ON A RELATION BETWEEN TWO CONJECTURES OF THE THEORY OF NUMBERS.

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§1. Let $r_{k,s}(n)$ denote the number of representations of n as a sum of s kth powers of integers $\geqq 0$. Since $r_{k,2}(n) = O(n^{\epsilon})$ for every positive ϵ, we have

$$r_{k,k}(n) = O\left(n^{1-\frac{2}{k}+\epsilon}\right)$$

and no better result is known. Denote by (A_s) the unproved result that

$$(A_s) \qquad r_{k,k}(n) = o\left(n^{\frac{s}{k}}\right)$$

where $k \geqslant 5$ and s is a positive integer $\leqq k-3$. Denote by (B_s) the conjecture that

(B_s) $\quad \sum_{m=1}^{k-s} x_m^k = \sum_{m=1}^{k-s} y_m^k$ *has a non-trivial solution in positive integers* $x_1, x_2,$ *etc.*, $y_1, y_2,$ *etc.*

Then we have the

*Theorem.** *If* (A_s) *is false then* (B_s) *is true, and conversely.*

In other words at least one of (two desirable results) (A_s) and (B_s) is true.

Proof. If (B_s) is false, then

$$r_{k,k-s}(n) = O(1)$$

whence

$$r_{k,k-s+1}(n) = O\left(n^{\frac{1}{k}}\right)$$

$$r_{k,k-s+2}(n) = O\left(n^{\frac{2}{k}}\right)$$

etc.

$$r_{k,k}(n) = O\left(n^{\frac{s}{k}}\right)$$

* The proof shows that the result is trivial; my object is to draw attention to the conjectures themselves.

i.e., *if* (B_s) *is false then* (A_s) *is true.* Conversely, if (A_s) is false, it follows that (B_s) must be true.

§2. Note the special case when $s = 3$ ($k \geqslant 6$). at least one of the following conjectures is true :

$$\text{(A)} \qquad a_1^k + a_2^k + a_3^k = b_1^k + b_2^k + b_3^k$$

has a non-trivial solution in positive integers, a_1, a_2, a_3, b_1, b_2, b_3.

$$\text{(B)} \qquad r_{k,k}(n) = O\left(n^{\frac{k-3}{k}}\right)$$

It seems to the writer that (A) is *not likely* to be proved in the near future. On the other hand, it is possible that (B), which asserts so much less than " Hypothesis K " of Hardy and Littlewood, is not a difficult result. We observe also that (A) *may be false, for solutions of* (A) are not known even for $k = 5$.