Dephasing of Aharonov-Bohm oscillations in a mesoscopic ring with a magnetic impurity

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(February 1, 2008)

We present a detailed analysis of the Aharonov-Bohm interference oscillations manifested through transmission of an electron in a mesoscopic ring with a magnetic impurity atom inserted in one of its arms. The electron interacts with the impurity through the exchange interaction leading to exchange spin-flip scattering. Transmission in the spin-flipped and spin-unflipped channels are explicitly calculated. We show that the spin-flipper acts as a dephasor in spite of absence of any inelastic scattering. The spin-conductance (related to spin-polarized transmission coefficient) is asymmetric in the flux reversal as opposed to the two probe conductance which is symmetric under flux reversal.

PACS Nos.: 73.23.-b, 05.60.Gg, 72.10.-d, 03.65.Bz

Quantum transport in open mesoscopic systems has attracted considerable attention in the last two decades[1–3]. In this area, the study of phase coherent transmission of electrons in the Aharonov-Bohm (AB) ring occupies a prominent place[1–6]. Study of dephasing[7–11] of electrons in this geometry is very timely[12] to understand basic issues related to quantum phenomena. By introducing a magnetic impurity atom (to be referred to as the spin-flipper, or the flipper, for short) in one arm of the ring, one can couple the spin of the electron ($\vec{\sigma}$) to the spin of the flipper ($\vec{S}$) via the exchange interaction[1,7]. This leads to scattering of the electron in which the spin state of the electron and the impurity is changed without any exchange of energy leading to reduction of interference. Let the electron be incident from the left reservoir with its spin pointing "up" (see Fig. 1). The spin of the electron passing through the upper arm may or may not be flipped by the flipper. In the case that the spin is unflipped, one would expect the usual AB-oscillations of the transmission due to interference of the partial waves passing through the upper and the lower branches of the ring. However, in the case that the spin is flipped, one would think, guided by naive intuition, that a path detection has taken place and hence one would be led to conclude that the interference pattern for the spin-down component would be wiped out. This is true provided we consider only two forward propagating partial waves. However, there are infinitely many partial waves in this geometry which are to be superposed to get the total transmission. These arise due to the multiple reflections from the junctions and the impurity site. Consider, for example, an incident spin-up particle moving in the upper arm which is flipped at the impurity site and gets reflected to finally traverse the lower arm before being transmitted. Naturally, this partial wave will interfere with the spin-flipped component transmitted along the upper arm. This results in non-zero transmission for the spin-flipped electron. Thus on taking into account the multiple reflections (more than just two partial waves) the presence of magnetic impurity does not lead to "which-path" information. However, the magnetic impurity acts as a dephasor[1,7].

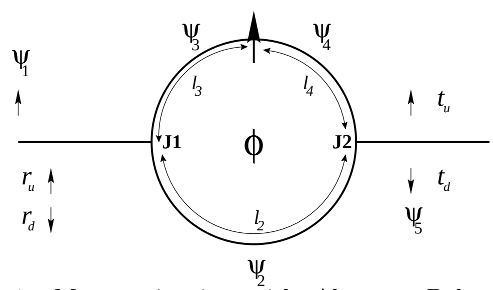

FIG. 1. Mesoscopic ring with Aharonov-Bohm flux ϕ threading through the center of the ring and a magnetic impurity in one arm of the ring.

In this work we show that the amplitude of AB-oscillations is reduced by the flipper causing dephasing. We study the problem using the quantum waveguide theory approach[6,13] and the spin degree of freedom of the electron is dealt with in line with ref. 14. We consider an impurity consisting of a flipper capable of existing in M different discrete internal spin states and located at a particular position on the upper arm of the ring (see Fig. 1). The spin $\vec{\sigma}$ of the electron couples to the flipper spin $\vec{S}$ via an exchange interaction $-J\vec{\sigma}\cdot\vec{S}\delta(x-l_3)$. The magnetic flux threading the ring is denoted by ϕ and is related to the vector potential $A=\phi/l$, l being the ring circumference[13]. During passage of the electron through the ring, the total spin angular momentum and its z-component remain conserved. We analyze the nature of spin-up/down and total transmission (reflection) coefficients. For this we consider the incident electron to be spin-polarized in the up-direction. Apart from dephasing we show that up/down-transmission coefficients are asymmetric in flux reversal, i.e., total spin polarization (related to spin conductance[15]) is asymmetric in flux reversal. As expected we find that the total transmission coefficient which is the sum of spin-up and spin-down transmission coefficients is symmetric in the flux reversal.

Let l_2 be the length of the lower arm of the ring and the impurity atom be placed at a distance l_3 from the junction J1, l_4 being the remaining segment length of the upper arm. The various segments of the ring and its leads are labelled as shown in Fig. 1 and the wave functions in these segments carry the corresponding subscripts. The wave functions in the five segments for a left-incident spin-up electron can be written as follows:

$$\begin{aligned}
\psi_1 &= (e^{ikx} + r_u e^{-ikx})\chi_m\alpha + \\
&\quad r_d e^{-ikx}\chi_{m+1}\beta, \\
\psi_2 &= (A_u e^{ik_1x} + B_u e^{-ik_2x})\chi_m\alpha + \\
&\quad (A_d e^{ik_1x} + B_d e^{-ik_2x})\chi_{m+1}\beta, \\
\psi_3 &= (C_u e^{ik_1x} + D_u e^{-ik_2x})\chi_m\alpha + \\
&\quad (C_d e^{ik_1x} + D_d e^{-ik_2x})\chi_{m+1}\beta, \\
\psi_4 &= (E_u e^{ik_1x} + F_u e^{-ik_2x})\chi_m\alpha + \\
&\quad (E_d e^{ik_1x} + F_d e^{-ik_2x})\chi_{m+1}\beta, \\
\psi_5 &= t_u e^{ikx}\chi_m\alpha + t_d e^{ikx}\chi_{m+1}\beta.
\end{aligned} \tag{0.1}$$

where $k_1 = k + (e\phi/\hbar cl)$, $k_2 = k - (e\phi/\hbar cl)$, k is the wave-vector of incident electron. The subscripts u and d represent "up" and "down" spin states of the electron with the corresponding spinors α and β respectively (i.e., $\sigma_z\alpha = \frac{1}{2}\alpha$, $\sigma_z\beta = -\frac{1}{2}\beta$) and χ_m denotes the wave function of the impurity[14] with $S_z = m$ (i.e., $S_z\chi_m = m\chi_m$). The reflected (transmitted) waves have amplitudes r_u (t_u) and r_d (t_d) corresponding to the "up" and "down" spin components respectively. Continuity of the wave functions and the current conservation[6,13,14] at the junctions J1 and J2 imply the following boundary conditions.

$$\begin{aligned}
\psi_1(x=0) &= \psi_2(x=0), \\
\psi_1(x=0) &= \psi_3(x=0), \\
\psi_1'(x=0) &= \psi_2'(x=0) + \psi_3'(x=0), \\
\psi_3'(x=l_3) - \psi_4'(x=l_3) &= G(\vec{\sigma}\cdot\vec{S})\psi_3(x=l_3), \\
\psi_3(x=l_3) &= \psi_4(x=l_4), \\
\psi_4(x=l_3+l_4) &= \psi_5(x=0), \\
\psi_2(x=l_2) &= \psi_5(x=0), \\
\psi_2'(x=l_2) + \psi_4'(x=l_3+l_4) &= \psi_5'(x=0).
\end{aligned} \tag{0.2}$$

Here $G = 2mJ/\hbar^2$ is the coupling constant indicative of the "strength" of the spin-exchange interaction. The primes denote the spatial derivatives of the wave functions. Equations (0.1) along with the boundary conditions (0.2) were solved to obtain the amplitudes t_u, t_d, r_u and r_d. Owing to the large length of the expressions in the following we confine ourselves to the graphical interpretation of the results. We have taken the flipper to be a spin-half object ($M = 2$) situated symmetrically at the center of the upper arm, i.e., $l_3 = l_4$. Now, depending upon the initial state of the flipper we have possibility of either spin-flip scattering ($\sigma_z = 1/2$, $S_z = -1/2$) or no spin-flip scattering ($\sigma_z = 1/2$, $S_z = 1/2$), as demanded by the conservation of the total spin and its z-component. In the case of no-spin-flip scattering ($\sigma_z = 1/2$, $S_z = 1/2$) the problem at hand reduces to that of simple potential scattering from the impurity. We have set $\hbar = 2m = 1$ and throughout the value of interaction strength G is given in dimensionless units. The parameters used for the analysis are mentioned in the figure captions.

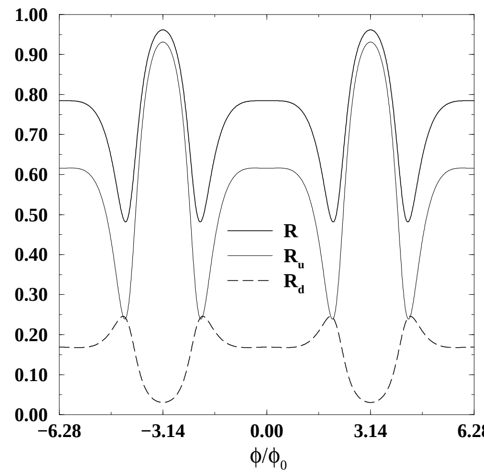

FIG. 2. Plot of total reflection coefficient R, spin-up reflection coefficient R_u and spin-down reflection coefficient R_d for the spin -flip scattering case. The parameters are $kl = 1.0$, $G = 10.0$.

To begin with we take a look at the symmetry properties of the transport coefficients in spin-flip scattering case where the electron spin is opposite to the flipper spin. It is worth noting that due to the presence of spin degree of freedom the problem in hand although one-dimensional becomes a multichannel problem. Figure 2 shows the spin-up reflection coefficient $R_u = |r_u|^2$, spin-down reflection coefficient $R_d = |r_d|^2$ and total reflection coefficient $R = R_u + R_d$ as a function of the magnetic flux parameter $\eta = \phi/\phi_0$, ϕ_0 being the flux quantum hc/e. We clearly see the AB-oscillations with flux periodicity[4] of $2\pi\phi_0$. All three reflection coefficients are symmetric in the flux reversal as expected on general grounds[16]. In Fig. 3 we plot the spin-up transmission coefficient $T_u = |t_u|^2$ (thin line), spin-down transmission coefficient $T_d = |t_d|^2$ (dashed line) and total transmission coefficient $T = T_u + T_d$ (thick line) versus η. It unambiguously shows that though the total transmission T (related to the two-terminal electrical conductance) is symmetric in flux reversal the spin-up T_u and spin-down T_d components are asymmetric under flux reversal. These transmission coefficients show AB-oscillations with flux periodicity of $2\pi\phi_0$. We have verified this behavior of the reflection and transmission coefficients for various values of wave vector k of the incident electron and impurity strength G. These observations are consistent with the reciprocity relations for transport in multichannel systems[16] and are a consequence of the general symmetry properties of the Hamiltonian[2]. The transmission coefficient at flux ϕ for the case when the incident particle is spin-up and the impurity is spin-down is equal to the transmission coefficient for the case when incident particle is spin-down and impurity is spin-up but the flux direction is reversed. For the spin-polarized transport the total polarization $T_u - T_d$ is related to the spin-conductance[15]. The above symmetry properties imply that the spin-conductance is asymmetric under the flux reversal. It should be noted that at zero temperature the total electrical conductance is calculated by summing up with equal weight-age the total transmission coefficients for all the four cases, i.e., $\sigma_z = \pm 1/2$ and $S_z = \pm 1/2$.

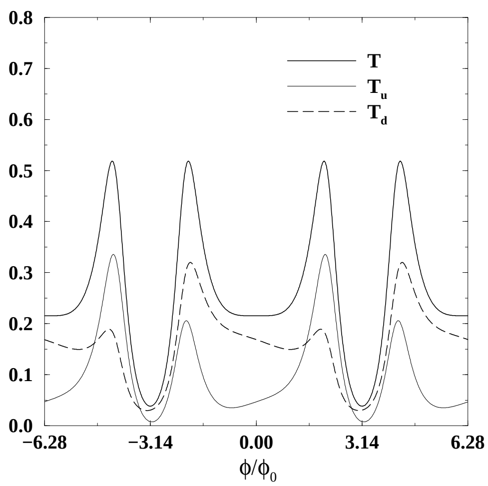

FIG. 3. Plot of total transmission coefficient T, spin-up transmission coefficient T_u and spin-down transmission coefficient T_d for the spin -flip scattering case. The parameters are $kl = 1.0$, $G = 10.0$.

As discussed in the introduction, due to multiple reflections the presence of a spin-flipper in one arm does not lead to "which-path" information. This would have implied the complete blocking of spin-down transmission. In contrast we clearly observe the AB-oscillations for the case of T_d originating from multiple reflections. We now address the question of dephasing due to the spin-flipper. To quantify dephasing , we calculate the amplitude of AB oscillations (also referred to as visibility factor) by taking the difference between the maximum and the minimum of total transmission coefficient as a function of flux ϕ over one period of the oscillation. A plot of the variation of the amplitude of oscillation of total transmission T with the interaction strength G for the two cases, no spin-flip scattering ($S = 1/2$ $m = 1/2$) and spin-flip scattering ($S = 1/2$ $m = -1/2$), is shown in figure Fig.4 with dashed line and thick line respectively. Note, however, the signature of dephasing is that the amplitude of AB oscillation of transmission coefficient for the spin-flip case is always smaller than that for the no spin-flip case for all non-zero values of coupling strength G. In other words the reduction of amplitude of AB oscillations is stronger for the spin-flip scattering case. We have verified the above observation for other parameters in the problem. Thus the presence of spin-flipper reduces or dephases the AB-oscillations and it acts as a dephasor[1,7,8].

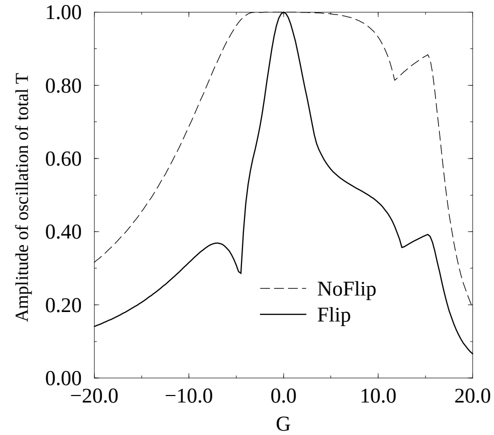

FIG. 4. Variation of amplitude of oscillation of total transmission coefficient with the interaction strength for the two cases of flip and no-flip scattering. The parameters are $kl = 1.0$ and $\eta = 1.0$.

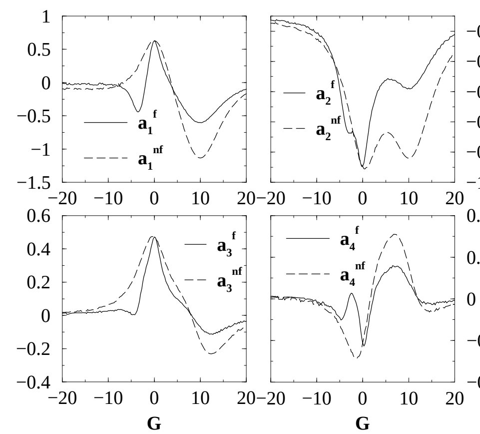

FIG. 5. Variation of n^{th} harmonic component a_n of the total transmission coefficient with the coupling strength G at $kl = 1.0$. Dashed lines are for the no-flip case and solid lines are for the flip case.

At this point we are inclined to think that the harmonic components of the total transmission $T(\eta)$ in $\eta = \phi/\phi_0$ might be able to shed more light on the issue. So, with the hope of extracting some systematics we plot the n^{th} harmonic component $a_n = \int_0^{2\pi} T(\eta) cos(n\eta) d\eta$ for $n = 1, 2, 3...$ as a function of strength G for the spin-flip scattering as well as no spin-flip scattering cases. The plots are shown in Fig. 5 for first four harmonic components. As can be seen the harmonic components do not show any systematics in the sense that the higher harmonic components can dominate over the lower harmonic components at certain values of strength G for spin-flip scattering as well as no spin-flip scattering cases. *Also, the n^{th} harmonic component for spin-flip scattering could dominate over that of the no spin-flip scattering component.* These features of the harmonic components are manifestations of the multiple scattering nature of the transport in such ballistic systems as against the observation of domination of lowest harmonic component ($n = 1$) in the case of transport in the presence of evanescent modes[17]. Guided by the naive intuition mentioned earlier we would have expected the lowest harmonic to dominate. This reiterates the important role played by the reflection at the impurity site. We would like to emphasize that irrespective of the behavior of the harmonic components (say for a particular case n^{th} harmonic component in the spin-flip case is dominant over the same n^{th} harmonic component for no-spin-flip case) the AB-oscillations of the total transmission are always dephased in the spin-flip case.

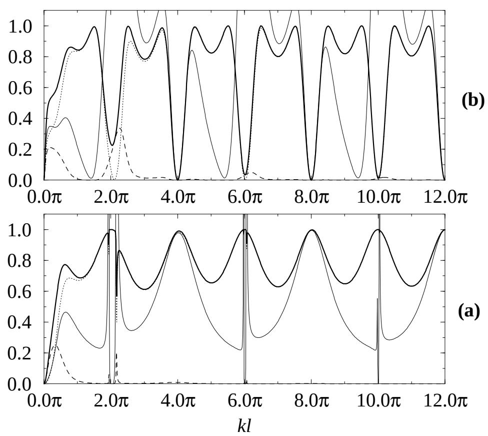

FIG. 6. The transmission spectrum of T(thick line), T_u(dotted line) and T_d(dashed line) for (a) $\eta = 0.0$ $G = 5.0$ and (b) $\eta = 1.3$,$G = 5.0$. Thin line shows the plot of the unnormalized electron probability $|\psi_3(l_3)|^2$ at the impurity site $x = l3$.

In order to make sure that nothing unusual happens at other energies we study the T, T_u and T_d as functions of kl for the case of spin-flip scattering. Figure 6 reveals an interesting fact, namely at $kl = 2\pi + 4n\pi,\ n = 0, 1, 2...$ the T_d component vanishes independent of the value of interaction strength G. In the $\eta \neq 0$ case this happens at $kl = 4n\pi,\ n = 1, 2, ...$ At these values of the incident wave-vectors the electron wave function at the impurity site happens to be zero. As a result the electron does not interact with the impurity at all and consequently there is no spin-flip scattering at these energies. However, these k-points are to be distinguished from those at which although T_d is zero but in addition $T_u = 1$, because at these resonant energies the restriction $T + R = 1$ forces T_d, R_u and R_d to be zero.

In conclusion, we have studied in detail the nature of AB-oscillations in mesoscopic ring in the presence of a spin-flipper in one of its arms. We have shown that this acts as a dephasor. The presence of magnetic impurity makes the polarized transmission coefficient asymmetric in flux reversal whereas the total transmission coefficient is symmetric in line with the theoretical expectations. Further case of asymmetrically placed flipper, spin-flipper with higher number of internal states and spin-flippers in both arms of the ring are under investigation for studying the nature of resonances, phase-shifts and dephasing.

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