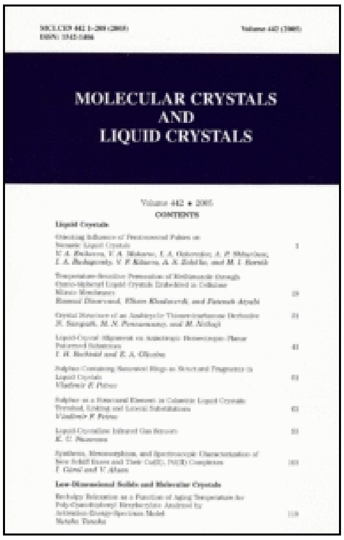

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Thermomechanical Effect in Cholesteric Liquid Crystals†

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We discuss some possible experimental geometries for studying Leslie's thermomechanical effect in cholesteric liquid crystals. The discussion includes the effect of a temperature gradient (i) along and (ii) perpendicular to the helical axis. The feasibility of observing the effect and of extracting the thermomechanical coefficients is examined.

I. INTRODUCTION

Lehmann[1] observed that droplets of cholesteric liquid crystals were set into violent rotatory motion when heated from below, implying thereby that there exists a coupling between thermal and mechanical effects in the medium. Leslie's theory[2] of cholesterics allows for such a coupling and provides an explanation of the rotation when the thermal gradient is along the helical axis. As far as we are aware, Lehmann's observations have never been confirmed subsequently. A number of attempts[3–5] have been made in recent years to study other aspects of thermomechanical coupling and to demonstrate its existence. In view of the importance of this problem we reconsider it here in two different situations.

In the first case, the cholesteric is in the Poiseuille flow geometry with its helical axis along the axis of the capillary.[6,7] Application of a temperature gradient parallel to the tube axis results in a flow with a flat velocity profile. The amount and direction of flow yields in principle Leslie's thermomechanical coefficient λ_3.

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In the second case, the cholesteric is confined between two flat parallel plates with the helical axis normal to the plates. Under the influence of a temperature gradient parallel to the plates the uniaxial symmetry of the cholesteric is destroyed. From the amount of induced birefringence one can evaluate the thermomechanical coefficient μ_8.

2. THEORY

We use the continuum theory of cholesteric liquid crystals as developed by Leslie.[2] Throughout our discussion we ignore the effects of the extra material constant α that figures in Leslie's theory.

For an incompressible cholesteric liquid crystal the governing equations are:

$$t_{ji,j} = \rho \dot{v}_i \tag{1}$$

$$g i + \pi_{ji,j} = \rho_1 \ddot{n}_i \tag{2}$$

The static and the dynamic parts t_{ji}° and t_{ji}' of the stress t_{ji} are given by

$$t_{ji}^{\circ} = -p\delta_{ij} - \frac{\partial F}{\partial n_{k,j}} n_{k,i} \tag{3}$$

Here p is the hydrostatic pressure and F is the elastic free energy density:

$$F = \frac{K_{11}}{2}(\nabla \cdot \mathbf{n})^2 + \frac{K_{22}}{2}(\mathbf{n} \cdot \nabla \times \mathbf{n} + q_0)^2 + \frac{K_{33}}{2}(\mathbf{n} \times \nabla \times \mathbf{n})^2 \tag{4}$$

K_{11}, K_{22} and K_{33} are the splay, twist and bend elastic constants. The spontaneous twist of the structure is $q_0 = 2\pi/P$.

$$\begin{aligned} t_{ji}' = {} & \mu_1 n_k n_p d_{kp} n_i n_j + \mu_2 N_i n_j + \mu_3 N_j n_i + \mu_4 d_{ji} \\ & + \mu_5 d_{ik} n_k n_j + \mu_6 d_{jk} n_k n_i + \mu_7 e_{ipq} n_j n_p T_{,q} \\ & + \mu_8 e_{jpq} n_i n_p T_{,q} \end{aligned} \tag{5}$$

Here $\mu_1 \ldots \mu_6$ are the viscosity coefficients, μ_7 and μ_8 the thermomechanical coefficients and $T_{,q}$ is the temperature gradient. Also

$$d_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}) \qquad \omega_{ij} = \frac{1}{2}(v_{i,j} - v_{j,i})$$

$$N_i = \dot{n}_i - \omega_{ij} n_j$$

The director body force g_i has a static part g_i° and a dynamic part g_i':

$$g_i^{\circ} = \gamma n_i - \beta_j n_{i,j} - \frac{\partial F}{\partial n_i} \tag{6}$$

γ and β_j being arbitrary, and

$$g'_i = \lambda_1 N_i + \lambda_2 d_{ij} n_j + \lambda_3 e_{ipq} n_p T_{,q} \tag{7}$$

Here

$$\lambda_1 = \mu_2 - \mu_3$$
$$\lambda_2 = \mu_5 - \mu_6$$
$$\lambda_3 = \mu_7 - \mu_8$$
$$e_{ijk} = \text{permutation tensor.}$$

Finally the director surface stress π_{ji} is given by

$$\pi_{ji} = \beta_j n_i + \frac{\partial F}{\partial n_{i,j}} \tag{8}$$

In addition to Eq. 1 and 2 we also have the following equation for entropy generation:

$$T\dot{S} = -q_{i,i} + t'_{ji} d_{ij} - g'_i N_i \tag{9}$$

where

$$q_i = K_1 T_{,i} + K_2 n_k T_{,k} n_i + K_3 e_{ipq} n_p N_q + K_4 e_{ipq} n_p d_{qr} n_r \tag{10}$$

Here $K_1 \ldots K_4$ are the thermal conductivity coefficient.

3. RESULTS

We apply the theory to two special situations. In either case the imposed temperature gradient is assumed to be small.

(a) Poiseuille geometry

The cholesteric is confined to a cylindrical tube with its twist axis along the axis of the cylinder and with a firm anchoring of the director at the walls. The imposed temperature gradient $T_{,z}$ is along the twist axis, i.e. z-axis.

Since $T_{,z}$ is taken to be small, we may assume the structural distortion and flow velocities to be small enough to get a first order perturbation solution. The approximations are similar to those found in Helfrich's permeation model.[6,7] Under this approximation we find no distortion of the unperturbed cholesteric structure, i.e.

$$n_x = \cos q_o z, \qquad n_y = \sin q_o z \qquad \text{and} \qquad n_z = 0 \tag{11}$$

Also the induced flow is primarily along the z-axis with a flat profile. To a very good approximation this velocity is given by

$$v_z = \frac{\lambda_3}{\lambda_1 q_o} T_{,z} \tag{12}$$

As $\lambda_1 < 0$, a negative λ_3 results in flow along the temperature gradient and a positive λ_3 in flow in the opposite direction.

The quantity of fluid flow/sec in a tube of radius R is given by

$$Q = \frac{\pi R^2}{q_o} \frac{\lambda_3}{\lambda_1} T_{,z} \tag{13}$$

Thus the direction of fluid flow and a measurement of Q yields λ_3 in sign and magnitude. The merit of this geometry is the demonstration of a simple flow phenomena induced by a temperature gradient. This process is physically the opposite of the one considered by Prost.[4] He worked out the implications of Eqs. 9 and 10 and showed that there should be a generation of a temperature gradient under permeation flow induced by an applied pressure gradient.

(b) Flow between parallel plates

The material is confined between two parallel plates at $z = \pm h$. The director orientation at the midplane is taken to be y-axis. The orientation at the two plates are given by $\varphi_o(+h) = q_o h$, $\varphi_o(-h) = -q_o h$. The imposed temperature gradient $T_{,x}$ is along the x-axis. Within the approximations indicated there is no distortion in the cholesteric twist q_o. However, there is a tilt Θ of the director towards z-axis. Also thermally induced flow has two components v_x and v_y (v_y being reminiscent of secondary flow). The differential equations governing these thermally induced variables Θ, v_x and v_y are:

$$\Theta_{,zz} + \frac{\lambda_1 + \lambda_2}{2K_{11}}(v_{x,z} \cos q_o z + v_{y,z} \sin q_o z) - \frac{K_{33}}{K_{11}} q_o^2 \Theta - \frac{\lambda_3}{K_{11}} T_{,x} \sin q_o z = 0 \tag{14}$$

$$v_{x,z}[\mu_4 + (\mu_3 + \mu_6)\cos^2 q_o z] + v_{y,z}(\mu_3 + \mu_6)\cos q_o z \sin q_o z - 2\mu_8 T_{,x} \cos q_o z \sin q_o z = A \tag{15}$$

$$v_{x,z}[\mu_3 + \mu_6]\sin q_o z \cos q_o z + v_{y,z}[\mu_4 + (\mu_3 + \mu_6)\sin^2 q_o z] - 2\mu_8 T_{,x} \sin^2 q_o z = B \tag{16}$$

A and B are constants to be obtained from the boundary conditions:

$$\Theta(\pm h) = 0, \quad v_x(\pm h) = 0, \quad v_y(\pm h) = 0 \quad \text{and} \quad v_z(\pm h) = 0 \tag{17}$$

The allowed solutions turn out to be

$$v_x = \frac{2T_{,x}\mu_8 h(\cos\xi - \cos 2q_o z)}{\xi\left[2\mu_4 + (\mu_3 + \mu_6)\left(1 + \dfrac{\sin\xi}{\xi}\right)\right]} \tag{18}$$

$$v_y = \frac{2T_{,x}\mu_8(z\sin\xi - h\sin 2q_o z)}{\xi\left[2\mu_4 + (\mu_3 + \mu_6)\left(1 + \dfrac{\sin\xi}{\xi}\right)\right]} \tag{19}$$

$$\Theta = CT_{,x}\frac{\sinh Q_z \sin q_o h - \sinh Qh \sin q_o z}{\sinh Qh} \tag{20}$$

Θ being written for large ξ with

$$A = 0$$

$$B = -\frac{2\mu_4\mu_8 T_{,x}\left(1 - \dfrac{\sin\xi}{\xi}\right)}{2\mu_4 + (\mu_3 + \mu_6)\left(1 + \dfrac{\sin\xi}{\xi}\right)}$$

$$C = \frac{\lambda_3(2\mu_4 + \mu_3 + \mu_6) - \mu_8(\lambda_1 + \lambda_2)}{q_o^2(K_{11} + K_{33})(2\mu_4 + \mu_3 + \mu_6)}$$

$$Q = q_o(K_{33}/K_{11})^{1/2} \qquad \xi = 2q_o h$$

The uniaxial symmetry along the z-axis is destroyed by the imposed temperature gradient. This induces a birefringence $\Delta\mu$ for light propagation along the z-axis. For an integral number of pitches between the plates

$$\Delta\mu = \frac{\bar{\mu}\varepsilon C^2}{8}T_{,x}^2 \tag{21}$$

Here $\bar{\mu} = \frac{1}{2}(\mu_x + \mu_y)$, μ_x, μ_y being the mean refractive indices for propagation along z and for light polarized along the x and y directions respectively, and

$$\Delta\mu = [\mu_x - \mu_y]$$

Here $\varepsilon = 2(\mu_\parallel - \mu_\perp)/\mu_\perp$, and $\mu_\parallel$ and $\mu_\perp$ are the principal refractive indices of any cholesteric layer which is locally like a nematic.

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