{RR} \tag{1.12}$$

Then the propagator with zero momentum along y^a is given by [4]

$$\begin{aligned} G^{(0)}_{ab,cd}(Z_1, Z_2) &= \int d^4y < B^*_{ab}(y, Z_1) B_{cd}(y, Z_2) > \\ &= \frac{1}{8\pi^3 |Z_1 - Z_2|^4} [a(Z_1, Z_2)(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc}) + b(Z_1, Z_2)\epsilon_{abcd}] \end{aligned} \tag{1.13}$$

where

$$\begin{aligned} a(Z_1, Z_2) &= (\frac{r_1}{R})^4 + (\frac{r_2}{R})^4 \\ b(Z_1, Z_2) &= i[(\frac{r_1}{R})^4 - (\frac{r_2}{R})^4] \end{aligned} \tag{1.14}$$

Note that the propagator depends on the individual brane locations. In flat space one has

$$G^{(0)}_{ab,cd}(Z_1, Z_2) = \frac{1}{4\pi^3 |Z_1 - Z_2|^4} [\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc}] \tag{1.15}$$

In a similar way, the couplings of these modes to the brane fields are also different in $AdS_5 \times S^5$ [9] - this is related to the fact that these couple to dimension six operators. The operator $(\tilde{O}_i^{ab})^{flat}$ is now modified to

$$(\tilde{O}_i^{ab})^{Ads} = \frac{\sqrt{\kappa}}{2}(F_i)^{ab} + \frac{\kappa^{3/2}}{2}(\frac{R}{r})^4[(F_i)^{ac}(F_i)_{cd}(F_i)^{db} - \frac{1}{4}(F_i)^{ab}(F_i)^{cd}(F_i)_{cd}] \tag{1.16}$$

This also depends on the individual brane locations.

Nevertheless it was shown in [4] that the final result for the interaction potential between the branes with constant gauge fields on them is identical to that in flat space and hence reproduced precisely by the Yang-Mills effective action. This happens due to cancellations which are not quite understood. In fact, even in flat space, the operator O^{ab} which couples to the 2-form field contains a term linear in the field strength and one might expect a term which is quadratic in F_{ab}. From the gauge theory point of view such a term is not present - a reflection of the nonrenormalization of the coupling constant. In the supergravity calculation this again comes about due to a cancellation between the NS-NS and R-R exchanges and this persists in $AdS_5 \times S^5$. These results provide rather non-trivial evidence for validity of the AdS/CFT correspondence in the Coulomb branch.

When the waves on the branes are not constant, supergravity modes which are exchanged will carry nonzero momentum in the brane directions and the result will be sensitive to causal propagation in the bulk. However two branes situated at the same point in S^5 and separated along the "radial" direction in AdS_5 correspond to the same location in the boundary gauge theory and one might wonder how the boundary theory knows about causal propagation in the bulk.

In [4] it was proposed that some of these effects are encoded in certain terms in the Yang-Mills effective action which involve derivatives of the gauge fields. We will refer to such terms as "acceleration terms" since similar terms with the gauge fields replaced by the Higgs field represent accelerations of the probe branes in the transverse space. In this context such terms were studied in [10]. The simplest such term involving only gauge fields in fact comes from the exchange of the 2-form field. For waves on branes situated in flat space which are dependent only on time (and not the other brane coordinates) it was shown in [4] that supergravity predicts a term proportional to $(\partial F)^2$ in the effective action. This comes from the expansion of the (brane) momentum space propagator of the 2-form field around the static propagator. Such a term is indeed present in the same multiplet as the F^4 term [11] [12] and is also one-loop exact, which makes a comparison with supergravity meaningful.

While these results are indicative, they do not establish the proposal even for this term. First, with the brane waves dependent on time alone, the detailed Lorentz structure of the term is invisible while the gauge theory predicts a specific Lorentz structure. Secondly the above calculation was performed in flat space - i.e. with no other brane present - where the relevant gauge theory is $SU(2)$. The nonrenormalization of this specific one loop term in the gauge theory predicts that the same answer should follow in $AdS_5 \times S^5$ and it is not clear whether the cancellations which made the F^4 terms work are operative in this context.

In this paper we provide further evidence in favor of this proposal. We perform an explicit calculation of the $(\partial F)^2$ term in the supergravity potential between branes in $AdS_5 \times S^5$. We show, again due to remarkable cancellations, that the result is the same as that between branes in flat space and in exact agreement with the predictions of Yang-Mills theory. This particular term vanishes when the gauge fields are on-shell (i.e. $\partial_\mu F_i^{\mu\nu} = 0$), but is present in an off-shell $N = 2$ supersymmetric effective action [12]. As we shall see, the supergravity calculation requires that the *total* current which couples to the 2-form field is divergenceless. This means that this particular $(\partial F)^2$ term cancels with terms higher order in F which come from other terms in the divergenceless current. Thus in SYM effective action there must be cancellations between terms with different powers of the gauge fields ! We then examine other possible derivative terms (which generally survive on-shell) in the Yang-Mills effective action and comment whether these terms should follow from supergravity.

2. The strategy

Consider the contribution to the force between the two probe 3-branes coming from the exchange of a single supergravity mode. Symbolically the interaction action is given by

$$S_{eff}^{sugra} = \int d^4y \int d^4y' \ \ [J^I(y,Z_1)\Delta_{IJ}(y-y',Z_1,Z_2)J^J(y',Z_2) + (Z_1 \to Z_2)] \tag{2.1}$$

Here $J^I(y,Z_1)$ denotes the current coupling to the supergravity mode labelled by I on the brane located at Z_1 and similarly for $J^J(y',Z_2)$. $\Delta_{IJ}(y-y',Z_1,Z_2)$ denotes the (retarded) propagator matrix of the supergravity modes between the points (y,Z_1) on the first brane and (y',Z_2) on the second brane. In writing down (2.1) we have used translation invariance along the brane directions y^a. With only gauge fields excited, these currents are some expressions made out of these gauge fields. We want to see whether S_{eff}^{sugra} is reproducible in a gauge theory calculation.

The expression S_{eff}^{sugra} is of course an integral of a bi-local quantity, whereas the gauge theory effective action is a sum of local terms arranged usually as a derivative expansion. To make a connection between the two it is easiest to go to momentum space on the brane but remain in the transverse position space. We thus write

$$\begin{aligned} J^I(y,Z) &= \int d^4p \ e^{-ip\cdot y} J^I(p,Z) \\ \Delta_{IJ}(y-y',Z,Z') &= \int d^4p \ e^{-ip\cdot(y-y')} \ \Delta_{IJ}(p,Z,Z') \end{aligned} \tag{2.2}$$

Then (2.1) becomes

$$S = \int d^4p \ (J^I)^*(p,Z_1)\Delta_{IJ}(p,Z_1,Z_2)J^J(p,Z_2) + (Z_1 \to Z_2) \tag{2.3}$$

We can now expand the propagator in powers of momentum and get an expression which (after transforming back to position space on the brane) becomes an expansion in derivatives of the currents.

$$S = \int d^4 y_0 [J^I(y_0, Z_1) \Delta^{(0)}_{IJ}(Z_1, Z_2) J^J(y_0, Z_2) \\ + \frac{1}{2} (\partial_{y^a} J^I(y_0, Z_1))(\partial_{y^b} J^J(y_0, Z_2)) \Delta^{ab}_{IJ}(Z_1, Z_2) + \cdots] \tag{2.4}$$

where

$$\Delta^{(0)}_{IJ}(Z_1, Z_2) = \int d^4 s \Delta_{IJ}(s; Z_1, Z_2) \tag{2.5}$$

is the zero momentum propagator and

$$\Delta^{ab}_{IJ}(Z_1, Z_2) = -[\frac{\partial^2}{\partial p_a \partial p_b} \Delta_{IJ}(p; Z_1, Z_2)]_{p=0} \tag{2.6}$$

The ellipses in (2.4) denote terms with higher derivatives on the currents.

For such a supergravity calculation to be reproducible in gauge theory one must be able to calculate at strong couplings in the gauge theory. At present the only possibility is to concentrate on terms in the effective action which are not renormalized, so that a perturbative calculation in the gauge theory actually yield the strong coupling answers. Indeed there is such a term in the $N = 2$ off-shell effective action which is quadratic in the field strength [12]

$$\int d^4 y \frac{1}{\rho^2} [(\partial_a F^{ab})(\partial^c F_{cb}) + (\partial_a \tilde{F}^{ab})(\partial^c \tilde{F}_{cb})] \tag{2.7}$$

where ρ is defined in (1.6) and $F_{ab} = (F_1)_{ab} - (F_2)_{ab}$ while

$$\tilde{F}_{ab} = \frac{1}{2} \epsilon_{abcd} F^{cd} \tag{2.8}$$

This term is in the same multiplet as the F^4/ρ^4 term and is also one-loop exact. As a result this is identical for $SU(2)$ and $SU(N)$ gauge theories. Therefore it should follow from supergravity calculation of the force between probe branes both in $AdS_5 \times S^5$ and in flat space. In the next section we will show that this is indeed true.

3. Acceleration term from 2-form exchange

Since the only supergravity mode which couples linearly to the gauge fields on the brane is the 2-form field, we need to evaluate terms like (2.4) where the indices (I, J) now refer to those of the 2-form. As already shown in [4], the term quadratic in the field strengths and containing no derivatives vanishes due to cancellations between the NS-NS and RR 2-form exchanges. We are interested in the second term of (2.4).

The piece of the 2-form propagator $< B^*_{ab}(p; Z_1) B_{gh}(p, Z_2) >$ which is quadratic in the brane momenta, which we denote by $G^{(1)}_{ab,gh}(p; Z_1, Z_2)$, is calculated in Appendix A with the final result

$$G^{(1)}_{ab,gh}(p; Z_1, Z_2) = \int d^6 Z \ (\frac{R}{r})^8 \ \frac{1}{|Z - Z_1|^4} \frac{1}{|Z_2 - Z|^4} M_{ab,gh}(p; Z, Z_1, Z_2) \tag{3.1}$$

where

$$\begin{aligned} M_{ab,gh}(p; Z, Z_1, Z_2) =& 2[p^2 a_1 a_2 (\eta_{ag}\eta_{bh} - \eta_{ah}\eta_{bg}) \\ &+ (a_1 a_2 + b_1 b_2)(p_a p_h \eta_{bg} - p_a p_g \eta_{bh} + p_b p_g \eta_{ah} - p_b p_h \eta_{ag}) \\ &+ (b_2 a_1 + b_1 a_2) p^2 \epsilon_{abgh} \\ &+ a_1 b_2 (p_a p^f \epsilon_{bfgh} - p_b p^f \epsilon_{afgh}) \\ &+ a_2 b_1 (p_h p^c \epsilon_{abcg} - p_g p^c \epsilon_{abch})] \end{aligned} \tag{3.2}$$

where we have used a shorthand notation :

$$\begin{aligned} a_1 &= a(Z_1, Z) = \frac{1}{R^4}(r^4 + r_1^4) \qquad a_2 = a(Z, Z_2) = \frac{1}{R^4}(r^4 + r_2^4) \\ b_1 &= b(Z_1, Z) = -i\frac{1}{R^4}(r^4 - r_1^4) \qquad b_2 = b(Z, Z_2) = -i\frac{1}{R^4}(r_2^4 - r^4) \end{aligned} \tag{3.3}$$

This piece of the propagator, like the zero momentum piece, depends on the individual locations of the two branes in a rather complicated way.

The coupling of the 2-form field to the probe three-branes may be read off from the DBI-WZ action of a single three brane in the $AdS_5 \times S^5$ background. This coupling term is [9]

$$\sqrt{\kappa} \int d^4 y \int d^6 Z \ [B^{ab}(y, Z) \mathcal{J}^*_{ab}(y, Z) + c.c.] \tag{3.4}$$

where, in brane momentum space

$$\begin{aligned} \mathcal{J}^{ab}(p) =& [(F_1)^{ab}(p) + \kappa(\frac{R^4}{r_1^4}) {G_1}^{ab}(p) + i(\tilde{F}_1)^{ab}(p)] \delta^6(Z - Z_1) \\ &+ [(F_2)^{ab}(p) + \kappa(\frac{R^4}{r_2^4}) {G_2}^{ab}(p) + i(\tilde{F}_2)^{ab}(p)] \delta^6(Z - Z_2) + \cdots \end{aligned} \tag{3.5}$$

and we have defined the quantity

$$G_i^{ab} = (F_i)^{ac}(F_i)_{cd}(F_i)^{db} - \frac{1}{4}(F_i)^{ab}(F_i)^{cd}(F_i)_{cd} \tag{3.6}$$

and the ellipses denote terms which are $O(F^4)$ and higher. We have to solve the classical equations of motion in the presence of this current using the above propagator. Consistency requires that $\partial_a \mathcal{J}^{ab} = 0$. It is easy to check that the equations of motion for the mixed components of the 3-form field field strength are obeyed by $B_{ij} = B_{ai} = 0$ (as has been assumed) and the solution for B_{ab} obtained from the above propagator, provided this condition is satisfied.

Since we are interested in the term which has the structure $(\partial F_1)(\partial F_2)$, we can ignore the G^{ab} piece in the current. Then the full interaction action, upto terms with two derivatives on the gauge fields is obtained by combining (3.4),(3.5),(2.4)and using the expressions (1.13),(2.6) and (3.1)

$$\begin{aligned} S_{eff}^{sugra} = \kappa \int d^4p[(F_1)^{ab}(p) - i(\tilde{F}_1)^{ab}(p)][G^{(0)}_{ab,gh}(Z_1,Z_2) + G^{(1)}_{ab,gh}(p;Z_1,Z_2)] \\ [(F_2)^{gh}(p) + i(\tilde{F}_2)^{gh}(p)] + c.c. \end{aligned} \tag{3.7}$$

It is easy to check that the term in (3.7) involving $G^{(0)}$ vanishes, as already noted in [4]. The calculation of the second term is outlined in Appendix B.

Remarkably, the dependence on individual brane locations neatly cancel in the final answer and we get

$$S_{eff}^{sugra} = \frac{\kappa}{|Z_1 - Z_2|^2} \int d^4y[(\partial_a F^{ab})(\partial^c F_{cb}) + (\partial_a \tilde{F}^{ab})(\partial^c \tilde{F}_{cb})] \tag{3.8}$$

Using the relationship between the gauge and Yang-Mills couplings and that between the physical distance and the Higgs values given in (1.7) and (1.8) the supergravity expression is exactly the same, upto a numerical factor which we did not calculate, as the gauge theory effective action (2.7).

The result (3.8) for branes separated in $AdS_5 \times S^5$ is in fact identical to the flat space answer. This may be seen by simply noting that in flat space we should put $a_1 = a_2 = 2$ and $b_1 = b_2 = 0$ in the above formulae, as seen from a comparison of the zero momentum propagators (1.13) and (1.14). On the other hand there is no factor of $(\frac{R}{r})^8$ in the term S_1 of the action. Thus we again get back precisely the same final expression (3.8). Even though we have not been careful about numerical factors, it is important to note that the flat space answer is *exactly* the same as the $AdS_5 \times S^5$ answer. We have not bothered to calculate an overall numerical factor in both.

4. Other acceleration terms

Once again remarkable cancellations conspired to yield a supergravity derivation of a specific acceleration term of the gauge theory effective action, providing an yet non-trivial evidence for the validity of AdS/CFT correspondence. There must be a simple reason behind these cancellations, probably related to the supersymmetry of the theory. However we have not been able to find that yet.

Do all such acceleration terms in the Yang-Mills effective action have a supergravity origin ? Consider for example terms with two factors of the gauge field. At one loop, a cursory look at the feynman diagram shows that one would have an expansion of the form

$$g_{YM}^2[F^2\log(\rho) + \sum_{n=1} \frac{(\partial^n F)^2}{\rho^{2n}}] \tag{4.1}$$

We know that the first term vanishes by supersymmetry and the term with $n = 1$ is the one which we found to have a supergravity interpretation. In fact it would be disastrous if the first term was non-zero. This is because while Yang-Mills theory knows about the coupling g and the Higgs value ρ, supergravity knows only about the gravitational coupling κ and the physical transverse distance (ΔZ). These are related by the expressions (1.7) and (1.8). Thus the only term in (4.1) which can be entirely expressed in terms of supergravity quantities is the term $n = 1$. When g_{YM}, ρ is traded for $\kappa, \Delta Z$ the first term in (4.1) would have *negative* powers of the string length; if nonzero this cannot have a sensible explanation. In a similar way the terms with $n > 1$ will now contain positive powers of l_s and could have a stringy origin. Indeed such terms would be renormalized. By the same token, a possible two loop correction to $(\partial F)^2/\rho^2$ would also have a negative power of l_s : indeed this is zero since the one loop contribution to this term is exact. It appears that the nonrenormalization theorems work precisely in a way which supports the gauge theory - supergravity correspondence.

A term in the SYM effective action with $2m$ factors of F and $2n$ derivatives at l loops would have a form (dictated by simple dimension counting) which may be schematically written as

$$g_{YM}^{2m+2(l-1)} \frac{(\partial^n F^m)^2}{f_{2(2m+n-2)}(z_1, z_2)} \tag{4.2}$$

In (4.2) $f_{2(2m+n-2)}(z_1, z_2)$ denotes a polynomial of z_1 and z_2 (these are defined in (1.1)) of weight $2(2m + n - 2)$ where we have assigned weight one to z_1 and z_2. Note that beyond one loop the answer will generally depend on z_1 and z_2 individually rather than just ρ. Converting this to κ, l_s and Z_1, Z_2 we get

$$\kappa^{m+(l-1)}\ l_s^{4(m+n-l-1)} \frac{(\partial^n F^m)^2}{f_{2(2m+n-2)}(Z_1, Z_2)} \tag{4.3}$$

Thus for $m+n < l+1$ we have negative powers of the string length, for $m+n > l+1$ we have positive powers of string length while the answer is expressible entirely in terms of κ and Z_1, Z_2 only for $m+n = l+1$. It is tempting to speculate that the first class of terms ($m+n < l+1$) vanish while there would be nonrenormalization theorems which ensure that a term with given (m,n) will be exact at the $(m+n-1)$-th loop level. Those with $m+n > l+1$ do not have an interpretation in supergravity. In [3] it was suggested that such terms could be interpreted in terms of a stringy uncertainity principle. Examples of exact terms at one loop are the ones dealt with in [3],[4], F^4/ρ^4 and the term discussed in this paper, $(\partial F)^2/\rho^2$. This also suggests that a term with say four factors of the gauge fields and two derivatives should appear at two loops. There are known renormalization theorems for such "v^6" terms as well. In flat space supergravity they cannot appear at the level of a single mode exchange. However, as we will soon see such terms can originate due to single mode exchange in $AdS_5 \times S^5$.

One could similarly start from the supergravity side and ask whether all the terms which appear as a result of expansion of the momentum space propagator can appear in the Yang-Mills theory. Let us consider, for example, terms which appear from a single dilaton exchange. The zero momentum piece is one of the F^4/ρ^4 terms considered above. If the branes are placed in flat space, the leading correction to this term along the lines of this paper will be of the form

$$\kappa^2 \frac{(\partial F^2)^2}{(\Delta Z)^2} \tag{4.4}$$

It is easy to see that this *cannot* be expressed in terms of Yang-Mills quantities.

The situation is quite different when the branes are placed in $AdS_5 \times S^5$. Recall that the zero momentum propagator for the dilaton in $AdS_5 \times S^5$ is identical to that in flat space, viz

$$G_\phi^{(0)}(p, Z_1, Z_2) = \frac{1}{|Z_1 - Z_2|^4} \tag{4.5}$$

However the nonzero momentum propagator is *not* the same as in flat space. The free part of the action of the dilaton field in a $AdS_5 \times S^5$ background reads

$$
\begin{aligned}
S_\phi &= S_\phi^{(0)} + S_\phi^{(1)} \\
S_\phi^{(0)} &= \int d^4y \; d^6Z \sum_{i=1}^{6} (\partial_{Z^i}\phi)^2 \\
S_\phi^{(1)} &= \int d^4y \; d^6Z (\frac{R}{r})^4 \sum_{a=1}^{4} (\partial_{y^a}\phi)^2
\end{aligned}
\tag{4.6}
$$

The nonzero brane momentum piece $S_\phi^{(1)}$ depends on the location r. Treating $S_\phi^{(1)}$ as a perturbation and using the methods of this paper it is easy to see that the leading correction to the dilaton propagator is given by

$$
G_\phi^{(1)}(p, Z_1, Z_2) = p^2 \int d^6Z \; (\frac{R}{r})^4 \frac{1}{|Z_1 - Z|^4} \frac{1}{|Z - Z_2|^4}
\tag{4.7}
$$

This leads to a contribution to the interaction action which is of the form

$$
S_{\phi \; eff}^{sugra} = \frac{\kappa^2 R^4}{f_6(Z_1, Z_2)} \int d^4y \; (\partial_a O_1^\phi)(\partial^a O_2^\phi)
\tag{4.8}
$$

where $f_6(Z_1, Z_2)$ is a polynomial of weight 6 (as defined below equation (4.2)) and the operator O_i^ϕ has been defined in (1.5). Now, the AdS scale R is related to g_{YM} and l_s by

$$
R^4 = (g_{YM}^2 N) l_s^4
\tag{4.9}
$$

Using this and the relations (1.7) and (1.8) this expression may be recast entirely in terms of Yang-Mills quantities

$$
S_{\phi \; eff} = \frac{g_{YM}^6 N}{f_6(z_1, z_2)} \int d^4y \; (\partial_a O_1^\phi)(\partial^a O_2^\phi)
\tag{4.10}
$$

Such a term could come from a two loop diagram in the Yang-Mills theory. It has an additional factor of $g_{YM}^2 N$ compared to a one loop term (with four external legs) and therefore has the expected behavior in the large-N expansion. It would be interesting to see if the detailed structure of this term is obtainable from gauge theory.

We have seen an example where only branes placed in $AdS_5 \times S^5$ and not in flat space seem to have a correspondence with Yang-Mills theory. This is what one expects. At one loop it is difficult to see the difference between $SU(N)$ and $SU(2)$ (i.e. between branes in

$AdS_5 \times S^5$ and branes in flat space) because of a combination of the structure of the terms and nonrenormalization theorems. In a similar way other acceleration terms are expected to come from higher loop processes in gauge theory. We hope that the problems which plagued similar comparisons in M(atrix) theory at higher loops [13] will not appear in our case.

Such acceleration terms have been a subject of discussion in the past. Note that the term $(\partial F)^2$ we have obtained vanishes on shell. In fact, for similar terms involving Higgs fields, this point has been discussed in detail in [10]. In spite of being zero on-shell, this term is automatic in an off-shell and explicitly $N = 2$ supersymmetric effective action. In [12] it was shown that such terms can be in fact removed by a suitable redefinition of the fields. Even though our physical situation is different from that in [10] and [12] it will be interesting to re-examine these issues in the light of our demonstration that at least one such term can be obtained from a supergravity calculation and reflects relativistic propagators in the bulk.

The supergravity calculation, however, requires that the divergence of the total current $\mathcal{J}^{ab}$ defined in (3.5) is zero. Extending our calculation to include the $O(F^3)$ terms in the current we find that there are nonzero contributions with the Lorentz structure $(\partial_c G_{1,ab})(\partial^c F_2^{ab})$ where the tensor G_{ab} has been defined in (3.6). Note that similar terms (with the derivatives contracted among themselves) cancelled out in our $O(F^2)$ calculation (see Appendix B, equation (8.2)). Furthermore the type of terms we have obtained above now generalizes to the structures $(\partial_a \mathcal{J}^{*\ ab})(\partial^c \mathcal{J}_{cb})$ and $(\partial_a \mathcal{J}^{*\ ab})(\partial^c \tilde{\mathcal{J}}_{cb})$. By consistency of the solution, these latter terms should vanish. Thus, at the end of the day supergravity predicts that the $O(F^2)$ contribution we have calculated in fact cancels with $O(F^4)$ terms. The acceleration terms which do survive from the 2-form exchange are higher order in field strengths. They come from higher loop effects in the Yang-Mills theory and do not vanish when the perturbative equations of motion are used. In addition there would be acceleration terms coming from the exchange of the other massless fields like the dilaton.

This is consistent with the fact that a field redefinition removes the $(\partial F)^2$ term from the effective action. However, this is a nontrivial prediction about the Yang-Mills theory. This would mean that in the off shell SYM effective action there would be precise relations between terms with different numbers of gauge fields and two derivatives. Furthermore, the supergravity terms we are talking about can be entirely expressed in terms of SYM quantities only when we are in $AdS_5 \times S^5$. This is because the factors of $(\frac{R}{r})^4$ in front of the cubic pieces of the current are necessary to get the right factors of l_s required to reexpress the coefficient in terms of g_{YM}, z_1, z_2. Thus these relations must be present for large-N SYM, but not for $SU(2)$.

5. Comments

In summary, we have provided quantitative evidence in favor of the proposal that the effects of nonzero (brane) momentum exchange between branes are encoded in a class of terms in the dual gauge theory effective action involving derivatives of the gauge field strengths. Such terms should be expressible entirely in terms of gravitational quantities and should therefore appear only at a particular order of the loop expansion. Known non-renormalization theorems seem to be consistent with this idea.

The piece of physics which is not transparent in this treatment is how gauge theory really encodes causal properties of the bulk. Since there is an integration over the coordinates of each brane one effectively gets a propagator which is the sum of the retarded and advanced propagators. Furthermore we have performed an expansion in powers of the brane momenta. It is difficult to see the causal structure in such an expansion. What we have seen, however is that the gauge theory effective action knows that light has a finite speed in the bulk - we havent been able to figure out whether it moves forward or backward in time. However other recent results on bulk causality [14] seem to show that the guage theory knows the latter as well.

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7. Appendix A : The propagator

The bulk action for the complex 2-form field in a $AdS_5 \times S^5$ background is

$$S = \int d^{10}y \sqrt{g}\frac{1}{12}[H^*_{\mu\nu\lambda}H^{\mu\nu\lambda} + iF^{\alpha\beta\mu\nu\lambda}(H_{\mu\nu\lambda}B_{\alpha\beta} - H^*_{\mu\nu\lambda}B_{\alpha\beta} - (c.c.))] \tag{7.1}$$

where

$$H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu} \tag{7.2}$$

and $F^{\alpha\beta\mu\nu\lambda}$ is the self-dual 5-form field strength. The background metric is given in (1.11) and the background 5-form field is (in the same notation as above)

$$F^{rabcd} = \frac{\epsilon^{abcd}}{R}(\frac{R}{r})^3 \tag{7.3}$$

We will be interested in field configurations in which only the longitudinal components of $B_{\mu\nu}$ are nonzero : $B_{ai} = B_{ij} = 0$. Then the action may be rewritten as

$$S = S_0 + S_1 \tag{7.4}$$

where S_0 is the part of the action which depends only on derivatives along the transverse direction, while S_1 is the part which depends on longitudinal derivatives.

$$\begin{aligned} S_0 &= \int d^{10}y\ \sqrt{g}\frac{1}{4}[(\partial_i B^*_{ab})(\partial^i B^{ab}) + iF^{rabcd}((\partial_r B_{ab})B_{cd} - (\partial_r B^*_{ab})B_{cd} - (c.c.))] \\ S_1 &= \int d^{10}y\ \sqrt{g}\frac{1}{4}(\partial_a B^*_{bc})[(\partial^a B^{bc}) + (\partial^b B^{ca}) + (\partial^c B^{ab})] \end{aligned} \tag{7.5}$$

S_0 is the part of the action which was used in [4] to obtain the zero brane momentum propagator given in (1.13). Note that the Chern-Simons term which mixes the NSNS and RR 2-forms is present only in S_0. Using the explicit form of the metric and utilizing translation invariance in the brane directions to perform a fourier transform into (brane) momentum space we get for S_1

$$S_1 = \frac{1}{4}\int [d^4p][d^6Z](\frac{R}{r})^8\ B^*_{ab}(p,Z)B_{ef}(p,Z)\ E^{cdabef}\ p_c p_d \tag{7.6}$$

where

$$B_{ab}(y^a, Z^i) = \int [\frac{d^4p}{(2\pi)^4}]\ B_{ab}(p,Z)\ e^{-ip_a y^a} \tag{7.7}$$

and the tensor E^{cdabef} is defined as

$$E^{cd,ab,ef} = \eta^{cd}\eta^{ae}\eta^{bf} + \eta^{ad}\eta^{be}\eta^{cf} + \eta^{bd}\eta^{ce}\eta^{af} \tag{7.8}$$

The propagators for massless modes in $AdS_5 \times S^5$ are quite involved and several of them have been obtained recently [15]. However, as is clear from the discussion of the last section, in order to compute the term in the interaction potential between branes which involves only two derivatives in the brane direction, all we need is the Taylor expansion of the momentum space propagator. In the above action, S_0 does not involve any power of the brane direction momentum while S_1 involves two powers of the momenta. Thus it is sufficient for us to treat S_1 as a perturbation to S_0. The term we are interested in is in fact obtained by a single insertion of S_0 into the zero momentum propagator (1.13).

The piece of the propagator quadratic in the brane momenta is therefore

$$G^{(1)}_{ab,gh}(p;Z_1,Z_2) = p_{c'}p_{d'}\int d^6Z\ (\frac{R}{r})^8\ G^{(0)}_{ab,cd}(Z,Z_1)\ E^{c'd',cd,ef}\ G^{(0)}_{ef,gh}(Z_2,Z) \tag{7.9}$$

Using (1.13) and (7.8), (3.1) follows after a long and straightforward manipulation.

8. Appendix B : The interaction energy

The second term in (3.7) evaluates to the following expression after a long and straightforward calculation

$$S_{eff}^{sugra} = \kappa \int d^6 Z \ (\frac{R}{r})^8 \ \frac{1}{|Z-Z_1|^4} \frac{1}{|Z_2-Z|^4} \int d^4 p \ \mathcal{F}(p; Z_1, Z_2) \tag{8.1}$$

where

$$\begin{aligned} \mathcal{F} =& 2(2a_1a_2 + -i(b_2a_1 + b_1a_2))(p^a p_a)((F_1)^{ab}(F_2)_{ab} + (\tilde{F}_1)^{ab}(\tilde{F}_2)_{ab}) \\ & - 8[(a_1a_2 + b_1b_2) - i(a_1b_2 - a_2b_1)][(p_a(F_1)^{ab})(p^c(F_2)_{cb}) + (p_a(\tilde{F}_1)^{ab})(p^c(\tilde{F}_2)_{cb})] \end{aligned} \tag{8.2}$$

Note that we have defined $\tilde{F}_{ab} = \frac{1}{2}\epsilon_{abcd}F^{cd}$ so that $(F_1)^{ab}(F_2)_{ab} = -(\tilde{F}_1)^{ab}(\tilde{F}_2)_{ab}$. Thus the first line in (8.2), which is proportional to p^2, vansihes. The second term has a coefficient which becomes, after using the definitions of a_1, a_2, b_1, b_2 in (3.3) and (1.14)

$$\begin{aligned} (a_1a_2 + b_1b_2) &= \frac{2}{R^8}(r^8 + r_1^4 r_2^4) \\ i(a_1b_2 - a_2b_1) &= -\frac{2}{R^8}(r^8 - r_1^4 r_2^4) \end{aligned} \tag{8.3}$$

Thus the terms which depend on the individual brane locations cancel neatly and one gets the result

$$\begin{aligned} S_{eff}^{sugra} = \kappa \int d^6 Z \ (\frac{R}{r})^8 \ & \frac{1}{|Z-Z_1|^4} \frac{1}{|Z_2-Z|^4} \\ & \int d^4 p \ 32(\frac{r}{R})^8 \ [(p_a(F_1)^{ab})(p^c(F_2)_{cb}) + (p_a(\tilde{F}_1)^{ab})(p^c(\tilde{F}_2)_{cb})] \end{aligned} \tag{8.4}$$

The powers of $\frac{R}{r}$ also cancel leaving with a Z integral which is

$$\int d^6 Z \ \frac{1}{|Z-Z_1|^4} \frac{1}{|Z_2-Z|^4} \sim \frac{1}{|Z_1-Z_2|^2} \tag{8.5}$$

Transforming back into (brane) position space we get (3.8).

In flat space $a_1 = a_2 = 1$ and $b_1 = b_2 = 0$ while the factors of $(\frac{R}{r})^8$ are not present in the action and in (8.1). Using (8.1) and (8.2) it is easy to see that in this case too we reproduce exactly (8.4).

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