PHYSICAL REVIEW A VOLUME 43, NUMBER 8 15 APRIL 1991

Dimensionality dependence in the singular dynamic scaling in the dilute Ising model

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(Received 31 August 1990)

The relaxation time τ and the thermal correlation length ξ_T in randomly diluted Ising magnets near the percolation threshold are related by the singular dynamic scaling $\ln\tau = f(\ln\xi_T)$, where $f(x) = Ax^2 + Bx + C$, with constants A, B, C. We investigate the dimensionality dependence of A by Monte Carlo simulation and compare our observations with the theoretical predictions.

The dynamic scaling hypothesis states that as the temperature T of a system approaches the critical temperature T_c the relaxation time τ and the corresponding thermal correlation length ξ_T are related through the generalized dynamical scaling relation

$$\ln\tau = f(\ln\xi_T) \quad (T \to T_c) , \tag{1}$$

where $f(x)$ is a function of its argument x. In most of the critical phenomena studied so far τ follows the "standard" form,[1] viz.,

$$f(y) = zy + \text{const} , \tag{2}$$

where the temperature-independent constant z is called the dynamic exponent. The numerical value of z depends not only on the space dimensionality d of the system but also on its dynamics. The dynamic universality class to which a system belongs depends crucially on the exponent z. In this paper, we are concerned with the nature of dynamic universality in a particular critical phenomenon in a class of random magnetic systems.

Theoretical activities in this field were triggered by the inelastic neutron scattering study[2] of the site-diluted antiferromagnet $Rb_2Co_pMg_{1-p}F_4$ with Co concentration p near the percolation threshold p_c. This system is a physical realization of (effectively) two-dimensional random Ising antiferromagnets where nonmagnetic Mg^{2+} ions substitutionally replace a fraction of the magnetic Co^{2+} ions. Fitting the experimental data for the spin relaxation time to the standard form (1) of dynamical scaling, Aeppli, Guggenheim, and Uemura[2] obtained $z \approx 2.4$, which is much larger than the theoretically expected value $z \approx 1.67$. Attempts[2–4] were made to reconcile theory with experiment by incorporating the fractal nature of the percolating clusters within the formalism *assuming*, however, that the standard form (1) holds near the bicritical point $p = p_c$, $T = 0$. On the other hand, subsequent theoretical works[5–7] claim that the standard form (1) of dynamical scaling breaks down at the bicritical point in randomly diluted systems of interacting Ising spins and the appropriate form is given by

$$f(y) = Ay^2 + By + C , \tag{3}$$

where A, B, and C are constants.

Equation (3) corresponds to a temperature-dependent effective dynamic exponent $A \ln\xi + B$, i.e.,

$$\tau \sim \xi^{z'} , \tag{4}$$

where $z' = (A \ln\xi + B) \to \infty$ as $\xi \to \infty$.

Experiments at lower temperatures have been designed for testing this claim.[8] However, already there are strong direct evidences in favor of the form (3) from Monte Carlo (MC) simulations.[9–11] There are also several indirect numerical evidences[12–16] supporting the quadratic form (3) instead of the linear form (2). It has been conjectured[17] that the coefficient A in (3) is "universal" in the sense that it depends only on the dimensionality and that in d dimension

$$A = \frac{d(d-1)}{2\nu_p} , \tag{5}$$

where ν_p is the exponent corresponding to the *percolation* correlation length ξ_p. Our main aim in this paper is to test the predicted form (5) for the dimensionality dependence of A. The original theoretical treatments[5,6] as well as the MC simulations in $d=2$ by Jain[9,10] were carried out at $p = p_c$ so that at all nonzero temperatures $\xi_T \ll \xi_p = \infty$. The numerical value of A in $d=2$ obtained from these MC simulations is in good agreement with the corresponding theoretical predictions. However, to our knowledge, A has not been estimated so far in $d=3$ by this method, one of the reasons being the prohibitively large computer time required in this approach. In this paper we suggest an alternative (and, computationally, more efficient) method for estimating A. We establish the reliability of this method by computing A in $d=2$ following this method and comparing its numerical value with the corresponding value obtained earlier by Jain. Then analyzing the existing MC data of Chowdhury and Stauffer[11] by the method proposed here we also get A in $d=3$. Finally, using the values of A thus obtained in $d=2$ and 3 we test the validity of da Silva and Lage's conjecture, viz., Eq. (5).

Let us now briefly describe our method of computing the relaxation time. The d-dimensional system simulated consists of L^d lattice sites with periodic boundary conditions where a fraction pL^d of sites are randomly occupied by Ising spins. For a given p, we take the temperature T of the system as that given by the relation

$$e^{-2J/k_BT} = 1 - \left[\frac{p}{p_c}\right] . \tag{6}$$

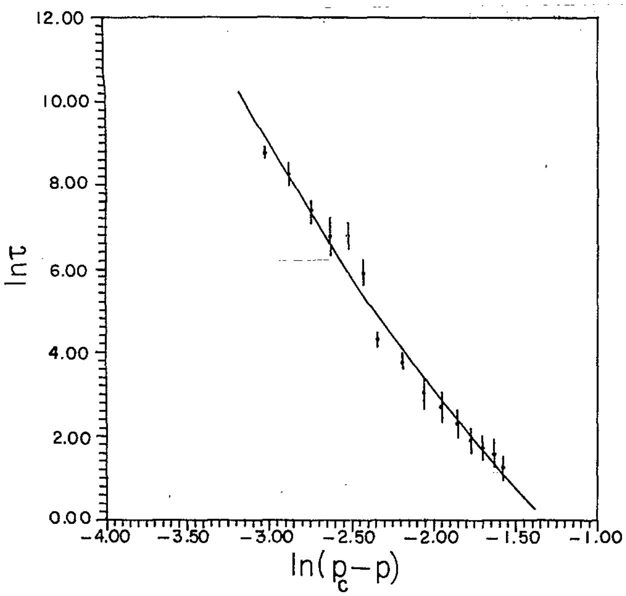

FIG. 1. $\ln\tau$ for the DIM on a square lattice plotted against $\ln(p_c-p)$. The curve is the best quadratic fit to the data.

We shall later use the fact that $\xi_T \simeq \xi_p$ along the curve (6). Beginning with a configuration where all the pL^d spins are up we monitor the magnetization per spin $m(t)$ as a function of time t as the system evolves following the Glauber single-spin-flip dynamics. If $m(t)$ vanishes for time of the order of t_0, we stop the simulation after a time $t_{max} \gg t_0$. The relaxation time τ is then computed from the definition

$$\tau = \sum_{t=0}^{t_{max}} m(t) . \tag{7}$$

Most of the data were generated for 50×50 systems using a main frame VAX 11/780 computer at the Jawaharlal Nehru University (JNU). A few data points were obtained for 100×100 systems using a CONVEX vector computer at the International Centre for Theoretical Physics (ICTP), Trieste. Since no significant difference in the values of τ for $L=50$ and 100 was observed, we have not attempted systematic study of the L dependence, if any, of τ. Each of the data points shown in Fig. 1 was obtained by averaging over a large number (typically 50) of impurity configurations. The relaxation times τ for both $d=2$ and 3 were found to fit well with the expression (see Fig. 1)

$$\ln\tau = A'[\ln(p_c-p)]^2 + B'[\ln(p_c-p)] + C' , \tag{8}$$

where A', B', C' are constants.

TABLE I. MC estimate of A vs theoretical prediction.

Dimension	A'	ν_p	A (MC)	A (Ref. 17)
2	0.86	1.33	0.48	0.75
3	1.66	0.9	2.05	3.33

As mentioned earlier, the singular dynamical scaling form (3) was originally derived under the condition $\xi_T < \xi_p = \infty$ whereas $\xi_T \simeq \xi_p$ throughout in our simulation. Therefore, instead of arguing that (8) follows from (3) when $\xi_T \simeq \xi_p$, we refer to Henley[18] for a derivition of Eq. (8) directly under the conditions of our simulation. It follows that

$$A' = A\nu_p^2 , \tag{9}$$

where A is the constant given in Eq. (3).

The numerical values of A' for $d=2$ and 3 are listed in Table I. From these values of A and the known values of ν_p from the literature, the numerical values of the constant A have been computed in $d=2$ and 3; these values are compared with the corresponding values of $d(d-1)/2\nu_p$ in Table I. Our estimate in $d=2$ is in good agreement with that predicted by Harris and Stinchcombe[6] as well as with the MC estimations of Jain.[9] The real-space renormalization-group (RSRG) technique used by da Silva and Lage[17] yields a much larger value of ν_p than the known exact value in $d=2$. Substituting the value of ν_p obtained from this RSRG analysis into Eq. (5) they obtained $A(d=2)=0.614$ which is not too far from the earlier MC estimates. However, Eq. (5) actually provides a much worse estimate if one uses the exact value $\nu_p=\frac{4}{3}$ in $d=2$ (see Table I). Our results convincingly demonstrate that da Silva and Lage's conjecture, viz., Eq. (5) is incorrect. Finally, we would like to point out that our MC data indicate $A'(d=3)\simeq 2A'(d=2)$ and $A(d=3)\simeq 4A(d=2)$. We hope that our observations would stimulate further theoretical work on the nature of universality in singular dynamic scaling.

One of us (D.C.) would like to thank Professor Abdus Salam and ICTP, Trieste, Italy, for hospitality and generous computer time in the initial stages of this work. We are indebted to Professor C. L. Henley for sending his unpublished works prior to publication. D.C. also thanks Professor C. L. Henley, Professor D. Stauffer, and Professor E. J. S. Lage for stimulating correspondences and useful suggestions and for a critical reading of the manuscript. Another author (B.B.) acknowledges financial support from Council for Scientific and Industrial Research (CSIR), India.

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