

Return to return point memory

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We describe a new class of systems exhibiting return point memory (RPM) that are different from those discussed before in the context of ferromagnets. We show numerically that one dimensional random Ising antiferromagnets have exact RPM, when configurations evolve from a large field. However, RPM is violated when started from some stable configurations at finite field, unlike in the ferromagnetic case. This implies that the standard approach to understanding ferromagnetic RPM systems will fail for this case. We also demonstrate RPM with a set of variables that keep track of spin flips at each site. Conventional RPM for the spin configuration is a projection of this result, suggesting that spin flip variables might be a more fundamental representation of the dynamics. We also present a mapping that embeds the antiferromagnetic chain in a two dimensional ferromagnetic model, and prove RPM for spin-exchange dynamics in the interior of the chain with this mapping.

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Many systems exhibit the remarkable phenomenon of “Return Point Memory” (RPM) [1]. This is most easily demonstrated by looking at hysteresis loops for ferromagnets, Figure 1, where an external field H is lowered from saturation to a minimum value H_{min} and then raised by some intermediate amount to H_{max} before being lowered again to H_{min} . In systems with RPM, the state of the system the second time the field reaches H_{min} is identical to the first. The result generalizes to more complicated variations in H , under the constraints shown in Figure 1.

One of the most well known demonstrations of this phenomenon is Barkhausen noise [2] where the noise observed in changing H of a ferromagnet is highly reproducible under repeated cycling of the field [3]. These experiments show that not only does the system return

to a state having the the same total magnetization but also gives evidence that the domain wall configurations are identical.

Many systems *do not* exhibit RPM. For example spin glass Hamiltonians with both ferromagnetic and antiferromagnetic bonds, violate return point memory and instead exhibit subharmonic limit cycles under the application of a periodic field [4]. Therefore from a theoretical point of view, it is of interest to try to find the conditions for a system to exhibit RPM. A major advance in this direction came from a proof of return point memory for systems with purely ferromagnetic interactions[5]. For a broad class of ferromagnetic models with zero temperature dynamics, RPM was elegantly proven [5] by building on the earlier no-passing theorem for charge density waves [6].

The essential ingredient in the proof [5] of RPM is that a partial ordering of spin configurations is preserved under the application of H . By partial ordering, we mean that if one has two spin configurations α, β corresponding to spins $s_1^\alpha, \dots, s_1^\alpha$ and $s_1^\beta, \dots, s_1^\beta$ then one says that $\alpha \geq \beta$ if $s_i^\alpha \geq s_i^\beta$ for all i . Preservation of this ordering means that if a time dependent field $H_\alpha(t)$ is applied to α , and a smaller field $H_\beta(t) \leq H_\alpha(t)$ (for all time t) is applied to β , then $\alpha \geq \beta$ for all times. That is, α cannot “pass” β . This no-passing constraint was proved to be valid for ferromagnetic systems with single spin flip dynamics[5].

An interesting question to ask is whether or not all systems satisfying RPM require similar conditions as were needed to prove the ferromagnetic case[5]. In more detail, is it necessary for *every* pair of two states, that satisfy $\alpha \geq \beta$, to satisfy the no-passing requirement? Here we show that, rather surprisingly, the answer to this question is no. This is shown by examining fully antiferromagnetic Ising chains in one dimension, with zero temperature (deterministic) dynamics identical to those used for ferromagnetic systems. We find that when started from a large H and fully saturated magnetization, the system always satisfies RPM. However, unlike the ferromagnetic case, if one starts in a random state that is stable at

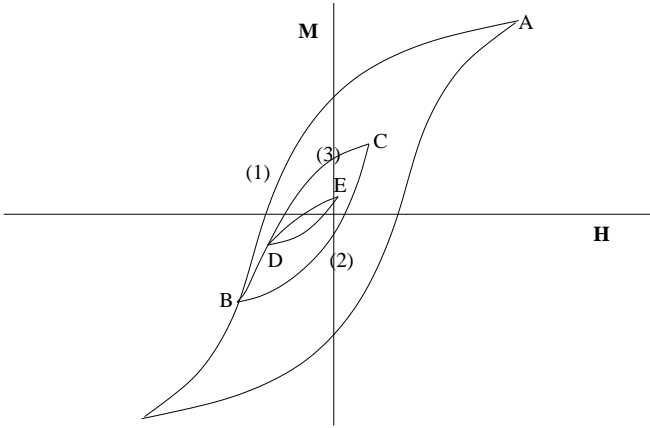


FIG. 1: Schematic of hysteresis loop for a ferromagnet, showing return point memory (RPM). RPM is seen on the trajectory ABCB, or ABCDED, or in general when H backtracks to a value that does not cross the previous extremum (e.g. E cannot cross C for the path ABCDED to show RPM). For zero temperature single spin flip dynamics, it can be proved [5] that the full spin configuration on branch (3) is bounded by (1) and (2), whence RPM follows.

some large field H_s , and lowers the field to H_{min} , then raising the field to $H_{max} < H_s$ and returning it to H_{min} changes the state. From a practical perspective, this is not a severe restriction, since if H is saturated in the distant past, RPM is valid for any subsequent evolution of $H(t)$ [7]. However, for the purpose of proving RPM, this implies that there is *no* definition of the bounding operator “ \geq ” (at least none for which $\alpha \geq \alpha$ is true of all α) with which a proof along the lines of the ferromagnetic case can be constructed.

Even for states descended from the saturated state, for which RPM is satisfied, we find that no-passing is violated for the spin configuration: in Figure 1, the spin configuration on branch (3) is not bounded above and below by (1) and (2) respectively. However we have been able to construct a new “spin flip” variable that does satisfy no-passing when starting from a high field. (This is not true starting from a random configuration; the remarks of the previous paragraph apply to any variable and bounding operator.) No-passing for the spin flip variable implies that it also satisfies RPM. (However, as with the spin configuration, RPM for a variable does not imply no-passing.) Since the spin configuration can be obtained as a projection of the flip state, this version of RPM is stronger, suggesting that this new variable may be a more fundamental way of understanding these systems.

We also investigate whether if one moves beyond single spin flip dynamics [9] it is possible for the antiferromagnetic chain to show RPM in the same sense as the ferromagnetic case. We were able to show that with spin exchange dynamics[10] that, aside from the ends, conserve magnetization, antiferromagnetic chains do show RPM in the same configuration-independent way that ferromagnets do. We did this by embedding this one dimensional antiferromagnetic problem in a two dimensional *ferromagnetic* system which has single spin flip dynamics and therefore shows return point memory. However this mapping must fail for single spin dynamics in a rather interesting way; because under this mapping, single spin flips become nonlocal and the standard proof[5] does not apply[11] Therefore it is not just the Hamiltonian, but the dynamics as well, that determine whether or not a system satisfies RPM.

We consider the random antiferromagnetic Ising model:

$$\mathcal{H} = - \sum_i [J_i s_i s_{i+1} + (h_i + H) s_i] \quad (1)$$

where the bonds J_i and local fields h_i are independent random variables. All the J_i 's are negative, and the h_i 's are equally likely to be positive and negative. H is the externally applied field. Initially, H is large and positive, and all the spins point up. Thereafter, the field is changed adiabatically. At any field, a spin is flipped if doing so reduces the energy \mathcal{H} of the system. This spin flip can render other spins unstable, in which case the process is repeated till there are no more spins to flip. If several spins are unstable, the one whose flipping re-

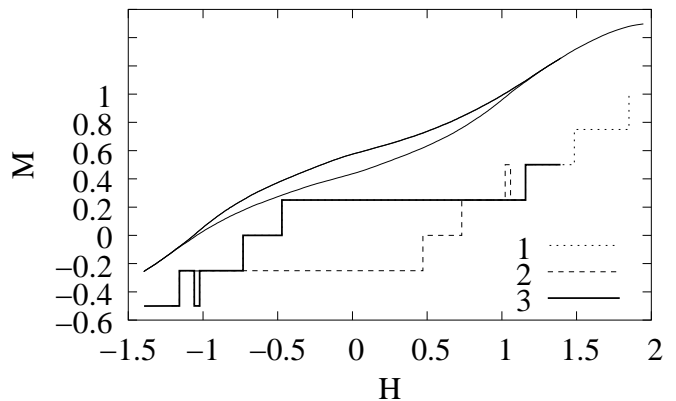


FIG. 2: Hysteresis curve for a one-dimensional Ising chain of length 8, where the bonds are all antiferromagnetic and of random strength. The lower curve shows the hysteresis curve for a single realization of randomness, with H lowered from ∞ to -1.4 (curve 1), raised to 1.4 (curve 2) and then lowered to -1.4 again (curve 3). The magnetization M changes contrary to H on (2) at $H \approx 1.05$ and on (3) at $H \approx -1$; apart from this excursion, (3) coincides with (1) from $-1.4 \leq H \leq 1.4$. The upper plot, shifted vertically by $M = 0.5$ for clarity, is a similar graph for a chain of length 64000. Curves (1) and (3) are different, but so close as to seem indistinguishable in this plot. However, return point memory (at $H = -1.4$) is exact.

duces the energy the most is flipped. However, because an avalanche propagates outwards from the original site, and the left and right propagating directions are disjoint, the same results would be obtained if all unstable spins were flipped simultaneously. For all the numerical results reported in this paper, $\sim 10^7$ random choices of $\{J_i, h_i\}$ were tested.

Figure 2 shows a typical hysteresis loop, with random bond disorder but no random fields ($h_i = 0$). The bonds are drawn from a distribution uniform over $[-1, 0]$. Return point memory is seen at $H = -1.4$ the hysteresis loop. RPM is also found when the h_i 's are drawn from a distribution uniform over $[-1, 1]$, if $J_i = -1$ for all i . In both cases, although it cannot be shown in the figure, RPM exists for the full spin configuration rather than just the overall magnetization. However, if the J_i 's are not equal *and* the random fields are non-zero, we find that RPM fails if $\delta h \gtrsim 0.01$ and $\delta J \gtrsim 0.01$ in the random bond and random field cases respectively. We therefore conclude that either δh or δJ must be zero for RPM. The results are the same for open and periodic boundary conditions [8]. (Even when δh and δJ are both non-zero, the deviation from RPM is quite small, and hard to detect if one averages the hysteresis loop over realizations of randomness. A similar phenomenon was observed earlier for Sherrington-Kirkpatrick spin glasses [12].)

An important difference between the ferromagnetic and antiferromagnetic cases is that a spin at a single site can flip several times while the magnetic field is varied monotonically. Thus if the field is lowered, a spin pointing up can be triggered and flip down; if its neighbors

have already flipped down, they can then be pushed back up by the new spin flip. As a result of this, the magnetization does not vary monotonically with H . This can be seen in the plot for a single realization of randomness in Figure 2. In more detail, it is possible to observe that i) an avalanche that starts from a site and destabilizes both its neighbors is only possible for a configuration $\downarrow\uparrow\downarrow\downarrow$ going to $\downarrow\downarrow\uparrow\downarrow$ (or its mirror image), where the initial site is in the middle. The next nearest neighbors are stabilized, and the avalanche only covers three sites. ii) an avalanche that starts from a site and destabilizes only one neighbor is only possible for a configuration $\uparrow\downarrow\downarrow$ going to $\downarrow\uparrow\downarrow$ (or its mirror image), where the initial site is at the end. The avalanche only covers two sites. Thus as H is varied, the chain evolves through single spin flips, two-site avalanches with $\Delta M = 0$ and three-site avalanches which have $\Delta M = 1$ for decreasing H and $\Delta M = -1$ for increasing H . These results and more have been proved earlier with random field disorder (without bond disorder), for the major hysteresis loop [13]; the full shape of the hysteresis loop is found analytically [13]. We have extended the results of [13] to prove ii) for the major loop and i) for the entire hysteresis curve [15].

Motivated by the observation of retrograde variation of the magnetization with H , we construct an alternative representation of the dynamics in terms of spin flips instead of the spin configuration. Initially, when all the spins point up, the flip variable is zero at each site. Thereafter, each time a spin at site i is reversed, the flip variable l_i is increased by 1 if this happens when the field H is increasing, and decreased by 1 if this happens when H is decreasing. Clearly, along any branch of the hysteresis loop, while H varies monotonically, so must each l_i . Also, $s_i = 1 - 2[l_i \bmod 2]$, and if two configurations α and β satisfy the condition that $l_i^\alpha - l_i^\beta$ is even for all i , they correspond to the same spin state. In our numerical simulations, we find that for the cases when RPM is valid, it also holds for the flip configuration. Since the configuration $\{s_i\}$ is a projection of $\{l_i\}$, this is a stronger result than RPM, and suggests that the underlying dynamics in terms of $\{l_i\}$ is fundamental to random antiferromagnetic chains.

With $m_i = \sum_{j \neq i} l_j$, we find that no-passing is satisfied: if H is decreased from H_{max} to H_{min} , increased to H_{max} and then returned to H_{min} , for any H and any site i the value of m_i on the third segment of this path is bounded below and above by the corresponding m_i 's on the first and second segments (see Figure 1). This is not true for the spin variables s_i [16]. However, as emphasized earlier, RPM is not satisfied if one starts from an arbitrary initial state at some H instead of the saturated state, so that unlike the ferromagnetic case [5] the proof of RPM must take into account the ancestry of a state.

In order to see to what extent RPM is influenced by the dynamics used for the model, we now consider spin-exchange dynamics [10] instead of single spin flip [9]. A pair of neighboring spins that are oriented opposite to each other are exchanged if it is energetically favorable

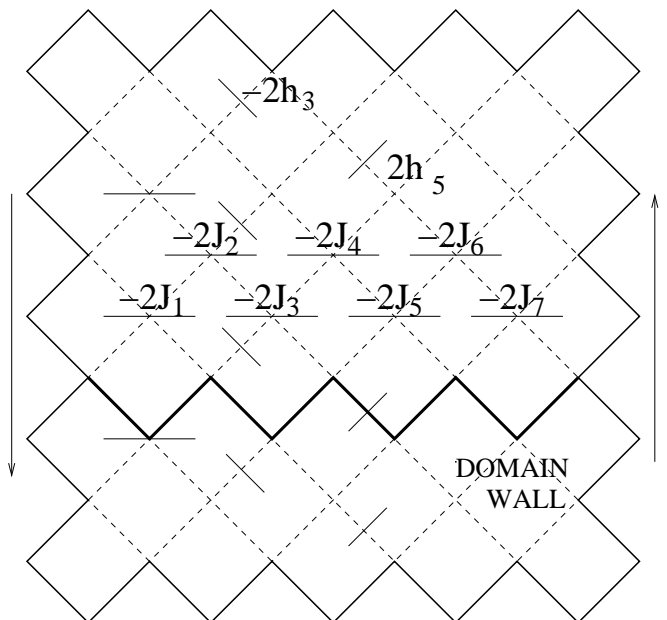


FIG. 3: Two dimensional lattice with ferromagnetic bonds. The dashed lines are to guide the eye; the spins are at the center of each dashed diamond. The spins are forced to be up and down at the top and bottom boundaries respectively. The domain wall in between maps to a one-dimensional spin chain. The case shown corresponds to a chain of (eight) alternating spins. The horizontal and diagonal bonds in the two dimensional lattice correspond to the random bonds and fields respectively of the chain; for the system shown, $h_3 < 0$ and $h_5 > 0$. All bonds in a vertical column are the same; for clarity, only some are shown. An external field H on the chain is equivalent to a field at the side boundaries, increasing as shown with a gradient H .

to do so. Since such a move does not change the overall magnetization, in order for there to be a response to a magnetic field, we allow single spin flips at the two ends of the chain (only open chains are considered).

This problem can be solved by embedding the anti-ferromagnetic chain in a two dimensional ferromagnetic model. We first consider the case when there is only random bond disorder. Figure 3 shows a two dimensional square lattice of spins, rotated by an angle $\pi/4$. Ferromagnetic bonds connect next nearest neighbors, but (without random fields) not nearest neighbors. As shown in the figure, the vertical bonds are all zero, and the horizontal bonds are identical within each vertical strip. At the top and bottom boundaries, the boundary conditions force all the spins to be up and down respectively. Free boundary conditions are used on the side walls. Thus in its ground state, there is one horizontal domain wall across the system. As shown in the figure, we adopt a convention in which the domain wall consists of line segments oriented at $\pm\pi/4$, i.e. along the principal directions of the square lattice. The mapping from the two dimensional system to the one-dimensional chain is as follows: if any line segment of the two dimensional domain

wall is oriented at $\pi/4$ or $-\pi/4$, the corresponding spin in the antiferromagnetic chain is 1 or -1 respectively. The two dimensional ground state corresponds to alternating spins in the chain, as is appropriate when H is zero.

For a general shape of the domain wall, whenever two successive segments point in the same direction, a (horizontal) bond is broken, whereas this does not happen when they point in opposite directions. By choosing the horizontal bond strengths to be $-2J_1, -2J_2, -2J_3 \dots$, correlated vertically, the energy of the antiferromagnetic chain is increased by $-2J_i$ when spins i and $i+1$ point in the same direction compared to when they are opposite, as desired for an antiferromagnetic chain. The magnetic field H couples to $\sum_i s_i$ for the chain, which is equivalent to the difference in height between the ends of the two dimensional domain wall. This is equivalent to a magnetic field H on the rightmost column of the two dimensional system, with the left end of the domain wall tethered. It is also possible to generalize the model to include random bond disorder for the chain: nearest neighbor bonds of strength $2|h_i|$ are introduced in the i 'th column, oriented at $\pi/4$ if h_i is positive and $-\pi/4$ if h_i is negative.

With this construction, all bonds are ferromagnetic for the two dimensional system. Further, the fields at the side boundaries vary monotonically with H . Further, spin-exchange for the chain is equivalent to single spin flips in the two-dimensional lattice. The results of Ref. [5]

can therefore be invoked. We conclude that, with these dynamics, RPM is valid for all configurations, and is valid for simultaneous random field and random bond disorder. As we have seen, neither of these statements is valid for single spin flip dynamics for the chain; the two dimensional analog of spin flip at a site on the chain is to move the entire domain wall to the right of the site up or down by one unit if the spin flips up or down [17].

In this paper we have shown that the hysteresis loop for a random Ising antiferromagnetic chains at zero temperature exhibits return point memory (RPM). For spin flip dynamics, the result is history dependent, being valid only for configurations that start from saturated magnetization and a large magnetic field. This is unlike the result for ferromagnets, where the result is valid for all configurations, indicating that the mechanism for RPM is different from the ferromagnetic case. (Also, RPM is only valid if either random field or random bond disorder is present, but not both, a restriction that does not apply to ferromagnets.) For spin exchange dynamics, we have proved RPM by mapping to a two-dimensional ferromagnetic model, and have therefore shown that it is as general: valid for all configurations, and with simultaneous random field and bond disorder. This implies that RPM depends on the Hamiltonian and the dynamics used.

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- [17] One could instead untether the left hand end of the domain wall, apply a field $-H$ to the leftmost column of the lattice, and adopt a convention that if a spin in the chain flips retrograde to the direction of the magnetic field, the domain wall to the *left* of the site moves appropriately. The displacement of the left end of the domain wall is equal to $\sum_i [l_i/2 - (s_i + 1)/4]$, so that this is a partial projection from $\{l_i\}$ (or $\{m_i\}$) to $\{s_i\}$. We observe that this partial projection from $\{m_i\}$ still yields no-passing (when the initial state is the saturated state).