Fermi National Accelerator Laboratory

FERMILAB-PUB-86/90-T
CALT-68-1352
June 1986

BRST Ghosts as Worldsheet Parameters in Gauge-Invariant String Field Theory

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ABSTRACT

We quantize the closed and open bosonic strings in the Tomonaga-Schwinger-Dirac (TSD) formalism. This leads to a gauge-invariant second-quantized free string field theory. The worldsheet parameters are dynamical variables which, in the quantum theory, are represented by anticommuting operators. This TSD quantization is seen to be formally identical to the $BRST$ quantization and provides a geometrical interpretation of the anticommuting $BRST$ ghosts as quantized worldsheet parameters.

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I. Introduction

In the past year several formalisms for gauge-invariant string field theories have been discussed [1-9]. It is now clear that in any such formalism the string field has to be a functional, not only of $x^\mu(\sigma)$ (the position of the string in spacetime), but also of some extra dynamical variables. For instance, in the $BRST$ approach [3,4] these variables are ghost fields, while in ref. [2] they are "loop space differential forms" which may be, however, reexpressed in terms of ghosts [7]. The $BRST$ approach indeed leads to a rather elegant formulation, and interacting field theories of open [8] and closed [9] bosonic strings and open superstrings [8] have been proposed.

What is lacking in all these approaches is a physical meaning for the extra dynamical variables.[4] These extra degrees of freedom play a crucial role in the gauge invariance of string fields. A physical understanding of their origin may prove instrumental in formulating nonlinear transformations of interacting string theories and thus throw light on the origin of gauge and general coordinate invariance in nature.

Another drawback of presently known formalisms is the asymmetric treatment of the two worldsheet parameters σ and τ. A formalism which treats σ and τ in a symmetrical fashion may lead to an explicitly dual theory.

This letter is a modest step toward removing both these deficiencies. In earlier communications [10] we formulated the first-quantized bosonic string in a Tomonaga-Schwinger-Dirac (TSD) [11] approach. In this formalism the worldsheet parameters are elevated to the level of dynamical variables $\sigma(\xi)$ and $\tau(\xi)$. The wave functional $\Psi[x^\mu(\xi), \sigma(\xi), \tau(\xi)]$ depends on $x^\mu(\xi)$, the position of the string in spacetime, as well as on $\sigma(\xi), \tau(\xi)$ which denote the position of the string in parameter space. The particular quantization of the classical TSD string discussed in ref. [10] does not lead to a gauge-invariant theory. In this paper we reexamine the quantization paying particular attention to quantum anomalies. We find that, in a consistent quantization, one treats $\sigma(\xi)$ and $\tau(\xi)$ as anticommuting operators. The resulting formalism automatically leads to a gauge-invariant second-quantized action with a

[4] See, however, ref.[13], which considers a modified bosonic string theory in which the 1^{st}- quantized action is supplemented by additional kinetic terms for the Liouville (conformal) mode of the Polyakov worldsheet metric.

formal structure equivalent to the *BRST* method.

II. The Classical Theory

Let us briefly review the classical *TSD* string formalism. We begin with the bosonic string action in an orthonormal gauge:

$$S = -\frac{1}{2}\int d^2\sigma \; \frac{\partial X^\mu}{\partial \sigma^i} \frac{\partial X_\mu}{\partial \sigma_i} \tag{1}$$

where $\sigma^0 = -\sigma_0 = \tau$ and $\sigma^1 = \sigma_1 = \sigma$ are the worldsheet parameters. (One may think of (1) as coming from the Nambu action, or from the Polyakov action in the critical dimension.) The gauge constraints are:

$$\left(\frac{\partial X}{\partial \sigma^0}\right)^2 + \left(\frac{\partial X}{\partial \sigma^1}\right)^2 = \frac{\partial X^\mu}{\partial \sigma^0} \frac{\partial X_\mu}{\partial \sigma^1} = 0 \; . \tag{2}$$

We introduce a new arbitrary curvilinear coordinate system on the worldsheet, $\xi^\alpha = \xi^\alpha(\sigma^i)$. The lagrangian density L becomes:

$$L = -\frac{1}{2} J g^{\alpha\beta} \frac{\partial X^\mu}{\partial \xi^\alpha} \frac{\partial X_\mu}{\partial \xi^\beta} \tag{3}$$

where J is the Jacobian of the transformation and

$$g^{\alpha\beta} \equiv \frac{\partial \xi^\alpha}{\partial \sigma^i} \frac{\partial \xi^\beta}{\partial \sigma_i} \; .$$

Defining canonical momenta with respect to the new time variable $\xi^0, \Pi_\mu \equiv \partial L/\partial(\partial x^\mu/\partial \xi^0)$, (3) becomes

$$L = \Pi_\mu \dot{x}^\mu - h \tag{4}$$

$$h \equiv \left(J \frac{\partial \xi^0}{\partial \sigma^i} T^i_j\right) \dot{\sigma}^j \tag{5}$$

(Dots denote differentiation with respect to ξ^0, primes with respect to ξ^1).T^i_j is the energy-momentum tensor of the $X^\mu(\xi)$ fields in "flat" coordinates (σ, τ).

The crucial point is that, in equation (5), the factor multiplying $\dot{\sigma}^j$ is <u>independent</u> of $\dot{\sigma}^j$. Consequently, the $\sigma^j(\xi)$ play the role of <u>dynamical variables</u>, provided their conjugate momenta $P_j(\xi)$ satisfy the constraints

$$P_i = -J \frac{\partial \xi^0}{\partial \sigma^j} T^j_i \quad . \tag{6}$$

Then

$$L = \Pi_\mu \dot{x}^\mu + P_i \dot{\sigma}^i \quad . \tag{4'}$$

From equation (4′) we see immediately that the new dynamical system has a vanishing Hamiltonian, so $x^\mu(\xi), \sigma^j(\xi)$ are independent of the "time" ξ^0.(Henceforth, ξ will denote simply ξ^1, unless otherwise indicated.) The entire dynamics is contained in the constraints (6) and the original gauge conditions (2).

The components of P_i normal and tangential to the spacelike lines of constant ξ^0 are referred to as the "superHamiltonian" and "supermomentum", respectively. For purposes of quantization it is, however, more convenient to work in terms of the null worldsheet quantities

$$\begin{aligned} \rho_\pm(\xi) &\equiv \frac{1}{\sqrt{2}}\left(\sigma^0(\xi) \pm \sigma^1(\xi)\right) , \\ P_\pm(\xi) &\equiv \frac{1}{\sqrt{2}}\left(P_0(\xi) \pm P_1(\xi)\right) . \end{aligned} \tag{7}$$

The constraints (6) may then be written as

$$G_\pm(\xi) \equiv \pm \rho'_\pm P_\pm + \frac{1}{4}\left(\Pi(\xi) \pm x'(\xi)\right)^2 = 0 \tag{8}$$

while the gauge conditions (2) become simply

$$P_{\pm}(\xi) = 0 \ . \tag{9}$$

The two functions $\sigma^i(\xi)$ trace out a spacelike parametrized curve in the world-sheet. The constraints $G_{\pm}(\xi)$ generate deformations of this line along the two null directions. Their Fourier components

$$G_m^{\pm} = \int_{-\pi}^{\pi} d\xi \ \ e^{\pm i m \xi} G_{\pm}(\xi) \tag{10}$$

satisfy the classical Virasoro algebra under Poisson brackets.

At this point one may substitute (9) in (8) to recover the standard formalism in terms of $x^\mu(\xi)$ alone. The whole point, however, is to retain the redundant variables $\rho_{\pm}(\xi)$ to allow for a gauge-invariant formulation.

For closed strings with $-\pi \leq \xi \leq \pi$ the functions $x^\mu(\xi)$ and $\rho_{\pm}(\xi)$ are periodic and equations (8) and (9) denote four independent equations. For open strings $0 \leq \xi \leq \pi$. However the interval may be doubled in the standard fashion to $-\pi \leq \xi \leq \pi$ by defining

$$x^\mu(\xi) = x^\mu(-\xi) \ . \tag{11}$$

The corresponding condition on $\rho_{\pm}(\xi)$ may be obtained by noting that ξ may be, in particular, chosen to be σ itself. This leads to

$$\rho_{\pm}(\xi) = \rho_{\mp}(-\xi) \ . \tag{12}$$

With this extension of the physical interval all functions may be taken to be periodic. The four eqns. (8) and (9) with "+" and "−" are now only two independent equations, which may be chosen, *e.g.*, to be (8) and (9) with the "+" sign.

III. The Quantized Theory

We now proceed to quantize the classical system described by the constraints (8) and (9). The dynamical variables $x^\mu(\xi), \rho_\pm(\xi)$ are now operators; since we are dealing with a two-dimensional field theory these operators may be chosen to obey either canonical commutation relations or canonical anticommutation relations [14]. We take $x^\mu(\xi)$ and its conjugate momentum $\Pi^\mu(\xi)$ to obey the usual canonical commutation relations:

$$\begin{aligned} [x^\mu(\xi), \Pi^\nu(\bar{\xi})] &= i\,\eta^{\mu\nu}\,\delta(\xi-\bar{\xi}), \\ [x^\mu(\xi), x^\nu(\bar{\xi})] &= [\Pi^\mu(\xi), \Pi^\nu(\bar{\xi})] = 0 . \end{aligned} \tag{13a}$$

However, to begin with, we will only specify that $\rho_\pm(\xi), P_\pm(\xi)$ satisfy <u>either</u> canonical commutation relations,

$$\begin{aligned} [\rho_\pm(\xi), P_\pm(\bar{\xi})] &= i\,\delta(\xi-\bar{\xi}), \\ [\rho(\xi), \rho(\bar{\xi})] &= [P(\xi), P(\bar{\xi})] = [\rho_\pm(\xi), P_\mp(\bar{\xi})] = 0 \end{aligned} \tag{13b}$$

<u>or</u> canonical anticommutation relations,

$$\begin{aligned} \{\rho_\pm(\xi), P_\pm(\bar{\xi})\} &= i\,\delta(\xi-\bar{\xi}), \\ \{\rho(\xi), \rho(\bar{\xi})\} &= \{P(\xi), P(\bar{\xi})\} = \{\rho_\pm(\xi), P_\mp(\bar{\xi})\} = 0 \end{aligned} \tag{13c}$$

We will see below that the <u>latter</u> of the two choices (13b), (13c), yields a consistent quantum theory.

The constraints (8) and (9), being functions of the dynamical variables, are now themselves operators; since the Hamiltonian is identically zero, the entire content of the quantum theory is contained in the statement that the constraints vanish when acting on the wave functional $\Psi[x^\mu(\xi), \rho_\pm(\xi)]$, *i.e.*

$$G_\pm(\xi)\,\Psi = 0 , \tag{8'}$$

$$P_\pm(\xi)\Psi = 0 \,. \tag{9'}$$

Here we encounter a second ambiguity in passing from the classical to the quantum theory. Since $P_\pm = 0$, we can add to $G_\pm$ any function, $f_\pm$ which is polynomial in $P_\pm$ and its derivatives with no $P_\pm$ -independent term. The only natural restrictions on such a term are that it have worldsheet dimension equal to two,[5] and that the modified constraints $\hat{G}_\pm = G_\pm + f_\pm, P_\pm$ remain "first-class"; that is, the classical Poisson brackets of $\hat{G}_\pm, P_\pm$ are proportional to $\hat{G}_\pm, P_\pm$, so that imposition of these constraints does not imply further independent constraints. If, for the sake of technical simplicity, we add the third requirement that $f_\pm$ contain neither $\rho_\pm$ nor $P_\pm$ to higher than linear degree, we find the most general $\hat{G}_\pm$ to be

$$\hat{G}_\pm(\xi) = \pm(J-1)P'_\pm(\xi)\rho_\pm(\xi) \pm J\,P_\pm(\xi)\rho'_\pm(\xi) + \frac{1}{4}\left(\Pi(\xi) \pm x'(\xi)\right)^2 \tag{14}$$

where J is an arbitrary constant. $\hat{G}_\pm(\xi)$ generate conformal reparametrizations of the two null coordinates; J is the conformal dimension [12] of $P_\pm(\xi)$.

The quantum operators must be normal-ordered with respect to some particular vacuum state. This can be done using the following mode expansions:

$$\Pi^\mu(\xi) \pm x'^\mu(\xi) = \frac{1}{\sqrt{\pi}} \sum_n \alpha_n^{\mu\pm} e^{\mp in\xi}, \tag{15a}$$

$$\rho_\pm(\xi) = \frac{1}{\sqrt{2\pi}} \sum_n \rho_n^\pm e^{\mp in\xi}, \tag{15b}$$

$$P_\pm(\xi) = \frac{i}{\sqrt{2\pi}} \sum_n P_n^\pm e^{\mp in\xi} \,. \tag{15c}$$

[5]This simply means that the new constraint $\hat{G}_\pm = G_\pm + f_\pm$ will transform homogeneously under a uniform rescaling of the parameters ξ^0, ξ^1, so as not to select a preferred scale. From (4) we see that if $\xi^i \to \Lambda^{-1}\xi^i$, where Λ is a constant, P_i and ρ^i must transform so that $P_i\dot{\rho}^i \to \Lambda^2 P_i\dot{\rho}^i$ in order for the action to remain unchanged. Then (8) tells us that $G_\pm \to \Lambda^2 G_\pm$, so we must have $f_\pm \to \Lambda^2 f_\pm$ as well.

The vacuum is annihilated by $\alpha_n^{\pm}, \rho_n^{\pm}$ and $P_n^{\pm}$ for $n > 0$ and by $P_0^{\pm}$ From the commutation relations (13) one has

$$\begin{aligned} (\alpha_n^{\mu\pm})^{\dagger} &= \alpha_{-n}^{\mu\pm} , \\ [\alpha_m^{\mu\pm}, \alpha_n^{\nu\pm}] &= m\eta^{\mu\nu}\delta_{m+n,0} , \\ [\alpha_m^{\mu\pm}, \alpha_n^{\nu\mp}] &= 0 . \end{aligned} \tag{16}$$

If $\rho_{\pm}(\xi)$, $P\pm(\xi)$ obey commutation relations, then

$$\begin{aligned} (\rho_n^{\pm})^{\dagger} &= \rho_{-n}^{\pm} , \quad (P_n^{\pm})^{\dagger} = -P_{-n}^{\pm} , \\ [\rho_m^{\pm}, P_n^{\pm}] &= \delta_{m+n,0} , \\ [\rho_m, \rho_n] &= [P_m, P_n] = [\rho_m^{\pm}, P_n^{\mp}] = 0 . \end{aligned} \tag{17}$$

If they anticommute one has, instead,

$$\begin{aligned} (\rho_n^{\pm})^{\dagger} &= \rho_{-n}^{\pm} , \quad (P_n^{\pm})^{\dagger} = P_{-n}^{\pm} , \\ \{\rho_m^{\pm}, P_n^{\pm}\} &= \delta_{m+n,0} , \\ \{\rho_m, \rho_n\} &= \{P_m, P_n\} = \{\rho_m^{\pm}, P_n^{\mp}\} = 0 . \end{aligned} \tag{18}$$

In either case, the Fourier components of $\hat{G}_{\pm}(\xi)$ may be written as

$$\begin{aligned} \hat{G}_m^{\pm} &\equiv \int_{-\pi}^{\pi} d\xi \, e^{\pm im\xi} \hat{G}_{\pm}(\xi) \\ &= \sum_{n=-\infty}^{\infty} \left(\tfrac{1}{2} g_{\mu\nu} \alpha_{m-n}^{\mu\pm} \alpha_n^{\nu\pm} + ((J-1)m-n) P_{m+n}^{\pm} \rho_{-n}^{\pm} \right) \end{aligned} \tag{19}$$

and the normal-ordered Fourier components, $:\hat{G}^{\pm}_m:$, satisfy the Virasoro algebra with a central term,

$$[:\hat{G}^{\pm}_m:, :\hat{G}^{\pm}_n:] = (m-n):\hat{G}^{\pm}_{m+n}: + a(m)\,\delta_{m+n,0}. \qquad (19)$$

For commuting $\rho's$[12]

$$a(m) = \left(J^2 - J + \frac{1}{6}\right)m^3 - \frac{1}{6}m + \frac{D}{12}(m^3 - m) \qquad (20)$$

while for anticommuting $\rho's$

$$a(m) = \left(-J^2 + J - \frac{1}{6}\right)m^3 + \frac{1}{6}m + \frac{D}{12}(m^3 - m) \qquad (21)$$

(D is the spacetime dimensionality.) It may be easily seen that if $\rho_\pm$ and $P_\pm$ obey commutation relations there is no choice of $D > 1$ and real J for which the central charge vanishes. On the other hand, for anticommuting $\rho_\pm$, $P_\pm$ one can make the central charge vanish by redefining

$$:\hat{G}^{\pm}_m: \rightarrow :\hat{G}'^{\pm}_m: = :\hat{G}^{\pm}_m: + \alpha_0\,\delta_{m,0} \qquad (22)$$

and choosing

$$D = 12\left(J^2 - J + \frac{1}{6}\right), \qquad \alpha_0 = \frac{1}{2}\left(\frac{1}{6} - \frac{D}{12}\right). \qquad (23)$$

We thus conclude that it is possible to construct a consistent quantum theory when $\rho_\pm$ and $P_\pm$ are anticommuting operators and the relations (13) are obeyed.

The quantum theory is defined by imposing the following conditions on the wave functional:

$$:\hat{G}'^{\pm}_m: \Psi = 0 \qquad (m \geq 0), \tag{24a}$$

$$P^{\pm}_m \Psi = 0 \qquad (m \geq 0). \tag{24b}$$

It may be checked that if the wave functional is further restricted by

$$\rho^{\pm}_m \Psi = 0 \qquad (m > 0) \tag{25}$$

eq. (24a) reduces to

$$(:L^{\pm}_m: + \alpha_0 \delta_{m,0}) \Psi = 0$$

which are the standard string equations in a formalism involving $x^\mu(\xi)$ alone. ($L^{\pm}_m$'s are the standard Virasoro operators $L_m, \tilde{L}_m$).

As noted earlier, J is the conformal dimension of the field $P_\pm(\xi)$. So far we have not specified what the value of J is. The original definition of $\rho_\pm(\xi)$ in eq. (7) as worldsheet parameters suggests that it is natural to take $\rho_\pm(\xi)$ to have conformal dimension -1. The anticommutation relations then fix J, the conformal dimension of $P_\pm(\xi)$, to be 2. From eq. (23) one sees that for $J = 2$, a consistent quantum theory is obtained for

$$D = 26, \qquad \alpha_0 = 1.$$

A remarkable structure now emerges. The content of the equations (24a) and (24b) may be summarized by the single equation

$$A \Psi = 0 \tag{26}$$

where the operator A is given by

$$A = \int_{-\pi}^{\pi} d\xi \Big\{ \rho_+(\xi)\big[\rho_+'(\xi) P_+(\xi) + \tfrac{1}{4}\left(\Pi(\xi) + x'(\xi)\right)^2 \big] + \rho_-(\xi)\big[-\rho_-'(\xi) P_-(\xi) + \tfrac{1}{4}\left(\Pi(\xi) - x'(\xi)\right)^2 \big] \Big\} . \tag{27}$$

A may be seen to be nilpotent, *i.e.*

$$A^2 = 0 .$$

Thus (27) has a gauge invariance,

$$\Psi \to \Psi + A\varepsilon , \tag{28}$$

where ε is an arbitrary functional of $x^\mu(\xi), \rho_\pm(\xi)$. It follows from the anticommutation relations that

$$\hat{G}'^{\pm}_m = \{ P^{\pm}_m , A \} . \tag{29}$$

So, if one fixes the gauge symmetry (28) partially by imposing

$$P^{\pm}_m \Psi = 0 , \quad m \geq 0$$

one obtains eq. (24a). The equations (24a) and (24b) thus come from a gauge-invariant system with (24b) playing the role of a gauge condition.

The connection of our approach to the *BRST* formalism is now clear. The worldsheet parameters $\rho_\pm(\xi)$ play the role of ghost fields while their conjugate momenta $P_\pm(\xi)$ are the antighosts. The operators $\hat{G}^\pm_m$ are the "total" Virasoro operators and A is the *BRST* charge. The ghost number operator is

$$N_g = \int_{-\pi}^{\pi} d\xi \left[\rho_+(\xi) P_+(\xi) + \rho_-(\xi) P_-(\xi) \right] .$$

It is evident that N_g generates global dilatations $\rho_\pm(\xi) \to e^\alpha \rho_\pm(\xi)$ of the variables representing the string on the world sheet:

$$\delta\rho_\pm(\xi) = \alpha \{ N_g , \rho_\pm(\xi) \} .$$

(For $J \neq 2$ the constraints $\hat{G}'^{\pm}_m$ cannot be obtained from a BRST charge A in the manner of eqs. (27), (28). It seems highly likely that, with $J \neq 2$ and/or $D \neq 26$, the theory contains negative-norm states in the physical spectrum, but as of this writing this has not been proved.)

One can now write down a free second-quantized action for the closed bosonic string [7,9]. The string functional $\Psi[x^\mu(\xi), \rho_\pm(\xi)]$ is restricted to the physical subspace

$$\begin{aligned} &N_g \Psi = -\Psi , \\ &\Delta N \Psi = 0 , \\ &(P_0^+ - P_0^-) \Psi = 0 . \end{aligned} \tag{30}$$

ΔN is the difference of the number of $+$ modes and number of $-$ modes. The action now reads [9]:

$$S = \int \mathcal{D}x^\mu \mathcal{D}c \mathcal{D}\tilde{c} \; \Psi^T \frac{1}{2} \frac{\partial}{\partial \tilde{c}_0} [\tilde{c}_0 , A] \Psi \tag{31}$$

where

$$\Psi^T \equiv \Psi[x^\mu(-\xi), c(-\xi), -\tilde{c}(-\xi)],$$

$$\tilde{c}(\xi) \equiv \frac{1}{\sqrt{2}}(\rho_+(\xi) - \rho_-(\xi)), \quad c(\xi) \equiv \frac{1}{\sqrt{2}}(P_+(\xi) + P_-(\xi))$$

and $\tilde{c}_0, c_0$ are the zero modes of $\tilde{c}(\xi)$ and $c(\xi)$, respectively. The equation of motion following from (31) is the same as $A\Psi = O$ provided the conditions (30) hold.

We have considered closed strings so far. For open strings, as discussed above, everything goes through with only the "+" variables present. In the field theory, the conditions (30) are replaced by

$$N_g \Psi = -\frac{1}{2}\Psi,$$

and the action is simply

$$S = \int \mathcal{D}x^\mu \, \mathcal{D}c \, \mathcal{D}\tilde{c} \; \Psi^T A \Psi,$$

$$\Psi^T \equiv \Psi[x^\mu(\pi-\xi), c(\pi-\xi), -\tilde{c}(\pi-\xi)]. \quad (0 \leq \xi \leq \pi)$$

Thus, using the TSD formalism, we have arrived systematically at a formal structure isomorphic to that obtained in the $BRST$ approach. However, the extra degrees of freedom necessary to write down a gauge-invariant theory have entered in a very natural manner and with a clear geometric meaning. They are simply the original world sheet parameters $\tau = \frac{1}{\sqrt{2}}(\rho_+ + \rho_-)$ and $\sigma = \frac{1}{\sqrt{2}}(\rho_- - \rho_+)$ which, as functions of ξ, specify the position of the string in parameter space, and which have been promoted to the status of dynamical variables in the TSD formalism. The total Virasoro operators $\hat{G}'^{\pm}_m$ are generators of conformal reparametrizations which deform the string in both coordinate and parameter space. Furthermore, the two worldsheet variables are treated on an equal footing and one might imagine that there is a realization of a "duality" transformation which interchanges these variables.

Our program should work also for superstrings. We expect the role of superconformal ghosts to be played by the Grassmann coordinates of the superworldsheet, promoted to commuting dynamical variables. This work is currently in progress and will be reported in a future communication.

Finally, we hope that the physical insight gained in the understanding of extra degrees of freedom in string theory will help us to understand interacting theories in a geometrical manner. In particular, the rather unnatural insertions of ghost factors in interaction terms of bosonic string theories may have a geometric meaning.

We would like to thank I. Angus for a very useful discussion.

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