# SOME STUDIES ON WAVE REFRACTION IN RELATION TO BEACH EROSION ALONG THE KERALA COAST

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## ABSTRACT

Using the British Admiralty bathymetric charts off the West Coast of India and employing the graphical method of constructing wave refraction diagrams, an attempt is made to study the behaviour of the short-period waves (4, 5 and 6 seconds) which are found to affect the coast generally in the neighbourhood of Cochin Port entrance. Nineteen stations, at intervals of roughly one mile, are chosen around the three-fathom line in this area. Considering a probable field of approach of deep-water waves, limited to a cone of  $90^{\circ}$ , five directions of approach are chosen at intervals of  $22\frac{1}{2}^{\circ}$  in the range of  $202\frac{1}{2}^{\circ}$  to  $292\frac{1}{2}^{\circ}$ . Refraction diagrams are prepared for these directions and periods, and from these, the refraction functions and directional parameters are evaluated for each station. The possible directions of flow of long-shore current and the areas vulnerable to erosion and sedimentation are investigated.

# INTRODUCTION

THE area under investigation is a twenty-mile stretch along the Kerala Coast, the neighbourhood of the entrance to the Cochin Port—which has to be maintained at a constant depth of 6 fathoms by frequent dredging as it often gets chocked with accretion materials. There is evidence of beach erosion at some points that are covered by this study.

The seaward stretch of the area of investigation extends from the coast to the fifty-fathom line. The bathymetric charts used for the purpose are British Admiralty Charts Nos. 749 and 750 (with recent corrections) which have been enlarged for convenience. A preliminary statistical analysis of the wave periods, reported in the *Indian Daily Weather Reports* for the years 192

percentage of the waves affecting the coast. Hence, this study is to waves of periods 5 and 6 seconds. However, 4-second period are also considered as the lower limit. Waves of periods less than ds are not considered because it is felt that their effect is small. Five ns for deep-water waves approaching the shore are chosen at intervals in the range of  $202\frac{1}{2}$ ° to  $292\frac{1}{2}$ °. Angles beyond these limits are left to the involved drafting errors and the possibilities of diffraction.

vals of one mile along and around the three-fathom line. Stations are to the south of Cochin Port entrance while G to S are on the northern it.

## METHOD OF STUDY

th the introduction of high speed electronic computers in recent years, struction of wave refraction diagrams has become much simplified stime-consuming (Dorrestein, 1960; Griswold, 1963; H. M. Iyer 1963). But the basic procedures remain almost the same as that in the present study, where the method suggested by Arthur, Munk acs (1952) and adopted by Pierson, Neumann and James (1955) is prepare refraction diagrams and to evaluate the refraction and directions  $\mathbf{R}$  and  $\theta$  respectively. These functions have been used to understanding of beach erosion and sedimentation along the coast.

this study, the ocean waves are represented as simple sine waves. : wave train is considered at a time in which the crests are parallel This is followed up to the coast in all successive refractions water. ent depths at intervals of one fathom applying Snell's/Law- $c_d/c$  = ine  $\alpha$ ; where  $c_d$  and c are the wave celerities in deep and shallow nd  $a_{\alpha}$  and  $\alpha$  are the angles made by the wave ray with the normal to our on the deep and shallow sides respectively. A wave ray is a line cular at every point to the successive wave crests and shows the of propagation of wave energy which again is supposed not to cross For a particular wave crest, as many wave rays or orthogonals on as practicable, which are equally spaced in deep water where all ogonals are parallel to each other. These are followed up to the f interest (stations A to S here), when, after successive refractions, ing between the adjacent wave rays and their directions are changed, ig upon the nature of the bathymetry. The changes in direction sing between adjacent wave rays are used to evaluate the refraction TABLE I
Refraction functions

Directions		2021			2250			24730			270°			29210		
Period→	<b>↓</b> 4 Sec.	5 Sec.	6 Sec.	4 Sec.	5 Sec.	Sec.	4 Sec.	5 Sec.	Sec.	4 Sec.	δ Sec.	6 Sec.	4 Sec.	5 Sec.	6 Sec.	
Stations		•														
¥	:	<b>:</b>	•	0.8208	•		0.8543	0.9871		0.8518	0.9727	1.2379	0.8241	0.8084	0.8020	
B	0.7273	•	•	0.7871	:	:	0.8342	0.7438	0.7317	0.8417	0.7780	0.9120	0.8202	0.7361	0.8846	
v	0.6861	0.6560	:	0.8302	0.7429	0.8485	0.8470	0.8618	0.9767	0.8374	0.8077	0.9509	0.8017	0.7336	0.8576	
D .	0.7327	0.6544	•	0.8119	0.7668	0.8655	0.8477	0.8292	1.1602	0.8292	0.7846	0.9451	0.7962	0.6709	0.8293	
<b>н</b>	0.7705	0.7158	:	0.8256	0.8225	1.0187	0.8490	0.8860	0906.0	0.8269	0.9805	1.0538	:	0.7443	1.0305	
Į.	0.8307	0.8172	0.9451	0.8946	0.8870	1.0987	0.8973	0.7896	1.0845	:	0.9412	1.2850	0.7752	:	:	
 5	:	:	:	•	· :	;	0.7775	:	•	0.7410	0.9152	1.0667	6099-0	0.5784	0.8357	
H	•	:	•	0.7831	0.8424	1.1176	:	0.7738	0.9898	0.7117	0.7459	0.8302	0.4360	0.4663	0.7642	
: H	0.8167	0.8454	0.9566	0.8541	0.9101	1.0110	0.8295	0.8441	1.0654	0.7646	0.8654	1.1325	0.7747	0.6580	8098.0	
:	0.8115	0.8226	0.8652	0.8537	0.8412	1.0256	0.8978	0.9894	1.4441	0.9314	0.9687	1.2168	0.8865	1.0055	1.0515	
ж ::	0.8146	0.7548	1.1153	0.9310	1.1420	1.6192	0.9179	0.9017	1.0732	0.8226	0.7238	1.0233	0.8470	0.7230	0.8010	
٦ :	0.5617	0.9370	0.7647	0.4875	0.6308	0.6564	0.7121	0.6785	0.5758	0.7644	0.8182	1.1683	0.7170	0.5856	0.8105	
	0.8567	0.5152	0.5958	0.7163	0.6193	0.5949	0.8344	0.8291	1.2365	0 • 7951	0.7294	0.8198	0.6954	0.6680	0.9331	
:	0.6519	0.5648	0.6138	8908.0	0.7739	0.9093	0.8313	0.7567	0.9357	0.7840	0.8252	0.9836	0.6933	0.7969	0.9501	
•	0.7871	0.8571	0.7709	0.8355	0.8633	1.3086	0.8765	0.9136	1.1408	0.8511	0.8148	1.0792	0.8074	1.1475	1.0480	
:	0.6371	0.3728	0.7720	0.7782	0.8334	0.8156	0.7597	0.6942	0.9598	0.8012	0.9442	1.2512	0.7298	0.7088	0.8240	
•	1.1225	0.8631	0.7420	0.9380	0.7191	0.8139	0.9483	0.9989	1.5191	0.9053	0.9165	1.1220	0.9273	0.9611	0.8410	
ж :	0.9459	1.0070	1.8940	0.8648	0.9425	1.2830	:	0.9882	1.0829	0.7398	0.7033	0.7289	0.7972	0.6864	:	
S	0.7545	0.8306	:	0.8653	1.0804	1.2310	:	0.7762	0.8950	0.8703	\$69 <b>8</b> ·0	1.4460	:	:	:	

TABLE II Direction functions

Directions		20230			S1	2250			2475°			2700			292 <u>1</u> 0	
Period→ Stations	4 Sec.	Sec.	6 Sec.	♣ Sec.	, ,	Sec.	Sec.	4 Sec.	Sec.	6 Sec.	4 Sec.	Sec.	6 Sec.	4 Sec.	5 Sec. (	6 Sec.
A	:	•	:	+	4.50	:	:	06.8+	+4.57	•	08.0-	+1.47	+ 0.48	- 2.10	- 3.94	6.79
, <b>:</b>	09:8 +	•	:	+	3.40	:	:	+1.60	+3.30	+ 6.63	:	-1.40	- 2.72	- 2.80	4.45	-11.24
: V	+ 7.10	7.10 +12.90	:	+	+ 02.2	8.56	+10.58	+1.16	+2.50	+ 2.46	-2.40	-2.90	02.9	- 3.40	- 9.83	-13.80
n,	+	6.40 +12.74	:	+ 2.	2.40 +	B-22	+ 7.65	+0.30	+0.98	+ 0.10	-3.10	-4.72	09.8	- 4.60	-12.28	-16.82
<b>=</b>	+	5.26 +10.13	•	+	1.60 +	4.33	+ 3.88	02.0-	-0.67	- 1.90	-5.02	09.9-	08.9 -	- 4.80	-12.24	-16.05
<b>:</b>	+	3.56 + 7.40 +12.05	+12.05	+	1.20 +	3.60	+ 3.30	-0.20	-0.33	- 2.83	:	-5.63	<b>8.</b> 40	. :	:	:
:	:	:	•	•		:		+4.10	:	•	-1.50	-4.00	- 8.15	- 7.80	-15.73	-18.37
н ::	:	•	:	+ 1.	1.50 +	3.90	+ 7.14	-3.80	-3.40	- 3.25	-7.30	-6.25	-13.70	-16.80	-23.27	-26.17
н	+ 2.00 +	86.8	8.98 +12.40	• +	0.15 +	3.68	+ 1.92	-3.60	-4.37	- 6.64	-9.42	-7.57	-17.95	-14.80	-21.47	<b>-24.</b> 20
Ţ.		+ 7.70 +10.80 +16.04	+16.04	÷ +	+ 00.8	3.08	+ 6.03	-0.30	-2.28	0.20	-2.90	-2.37	92.1	00.4	-11.10	-12.50
Ж ::		+13.37 +15.47	+28.50	+10.55	+ 99	9.70	+18.25	09.9+	+5.73	+13.82	+3.10	-8.00	+ 1.72	- 1.90	- 5.60	- 8.30
		+13.40 +24.57 +34.70	+34.70	+13.60	+ 09	+15.00	+23.00	+6.30	+3.80	+ 9.10	+0.63	-1.67	- 3.67	- 5.10	-10.27	-16.02
M		+11.40 +20.48 +23.60	+23.60	÷ +	<b>6.</b> 10 +	9.50	+11.28	-0.40	+0.63	- 1.03	-4.70	-4.42	-10.92	-11.52	-16.75	-21.67
:		+ 9.10 +14.75 +20.20	+20.20	+	+ 55.5	5.10	+ 6.84	-0.50	-1.00	- 2.12	-6.04	-6.15	-10.02	-11.10	-18.05	-22.67
0	+ 9.48	+18.58	+18.50	+ 4.	4.06 +	3.60	+ 7.60	+1.00	-1.22	- 1.04	-3.30	-4.38	-10.30	08.80	- 16.07	-17.90
: H		+11.10 +15.75 +17.10	+17.10	+	5.20 +	90.9	+10.78	+0.08	-1.45	+ 0.87	-4.70	-4.10	-10.76	:	-14.25	-17.25
	+	7.40 +10.78 +14.40	+14.40	+ 5.6	2.90 +	4.25	+ 4.60	90.0+	-1·55	- 2.52	-2.76	-0.30	- <b>6</b> ·30	:	-10.40	-14.35
ж :	+	<b>8</b> ·20 +19·00 +21·60	+21.60	+ 8:	+ 00.8	<b>₹.</b> 88	+11.60	+2.50	+1.64	+ 9.8	-1.40	-1.80	- 7.02	:	-11.62	:
s s	+10.88	+16.98	*	+ 6.5	+ 06.9	7.74	+11.70	+0.50	+1.84	+ 8.00	-3.80	-2.12	- 7.50	:	:	<b>:</b>
	_		-													

function R and the direction function  $\theta$  at the point of interest. The refraction function R is given by

$$R = [K (F, \theta)]^2 = \frac{b_d/b}{\frac{c}{c_d} + \frac{4\pi d}{L} \frac{1}{\sinh \frac{4\pi d}{L} \frac{c_d}{c}}}$$

where L is the wavelength, d the depth at the point of interest,  $C_d$  and C the wave velocities in deep water and at depth d respectively,  $b_d$  and b the distances between two adjacent orhogonals in deep water and at depth d respectively and F is the frequency of the wave. The refraction parameters are evaluated for each wave period and direction from the refraction diagrams prepared for each individual direction and period.

#### DISCUSSION

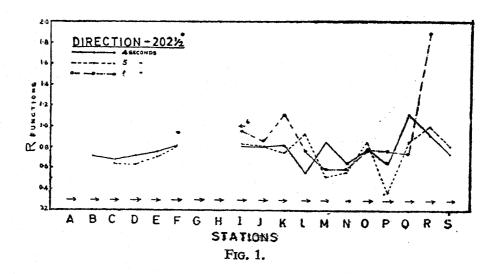
In Tables I and II are presented the refraction and direction functions, worked out at each of the stations for varying directions and periods. Table III presents the angles that the refracted rays make with the normal drawn to the contour at the point of interest. Figures 1 to 5 show the variation of R values from stations A to S with different directions and periods, while the arrow marks at each station show the probable direction of flow of long-shore currents derived from the direction function  $\theta$ .

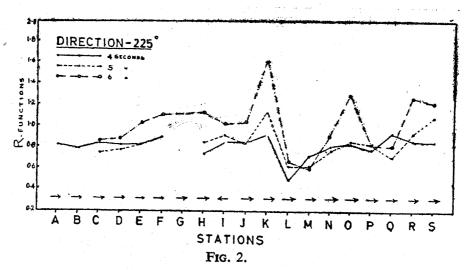
The distribution of R values along the stations is directly related to the amount of energy associated with the changed wave height which can be shown to be equal to  $\sqrt{R}$  times the deep-water wave height. Hence the convergence and divergence of wave rays, deciding R and  $\theta$  values at a particular station, speak for the accumulation and dissipation of wave energy as no energy crosses wave rays. The change in wave direction at the station, seen as the change from the deep-water direction, and the angle made by the wave ray with the normal drawn at the station to the mean contour on the landward side, gives us an idea about the direction of flow of the longshore current. The accumulated water, after the breaking of the waves, goes back as rip current and the like, only after an alongshore flow for a certain distance, depending upon the angle at which the wave breaks with the shore line (Per Bruun, 1963). Suggestions are therefore made regarding the longshore drift of beach materials along with the longshore currents at each station. The general pattern of erosion and accumulation of beach material follows from this.

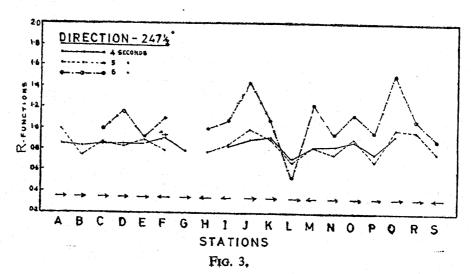
TABLE III

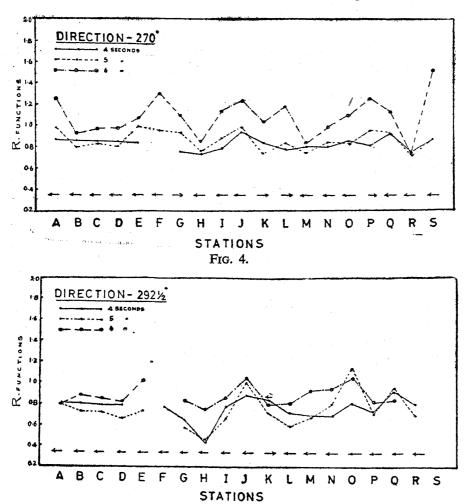
Direction functions
(With respect to the Normal at the Station of

					<u>₹</u>	(With respect to the Normal	t to the N		at the Station contour)	ion cont	our)				
Directions		2023°			225°			24150			2700			29240	-
Period→	4 Sec.	5 Sec.	6 Sec.	4 Sec.	S Sec.	6 Sec.	4 Sec.	5 Sec.	6 Sec.	♣ Sec.	5 Sec.	6 Sec.	4 Sec.	5 Sec.	6 Sec.
Stations															
 4	•	•	•	+34.50	:	:	09.4 +	+11.93	:	- 5.20	7.47	- 6.48	-26.40	-24.56	-21.11
· A	+55.90	•	:	+38.60	:	:	+ 7.90	+15.20	7.90 +15.20 +12.87	•	- 4.60	- 0.28	-22.70	-21.05	-14.26
:		+47.40 +41.60	•	+ 29.30	+23.44	+21.42	+ 8.34	+ 7.00	40.7 + 0	-10.60	-10.10	- 7.30	-32.10	-25.67	-21.70
Ð.	. +44.10	+44.10 +37.76	:	+25.60	+21.78	+20.35	+ 5.20	+ 4.52	3 + 5.40	- 13.90	-12.28	07.8 -	- 34.90	27.22	-22.68
Ħ:		+45.24 +40.37	:	+26.40	+23.67	+24.12	+ 6.13	+ 6.17	07.4	-14.98	-10.40	-10.20	-34.70	-27.26	- 23.45
: F-		+25.94 +22.10 +17.45	+17.45	+ 5.80	+ 3.40	+13.70	+ 4.70	-15.17	-12.73	•	-32.37	- 29.60	:	:	:
្ <b>ះ</b> ប		:	:	:	•	•	+19-40	:	•	+ 2.50	+ 5.00	+ 9.15	-13.70	- 6.77	- 3.13
н		•	•	+17.50	+15.10	+11.86	+ 0.30	- 0.10	- 0.25	-18.70	-19.75	-12.30	-31.70	-25.23	-22.33
н	+ 09.9 +	+ 2.52	06.0 -	-15.15	-14.68	-12.92	- 29.90	-29.13	-26.87	-46.58	-48.43	-38.05	-63.70	-57.03	-54.30
· ·	+40.80	+40.80 +37.70 +32.46	+32.46	+23.00	+22.92	+19.97	08.8 +	+ 5.78	+ 3.70	-16.10	-16.63	-11.24	-34.50	-30.40	-29.00
ж		+73.13 +71.03 +58.00	+ 58.00	+53.48	+54.30	+45.75	+34.90	+35.77	+17.68	+15.90	+15.90 +16.00	+17.28	- 1.60	+ 2.10	+ 4.80
H H		+58.10 +46.93 +33.80	+33.80	+35.40		+34.00 +26.00		+22.70	+20.20 +22.70 +17.40	+ 3.37	+ 5.67	+ 7.67	-13.40	8.23	3.40
M	+23.10	+23.10 +14.02	+10.90	06.9 +	+ 2.50	+ 0.72		-10.10 -11.13	- 9.47	- 28.30	- 28.58	-22.08	-43.98	-38.75	-33.63
z		+45.40 +39.75 +34.13	+34.13	+27.56	+26.90	+25.16	+10.00	+10.60	+10.00 +10.60 +11.62	96.9 -	- 6.85	20.0 +	-24.40	-17.45	-12.83
. 0		+42.02 +35.92 +33.00	+33.00	+24.94	+25.40	+25.40 +21.40	+ 5.50	+ 7.72	+ 7.54	-12.70	-11.62	- 5.70	- 29.70	-22.43 -	-20.60
A		+57.40 +52.75 +51.40	+51.40	+40.80	+40.94	+35.22	+23.42	+24.95	$+22 \cdot 63$	+ 5.70	+ 5.10	+11.76		- 7.25 -	- 3.25
:		+43.10 +39.77 +36.10	+36.10	+22.10	+83.75	+23.40	+ 5.44	+ 7.00	+ 8.02	-14.24	-16.70	-10.70	:	-29·10 -	-25.15
		+56.20 +45.50 +42.90	+42.90	+34.00	+37.12	+30.40	+17.00	+17.86	4 9.67	- 1.80	- 1.20	+ 4.02	:	-13.88	:
: :	+32.62	+26.55	•	+16.10	+13.26	+ 9.30	- 2.30	- 3.34	- 4.50	-20.20	-21.88	-16.50	:	:	:
	-					1			-			,			









As has been reasoned before, the lower period waves, like the ones considered in this study, are the most expected to have any continuous action on the shore line, barring the occasional storms that may send swells or longperiod waves to make sudden impacts on it. In the latter cases, however, the energy associated with the waves at breaking would be very high, compared with those of the lower periods studied here. The refraction functions therefore would be much higher than those found in the cases of 4, 5 and 6second waves. The waves however will break at greater depths and the total effect may vary much from those expected from the wave actions of lower periods.

FIG. 5.

In general, the refraction functions show increasing values from 4-second to 6-second waves as expected. The interesting point, however, is that while this is gradual between 4 to 5 seconds it is abrupt between 5 and 6-second periods. Thus, it is felt that any significant wave action in this region may be due to waves around 5 seconds and above as the energy associated with still lower periods will be too low to have much bearing on this. This is readily verifiable from Figs. 2, 3 and 4 where the R values for 4 and 5 seconds seem to lie close to each other and follow closely for the directions 225°,  $247\frac{1}{2}$ ° and 270°. In the limiting cases, *i.e.*, for angles  $202\frac{1}{2}$ ° and  $292\frac{1}{2}$ ° (as can be seen from Figs. 1 and 5) this trend in R values is not well defined, probably because of the drafting errors associated with the acute angles of approach of the waves.

The R functions as can be seen from Table I are mostly less than 1.0 for 4 and 5-second waves for all the deep-water directions. This shows a lessening of wave height in general and therefore dissipation of the energy associated, which may be aggravated by local wind actions, bottom percolations and the like. The 5-second waves, however, show R function to be more than the value of 1.0 at certain stations with varying directions such as at K and S for  $225^{\circ}$  and at J and O for  $292\frac{1}{2}^{\circ}$ . This, however, will be discussed later along with the station characteristics. For 6-second period, R mostly shows a value of more than 1.0 for all the five directions of approach and at particular stations (Table I) this is seen to be persistent as at stations E and F on the southern side of the fairway channel and at J, K and O on the northern side of it. The maximum and minimum R values for all the periods and the directions are found to be 1.89 (at station R for period 6 seconds and direction  $202\frac{1}{2}^{\circ}$ ) and 0.37 (at station P for period 5 seconds and direction  $202\frac{1}{2}^{\circ}$ ).

In Table II are presented the net changes in directions, brought in by refraction, indicated with positive and negative signs so that when added algebraically to the corresponding deep-water directions, the direction of arrival of the wave at the point of interest is obtained. It is seen from this table that up to a deep-water direction of 225° the changes in direction are positive whereas beyond 270° the changes are negative for all stations, i.e., up to 225° the angles increase and beyond 270° they decrease. The deep-water direction 247½° shows the characteristics of transition and the changes in angles here are mixed indifferently for different periods. There are positive and negative values for each of the periods which vary stationwise. Table III shows the angles that the refracted wave rays make with the landward drawn normals to the depth contours of the station. A rough idea of the alongshore currents can be had from the values given in the table.

In the perspective of the above findings the following suggestions, regarding the longshore flow, may be offered. For  $202\frac{1}{2}^{\circ}$  deep-water direction, the northerly component of longshore current is predominant for all the periods excepting a singular case in station I for 6-second waves. For the direction  $292\frac{1}{2}^{\circ}$  the direction of flow of longshore currents is towards south

in general excepting at station K where for 5 and 6-second wave periods the flow is found to be northerly. While these are as expected, being the two marginal directions, the refracted wave rays for the other deep water directions show very interesting behaviour as seen from Figs. 2, 3 and 4. Here, the longshore current is not northerly or southerly at all stations as in the marginal cases. The changes in longshore currents associated with the changes in peak and trough values of refraction functions give very plausible hints for the erosion and accumulation patterns for different waves.

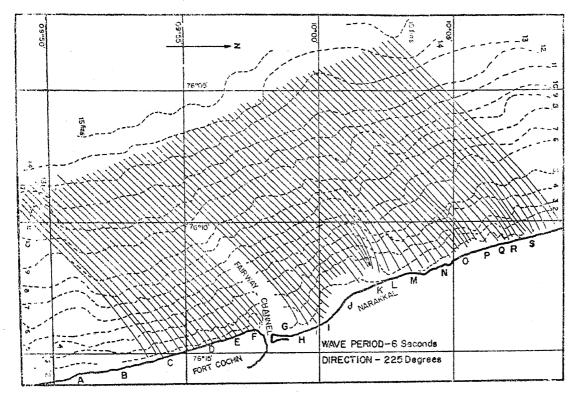


Fig. 6.

As seen from Figs. 1 to 5 there exist alternate peaks and troughs for R in the northern side of the fairway channel, *i.e.*, from stations G to S. This however is less true for the southern side, *i.e.*, for the stations A to F. Thus the southern side is likely to be less disturbed by the wave refraction and this is more so as there are few cases of energy concentration due to convergence of wave rays. The situation at station F is somewhat different and the high value of R here is mostly associated with an increased wave height which is due to the shoal immediately south of the channel (Fig. 6). Considering the directions  $225^{\circ}$ ,  $247\frac{1}{2}^{\circ}$  and  $270^{\circ}$  for stations F to S it is found that the longshore current directions diverge out from certain stations and converge to certain others (Figs. 2 to 4). And this whole process

keeps changing with changing deep-water directions and periods. These factors can most presumably be utilised to explain the facts that (i) the fairway channel gets filled up with materials carried in from both sides and (ii) erosion takes place near the stations J and K (Narakkal area). In Fig. 6 it can be seen that at station K the wave rays converge strongly.

Station O is another point where a peak for R exists due to convergence of wave rays for all the directions and periods with the only exception for a deep-water direction of 270.° Hence, this point is also susceptible to denudation due to wave action.

#### SUMMARY

The investigations on wave refraction along the coast near Cochin reveal the possible existence of areas of concentration of wave energy at certain points and removal of the material of the beach, which is disturbed near areas of concentration of wave energy by diverging longshore currents and the accretion of materials at certain regions due to converging longshore currents. The accretion of material in the fairway channel near Cochin and the reported beach erosion near Narakkal (situated near station K in Fig. 6) could be clearly explained with the help of this investigation.

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