LAMINAR DIFFRACTION AND THE BECKE PHENOMENON.

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1. *Introduction.*

WE shall concern ourselves in this paper with the optical effects arising from the passage of light through a thin plate of transparent material, the thickness or the refractive index of the substance of which exhibits more or less abrupt variations along its surface. The relation between the diffraction effects produced by such variations of thickness or refractive index and the appearance of the boundaries at which they occur when examined under the microscope is the special topic which will receive attention.

It is obvious that the effects occurring at the laminar "edges" must depend on the precise configuration of the wavefront of light after its passage through the plate. Lummer and Rieche[1] and more recently N. K. Sur,[2] who have applied the theory of microscopic vision to the case of laminar diffraction, assume that the wavefront after the passage through the plate exhibits an abrupt discontinuity on either side of the "edge". This can however be the case only if the variation in the thickness, or in the case of the variation of the refractive index the total thickness of the plate, be small compared with the wavelength of the light used. When these conditions are not satisfied, various complications will arise and the change of phase instead of being quite abrupt will be distributed over an appreciable part of the wavefront. P. N. Ghosh[3] has considered cases in which the variation in thickness takes place gradually from one point to another and has worked out the consequent diffraction effects at a distance from the plate. The influence of the shape of the laminar edge is particularly well exhibited in the case of "mixed plates," studied in detail by Prof. Raman[4] and his collaborators,

[1] Lummer and Rieche, *Bildenstehung im Mikroskop, Fr. Vieweg und Sohn* (1910), p. 75.

[2] N. K. Sur, *Proc. Ind. Ass. Cult. Sci.*, 1922, **7**, 125.

[3] P. N. Ghosh, *Proc. Ind. Ass. Cult. Sci.*, 1921, **6**, 61.

[4] Raman and Banerji, *Phil. Mag.*, 1921, **41**, 338, 860; 1921, **42**, 679; and K. Seshagiri Rao, *Proc. Ind. Ass. Cult. Sci.*, 1923, **8**, 243.

where the edges are drawn inwards under the action of surface tension and diffract a considerable amount of light through large angles in a markedly unsymmetrical manner.

N. K. Sur, as also Lummer and Rieche proceeding on the assumption of an abrupt phase change have worked out the appearance of the edge under the microscope in accurate focus. They deduce that the edge will appear as a fine dark line bordered symmetrically on either side by a set of fine equidistant fringes whose visibility quickly fades off with increasing distance from the edge. Sur himself reports, however, that his experiments show that the fringes are distinctly asymmetric in character, the visibility and the number being greater on the retarded side than on the other. This asymmetry becomes more pronounced as the microscope is put out of focus.

2. *The Becke Phenomenon.*

When a low-power microscope is focussed on a laminar "edge" the latter is seen in focus as a dark line. As the microscope is gradually disturbed from the focus a bright line detaches itself from the dark line and moves laterally: it moves towards the retarded side as the microscope is raised from its position of focus and moves towards the other side when the microscope is lowered. These effects constitute what is known the Becke Phenomenon, the bright line being referred to as the "Becke line". When the line is caused by a variation in thickness it may be used to determine the refractive index of the specimen. Liquids of known different refractive indices are tried as films on the plate till the "Becke line" vanishes. This happens when the refractive indices of the specimen and the liquid agree completely.

The usual explanation of the Becke phenomenon is based on geometrical optics. When a convergent beam of light is used to illuminate the specimen, some of the rays striking the "edge" from the side of the greater refractive index will undergo total internal reflection. Therefore over a narrow region on the retarded side there will be more rays entering the microscope than on

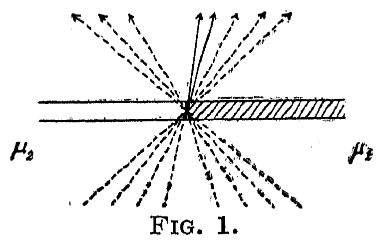

FIG. 1.

the other side. When the microscope is focussed at the edge itself the light will be brought to a point conjugate with the edge itself. But when raised from that position the microscope is really focussed on a plane above the plate and the excess of light, resulting from total internal reflection, is focussed to a point in the image plane conjugate to a point *near* the "edge". Its distance from the edge increases as the distance of the "focussed" plane from the specimen increases.

This explanation demands the disappearance of the phenomenon in parallel light incident normally to the plate. But the phenomenon is found to persist here as well. It seems, therefore, that the simple geometrical explanation is inadequate. In what follows an attempt is made to explain the asymmetry observed both in and out of focus as diffraction effects produced by the edge, by adopting a certain modification in the form of the emerging wavefront, as suggested by Prof. Raman[5] in a recent paper.

3. *The Theory.*

We consider plane waves incident normally on the specimen. Denoting by 2ρ the excess of path retardation suffered by one half of the wave over the other in its passage through the plate, we have

$$2\rho = (\mu_1 - \mu_2)\,t$$

where μ_1 is the greater refractive index, μ_2 the other refractive index and t the thickness. As mentioned before, N. K. Sur assumes that the wavefront after its passage through the plate exhibits an abrupt discontinuity on either side of the edge (Fig. 2). This can hold good only when the thickness is very small. With a greater thickness comparable with the wavelength of light, it is not likely that the discontinuity of the wavefront will be so sharp; the discontinuity will extend over a range and may be tentatively taken to have a straight slope from $-X_1$ to $+X_1$ (Fig. 3).

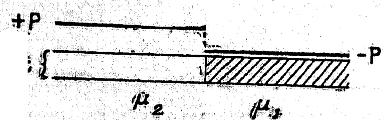

FIG. 2.

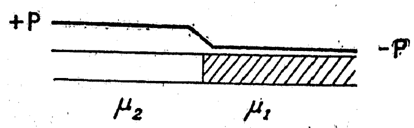

FIG. 3.

Now the phase of the wavefront as it emerges from the specimen will be represented by

$$+\rho \quad \text{from } X = -\infty \text{ to } -\frac{\rho}{k},$$

$$-kX \quad \text{from } X = -\frac{\rho}{k} \text{ to } +\frac{\rho}{k}$$

and

$$-\rho \quad \text{from } X = \frac{\rho}{k} \text{ to } +\infty.$$

$-k$ represents the slope of the wavefront at the "edge".

The expression[6] for the resultant amplitude of light wave at a point in

[5] Sir C. V. Raman, *Proc. Ind. Acad. Sci.*, 1935, 1, 585.

[6] Lummer Rieche, *Loc. cit.*, p. 90.

the image plane of the microscope conjugate to a point (x, y) in the object plane due to a luminous point (X,Y) in the object plane is given by

$$S = \frac{k}{\lambda^2} \iint_{\text{Aperture}} d\xi\, d\eta \iint_{\text{Object}} \phi(X, Y) \,.\, \sin \frac{2\pi}{\lambda} \{Vt - (x-X)\ \xi - (y-Y)\ \eta + \psi(X, Y)\}\, dX.\, dY.$$

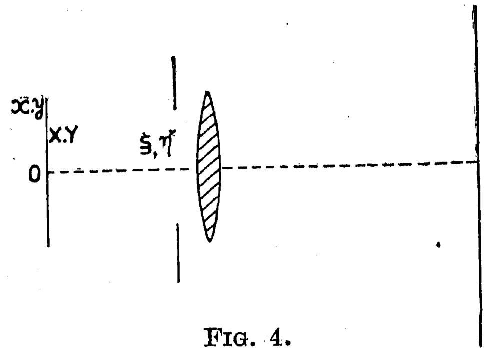

FIG. 4.

ξ, η are the angular co-ordinates of a point in the aperture with respect to O, ϕ (X, Y) is the transmission coefficient at the point (X, Y), and ψ (X, Y) the corresponding phase.

When we consider an object and a rectangular aperture both symmetric with respect to the Y axis, the expression reduces to

$$S = \frac{e}{\lambda} \int_{\text{Object}} dX \int_{-\xi}^{+\xi} \phi(X) \cdot \sin \frac{2\pi}{\lambda} \{Vt - (x-X)\ \xi + \psi(X)\}\ d\xi$$

We may take the transmission coefficient to be the same for both the media, so that ϕ (X) is a constant. The factor $\frac{2\pi}{\lambda}$ may be omitted for convenience only to be restored for numerical calculations. The variation of the phase ψ (X) over the wavefront has already been given. Then, omitting constants,

$$S = \int_{-\infty}^{-\frac{\rho}{k}} dX \int_{-\xi}^{+\xi} \sin\ \{Vt - (x-X)\ \xi + \rho\}\ d\xi$$

$$+ \int_{-\frac{\rho}{k}}^{+\frac{\rho}{k}} dX \int_{-\xi}^{+\xi} \sin\ \{Vt - (x-X)\ \xi - kX\}\ d\xi$$

$$+ \int_{\frac{\rho}{k}}^{\infty} dX \int_{-\xi}^{+\xi} \sin\ \{Vt - (x-X)\ \xi - \rho\}\ d\xi.$$

If we represent

$$\int_0^x \frac{\sin x}{x}\,dx \text{ by Si }(x), \text{ and } \int_\infty^x \frac{\cos x}{x}\,dx \text{ by Ci }(x)$$

we have

$$S = \sin Vt \cdot \cos \rho \left[\pi - \mathrm{Si}\left(x + \frac{\rho}{k}\right)\xi + \mathrm{Si}\left(x - \frac{\rho}{k}\right)\xi\right] + A \sin Vt$$

$$+ \cos Vt \cdot \sin \rho \left[\mathrm{Si}\left(x + \frac{\rho}{k}\right)\xi - \mathrm{Si}\left(x - \frac{\rho}{k}\right)\xi\right] - B \cos Vt,$$

$$\text{where}\quad A = \int_{-\frac{\rho}{k}}^{+\frac{\rho}{k}} \frac{\sin (x-X)\,\xi}{(x-X)\,\xi} \cdot \cos (kX)\, dX,$$

$$\text{and}\quad B = \int_{-\frac{\rho}{k}}^{+\frac{\rho}{k}} \frac{\sin (x-X)\,\xi}{(x-X)\,\xi}\, \sin (kX)\, dX.$$

When integrated

$$A = \mathrm{Si}\,(k+\xi)\left(x + \frac{\rho}{k}\right) - \mathrm{Si}\,(k+\xi)\left(x - \frac{\rho}{k}\right)$$

$$+ \mathrm{Si}\,(k-\xi)\left(x + \frac{\rho}{k}\right) - \mathrm{Si}\,(k-\xi)\left(x - \frac{\rho}{k}\right)$$

and

$$B = - \mathrm{Ci}\,(k+\xi)\left(x + \frac{\rho}{k}\right) + \mathrm{Ci}\,(k+\xi)\left(x - \frac{\rho}{k}\right)$$

$$+ \mathrm{Ci}\,(k-\xi)\left(x + \frac{\rho}{k}\right) - \mathrm{Ci}\,(k-\xi)\left(x - \frac{\rho}{k}\right).$$

The intensity is given by the sum of the squares of the coefficients of sin Vt and cos Vt in the expression for S. The intensity curves are drawn for different values of the path retardation 2ρ, using different values for k, with the help of tables for Ci and Si functions computed by Glaisher.[7]

In the calculations the value of ξ the semiangular aperture of the objective of the microscope is taken as 0·5. The abscissa of the curves are values of $\frac{2\pi}{\lambda} x\xi$.

[7] G. W. L. Glaisher, *Phil. Trans. Roy. Soc.* A, 1870, p. 367.

Slope $k = 1$.

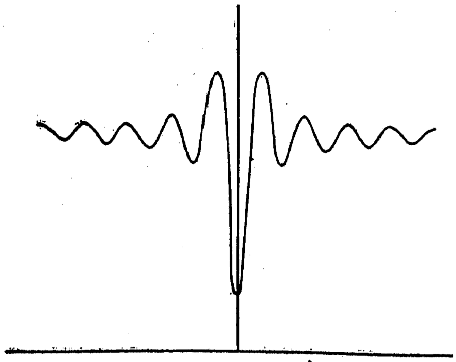

FIG. 5. $2\rho = \frac{\lambda}{3}$.

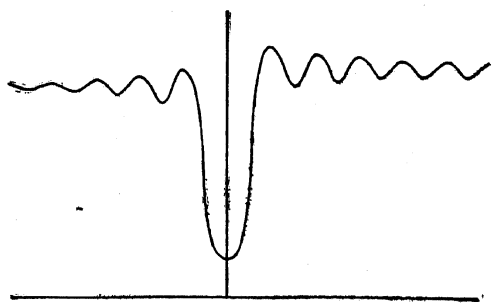

FIG. 6. $2\rho = 2\frac{1}{3}\lambda$.

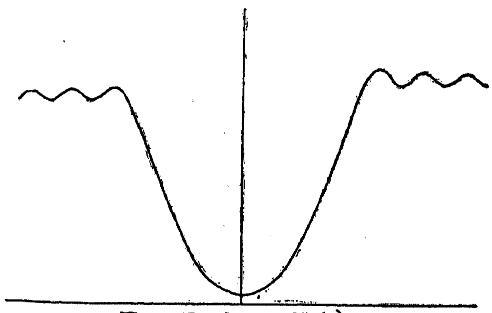

FIG. 7. $2\rho = 6\frac{1}{3}\lambda$.

Fig. 5 is drawn for the value of $2\rho = \frac{\lambda}{3}$, a fraction of a wavelength. It shows no asymmetry. The maxima and minima are sharp and equispaced. Fig. 6 is drawn for the value of $2\rho = 2\frac{1}{3}\lambda$, of the order of a few wavelengths. It shows some asymmetry on the right side. But the minimum at the edge is smaller than in the previous case and is distinctly wider. Fig. 7 represents the illumination when 2ρ is much greater. The central minimum widens out considerably.

Slope $k = \frac{1}{4}$.—The slope is small enough to allow the majority of the diffracted rays into the microscope.

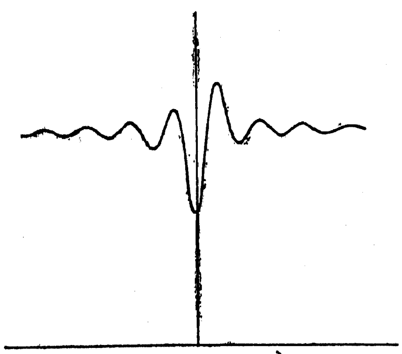

FIG. 8. $2\rho = \frac{\lambda}{3}$.

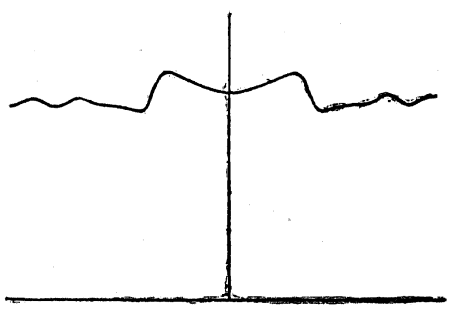

FIG. 9. $2\rho = 2\frac{1}{3}\lambda$.

Fig. 8 is drawn for the value of $2\rho = \frac{\lambda}{3}$, a fraction of a wavelength. The fringes are asymmetric with a sharp central minimum. But as the retardation increases the extent of the sloped part of the wavefront increases and the fringes are practically washed out. Fig. 9 is drawn for the value of $2\rho = 2\frac{1}{3}\lambda$.

P. N. Ghosh[8] has worked out the Fresnel diffraction pattern, caused by a wavefront of the form we have postulated, at a large distance in front of it. When we consider a pattern close to the edge, only the size of the pattern is diminished, so that it is not necessary to work out the details here. The microscope when out of focus with respect to the laminar edge is really focussed on this Fresnel pattern. The general form of the pattern is as reproduced below.

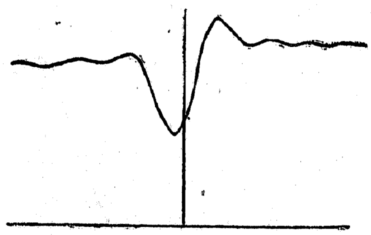

FIG. 10.

This explains why the asymmetry becomes prominent as the microscope is put out of focus. It merely magnifies the Fresnel diffraction pattern produced by the deformed wavefront.

The form of the wavefront postulated evidently succeeds in accounting for a certain amount of asymmetry in the pattern observed in a microscope in accurate focus. It also accounts for the asymmetry of the pattern when the microscope is put out of focus. A rigorous solution of the problem is, however, desirable which takes into consideration the boundary conditions at the two surfaces and also at the "edge" of the transparent medium. The treatment given by Raman and Rao[9] on the lines of Sommerfeld's solution of the problem of diffraction by a straight edge explains the existence of large angle diffraction by the laminar boundary but not the asymmetry considered in the present paper.

4. *Visibility of Laminar Boundary.*

It is interesting to calculate the minimum path retardation that may be detected by the observation of a dark line in the microscope under direct illumination. Following Michelson let us call the expression

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

[8] P. N. Ghosh, *Proc. Ind. Ass. Cult. Sci.*, 1921, **6**, 61.
[9] Raman and Rao, *Proc. Phy. Soc.*, 1927, **39**, 453.

the visibility of the fringe system. In the present case

$$I_{max} = \pi \cos \rho + \left(\frac{\pi}{2} + 0{\cdot}28\right) . 2 \sin \rho.$$

and

$$I_{min} = \pi \cos \rho.$$

According to the criterion of visibility due to Lord Rayleigh, the fringe system will cease to be visible when its visibility as defined before, comes down to $\frac{1}{40}$. Solving for ρ, we get in the limiting case

$$\rho = 2\tfrac{1}{2}^{\circ} \text{ (approximately)}$$

i.e., $2\rho = 5^{\circ}$ or corresponds to a retardation of $\frac{\lambda}{72}$.

This gives a lower limit to the detectable path retardation when viewed in direct light. When, however, the edge is viewed in oblique illumination and appears as a bright line, its visibility will be limited only by the background illumination if any, and much smaller changes of thickness should be visible, provided they are sufficiently abrupt.

Let me record here my humble thanks to Sir C. V. Raman, Kt., F.R.S., N.L., for the suggestion of the problem and the keen interest he took in the work.

5. *Summary.*

A sharp laminar edge appears as a dark line bordered by asymmetric fringes when examined in direct illumination under a microscope. The theories of Lummer and Rieche as also Sur, based on the assumption of an abrupt change of phase in the emerging wavefront, fail to account for this asymmetry. The Becke phenomenon—the appearance of a bright line close to the dark line corresponding to the " edge " when the microscope is put out of focus—merely represents the strongly asymmetric character of the pattern in the out-of-focus position as well. The simple geometrical explanation of the phenomenon as usually given is shown to be inadequate as it fails with normal parallel light. It is postulated that the phase change caused by the edge is not abrupt but is gradual and extends over a part of the emerging wavefront. The theory of microscopic vision as applied to such a deformed wavefront shows the pattern as observed in focus to be asymmetric. The same postulate also explains the asymmetry in the out-of-focus position—the Becke Phenomenon. Thus the asymmetric microscopic appearances both in and out-of-focus are explained as due to asymmetric diffraction effects at the edge. An estimate of the smallest path retardation that may be detected under direct illumination is also made.