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Minimum norm inversion of observed ground elevation changes for slips on the causative fault during the 1905 Kangra earthquake

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Abstract. We estimate the distribution of slip in the dip section of the causative fault for the 1905 Kangra earthquake by applying the minimum norm inversion technique to differences in pre- and post-earthquake levelling data collected along the Saharanpur-Dehradun-Mussoorie highway. For this purpose it is assumed that the causative fault of the 1905 Kangra earthquake was planar with a dip of 5° in the northeast direction and that it had a depth of 6 km at the southern limit of the Outer Himalaya in Dehradun region. The reliably estimated maximum slip on the fault is 7·5 m under the local northern limit of the Outer Himalaya. Using the inverted slip distribution we estimate that the maximum permanent horizontal and vertical displacements at the surface due to the Kangra earthquake were about 4 m and 1·5 m respectively. The maximum transient displacements at the surface should have exceeded these permanent displacements. These estimates of maximum slip on the causative fault and the resultant maximum permanent and transient displacements at the surface during the Kangra earthquake may be taken tentatively as being representative of the great Himalayan earthquakes.

Keywords. Great earthquakes; Dehradun; fault slip; inversion; ground elevation change.

1. Introduction

Four great earthquakes have occurred in the past 100 years along the Himalaya in response to the convergence of Indian and Eurasian plates. Such earthquakes will continue to occur along this convergent plate margin, at least in the foreseeable future. Lack of instrumental data pertaining to these earthquakes has posed a serious problem for estimating their source parameters, especially the spatial estent of their causative ruptures (Oldham 1899; Middlemiss 1910; Dunn *et al* 1939 and Ramchandra Rao 1953). The rupture extent of the 1950 Assam earthquake, was identified mainly on the basis of its aftershock zone (Chen and Molnar 1977 and Molnar and Pandey 1989), while observations of flooding, tilting and other qualitatively observed phenomena proved useful in defining the rupture extent of the 1897 earthquake (Gahalaut and Chander 1992a). The rupture zones of the 1934 Bihar-Nepal and 1905 Kangra earthquakes were poorly constrained by the intensity data (Molnar 1987 and Chander 1989a, b). But, while in three out of these four cases the magnitudes of slip on the causative faults are not known and cannot be determined now, the pre- and post-earthquake levelling data along Saharanpur-Dehradun-Mussoorie highway (Middlemiss 1910 and Rajal *et al* 1986)provide us an opportunity to simulate fault slip during the 1905 Kangra earthquake. Chander (1988) and Gahalaut and Chander (1992b) estimated the rupture extent and the fault slip involved by using Mansinha and Smylie's (1971) solution to the forward problem in the trial and error method. In this article we apply the minimum norm inversion technique on the same data.

2. Observations

The levelling benchmarks along the Saharanpur-Dehradun-Mussoorie highway (figure 1) were first surveyed from saharanpur to Dehradun in 1861–62 and from Dehradun to Mussoorie in 1903–04. The complete line from Saharanpur to Mussoorie was relevelled after the 1905 Kangra earthquake. The differences in the heights of benchmarks (figure 2a) between the pre- and post-earthquake surveys are reported by Middlemiss (1910) and Rajal *et al* (1986). These differences were estimated on the assumption that the benchmark at Saharanpur remained unaffected by the Kangra earthquake. The maximum measured uplift was about 14 cm at a benchmark in Dehradun. These differences have been attributed to coseismic elevation changes during the 1905 Kangr earthquake by Middlemiss (1910), Rajal *et al* (1986), Chander (1988) and Gahalaut and Chander (1992b). We too adopt this view and use the data to simulate slip on the causative rupture plane of the Kangra earthquake.

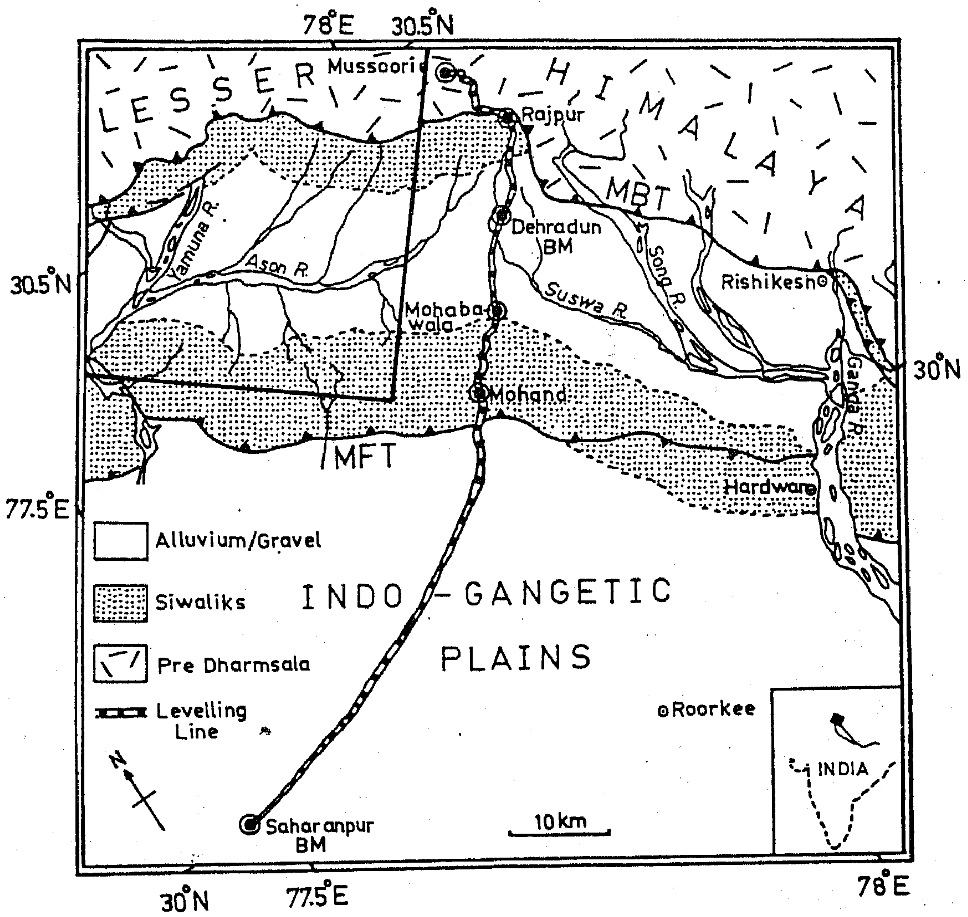

Figure 1. The levelling line from Saharanpur to Mussoorie. Some of the important benchmarks along the line are also identified. The inferred location of the southwestern and southeastern edges of the 1905 Kangra earthquake rupture is also shown. Geology and tectonic features adopted from Raiverman *et al* (1983). The abbreviations used are MBT – Main Boundary Thrust, MFT – Main Frontal Thrust.

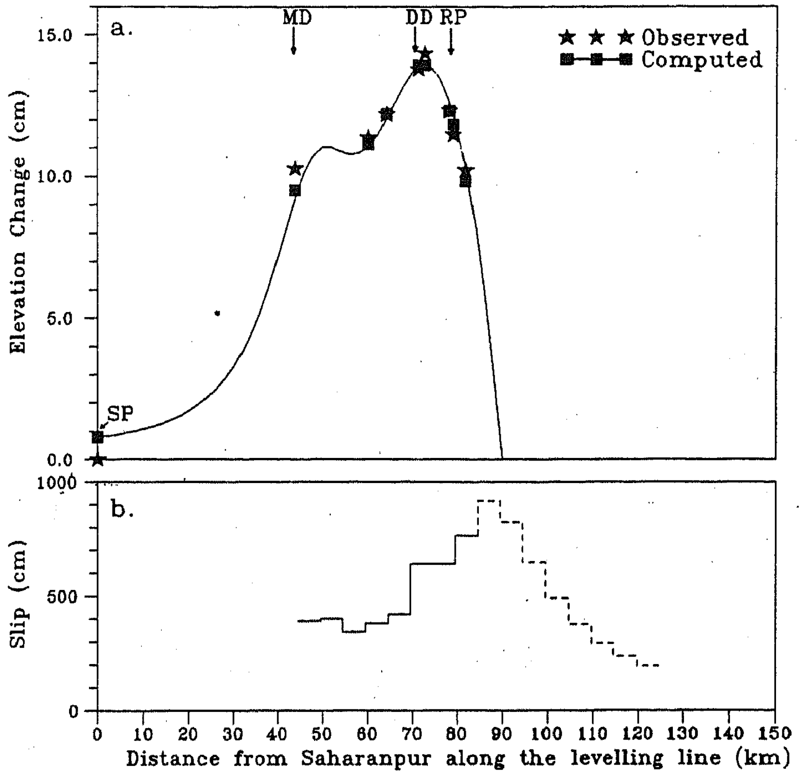

Figure 2. (**a**) Observed and computed elevation changes during the 1905 Kangra earthquake along the levelling line are shown in the upper part. Positions of benchmarks at Saharanpur (SP), Mohand (MD), Dehradun (DD) and Rajpur (RP) are also indicated. The estimated distribution of slip in the dip section of the planar fault is shown in the lower part. (**b**). The slip estimates shown by solid lines are well resolved by the data and those with the dashed lines are poorly resolved.

3. Theory

The problem of inverting observed coseismic elevation changes $y_i (i = 1 \ldots m)$ at m benchmarks due to slips $x_j (j = 1 \ldots n)$ on n rectangular ruptures on the causative fault of specified spatial orientation and location is linear (Mansinha and Smylie 1971) and may be stated as follows:

$$y_i = \sum_{j=1}^{n} a_{ij} x_j, \quad i = 1, \ldots m \tag{1}$$

or in matrix form as,

$$Y = AX \tag{2}$$

Here, X is the column vector of prescribed slips on different rupture segments and Y the column vector of observed elevation changes. Elements a_{ij} of the coefficient matrix A are defined by the theoretical formulation of Mansinha and Smylie (1971) which predicts the elevation changes due to uniform slip on a single rectangular rupture in a planar fault which is buried in an elastic half space.

If the number of parameters to be determined is much larger than the number of available observations, as usually happens in geophysical problems, then the minimum norm solution for a matrix equation such as that given in equation (2) is

$$X = A^T(AA^T)^{-1}Y. \tag{3}$$

In the presence of random errors in observational data, the solution may be obtained using the regularized solution,

$$X = A^T(AA^T + \varepsilon)^{-1}Y, \tag{4}$$

where ε is the regularising parameter.

The parameter resolution (R) and data information density (S) matrices as defined in the literature (e.g. see Menke 1984) are given by

$$R = A^T(AA^T + \varepsilon)^{-1}A, \tag{5}$$

and

$$S = AA^T(AA^T + \varepsilon)^{-1}. \tag{6}$$

The closeness of the R and S matrices to the identity matrix serves as a pointer to the resolution quality of inverted parameters and to the extent that various data points contribute to the clarity of the solution.

4. Model and results

In view of the smallness of observed elevation changes and the limited number of benchmarks for which data were available, we felt it desirable to carry out the above inversion with as few assumptions about the buried causative fault as possible: that it was planar and that, following Molnar (1987), Chander (1988) and Gahalaut and Chander (1992b), it had a dip of 5° in the northeast direction and a rupture area of $280 \times 80\,\mathrm{km}^2$. The ruptured portion of the fault was divided into 16 segments each of length 280 km along the strake and of width 5 km along the dip direction. The slip on each segment was assumed constant but it was allowed to vary from segment to segment. Thus, the slip varied stepwise in the dip direction but remained constant along the strike in the ruptured section of the planar fault.

4.1 *Location of rupture zone relative to the levelling line*

The slip estimates are sensitive to the relative locations of rupture in the fault as well as of the levelling line on the earth's surface. While Chander (1988) had assumed that the horizontal distance between the levelling line and the southeastern edge of the rectangular rupture zone was about 15 km, Gahalaut and Chander (1992b) assumed that it varied between 1 to 10 km. Figure 3 displays the rms errors for several inversions based on different possible separations between the rupture zone and the levelling line. The minimum in the curve corresponds to a distance of 10 km.

4.2 *Trade off*

A trade off exists between the closeness of fit of observed and computed estimates of elevation changes on the one hand and the smoothness of variation of slip along the dip section of the fault on the other. This is an important consideration because

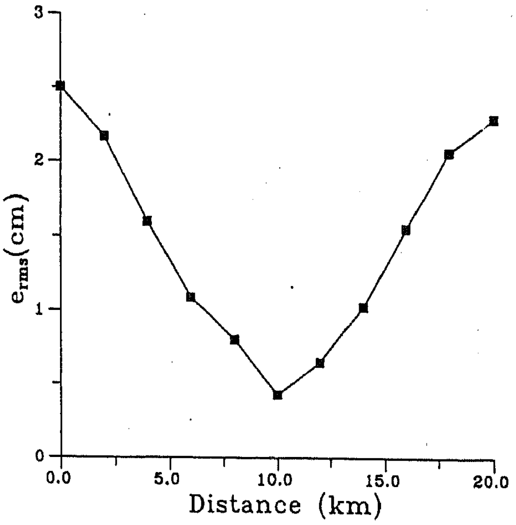

Figure 3. The graph displays the variation of rms errors with the distance between the levelling line and the southeastern edge of the rupture.

the choice of discrete jumps of slip along the fault, although convenient for mathematical computations related to superposition, implies violation of the concept of conservation of matter in a strict theoretical sense. The severity of this problem increases with the increase in magnitudes of jump in slips on adjoining fault segments. Commensurate with the quality and quantity of observations available we have opted for a subjective assessment of the best compromise in this trade off.

4.3 *Model*

Figure 2a shows a comparisonof observed elevation changes with the computed values corresponding to the set of slip values obtained from this inversion and shown in figure 2b. The e_{rms} is $\pm 0{\cdot}43$ cm and the magnitude of regularizing parameter ε is $2{\cdot}5 \times 10^{-6}$.

4.4 *Quality of inversion*

It is seen from the R matrix for this solution that although the maximum value of the estimated slip is 9·2 m (figure 2b), It is poorly resolved by the available data. The highest value of estimated slip that is well resolved by the data is 7·6 m. This value refers to slips of points on the fault at distances of 40–45 km from the southern edge of the rupture zone (figure 2b). An examination of information density matrix, in turn, confirms that all data points in respect of observed elevation changes are significantly influenced by the various elements of the vector X (equation 2) of fault slips.

5. Discussion

5.1 *Observational data*

The RF (short for Rossi-Forel) VII isoseismal drawn by Middlemiss (1910) on the basis of damage caused by the Kangra earthquake is highly elongated, with the long axis extending from northwest of Kangra to southeast of Dehradun. Within it, the intensity reached RF X levels in the Kangra region and only RF VIII level around the Dehradun region. This strongly suggests that the slip may not have been uniform in the ruptured section of the causative fault. But since the observations of elevation changes ara available only at nine points along a profile at the south eastern end of the RF VII isoseismal, it is not possible to simulate variations of slip along the strike of the causative fault. Similarly the pre-earthquake levelling of five benchmarks between Saharanpur and Dehradun was carried out about 43 years before the earthquake and observed elevation changes may also have been partly contributed by pre-seismic strains in the crust. Thus the quantity and quality of data available leave much to be desired. But these are still the best available data for a great Himalayan earthquake for modelling the slip on the corresponding causative fault.

5.2 *Choice of the thrust fault model for inversion*

Available fault plane solutions for moderate earthquakes in Himalaya and their revised estimates of focal depths (Fitch 1970; Chandra 1978; Baranowski *et al* 1984; Ni and Barazangi 1984; and Molnar and Lyon-Caen 1989) make it attractive to consider that they all occurred on the same thrust fault; specifically a narrow thrust zone dipping gently to the NE beneath the NW Himalaya (Ni and Barazangi 1984; and Molnar 1990). This thrust fault may coincide with the detachment surface separating the underthrusting Indian shield from the rocks of the Himalayan wedge (Seeber and Armbruster 1981; Ni and Barazangi 1984 and Molnar 1990), although in the absence of precise estimates of focal depths and fault plane solutions for the four great earthquakes which have occurred in the past 100 years in the Himalaya, this assumption may be conjectural (Seeber and Armbruster 1981; Chander 1988, 1989a; Gahalaut and Chander 1992a and 1992b). Seeber and Armbruster (1981) suggested that the northern limit of the ruptures of these great earthquakes coincide with the belt of moderate magnitude earthquakes. In accord with these views we have assumed that the causative ruptive of the 1905 Kangra earthquake had an areal extent of $280 \times 80\,\mathrm{km}^2$ and occurred on a shallow, thrust type gently dipping fault.

5.3 *On the choice of the method of inversion*

Typically, a rectangular rupture with uniform slip on the fault is assumed for modelling coseismic deformations (Stein and Lisowski 1983; Chander 1988; Lin and Stein 1989; Gahalaut and Chander 1992a, b). Analysis for variable slips was first attempted by ward and Barrientos (1986) for the 1983, Borah Peak, Idaho earthquake. The method was later applied to the 1985 Central Chile (Barrientos 1988) and the 1915 Avezzano, Italy (Ward and Valensise 1989) earthquakes. Recently Barrientos and Ward (1990) used the surface deformation data of the 1960 Chile earthquake to invert for the fault geometry as well as the variable slip on it. All of these studies involve rigorous inversion procedures. Here, in view of the limited quantity and quality of data, we have assumed a relatively simple fault model and used the well known minimum norm inversion technique to investigate variation of slip in the dip direction of the fault.

Connected with this choice of inverse method is the problem of estimating the location of the rupture zone relative to the levelling line. We have used a grid search procedure whose results are displayed in figure 3. A more objective method of estimating this seperation would be to regard it as an unknown parameter to be estimated alongwith the slips on the fault segments. Since this parameter enters the elements of matrix A of equation (2) in a non-linear fashion, this morecomppREhensive inversion would have to be formulated as a non-linear optimization problem. The quantity and quality of data available does not warrant that exercise at this stage.

5.4 *Estimate of maximum slip on the fault*

Barrientos amd Ward (1990) inferred a uniform slip of 17 m for the great 1960 Chile earthquake ($Ms = 8{\cdot}3$). They identified some pockets of slips as high as 40 m in their variable slip model. A similar magnitude of slip was also inferred for the great ($Ms = 8{\cdot}4$) Alaska earthquake of 1964 (Plafker 1972). In view of such high estimates of slips during great earthquakes of the world, our estimate of 7·5 m for the 1905 Kangra earthquake ($Ms = 8{\cdot}0$) is on the lower side. As in the case of Barrientos and Ward's (1990) estimates for constant and variable slip models of the Chile earthquake, the estimated maximum slip of 7·5 m using the variable slip analysis is only norminally greater than Chander's (1988) and Gahalaut and Chander's (1992b) estimates of 5 m and 3 m respectively, based on uniform slip models.

5.5 *Hazard assessment*

We estimated the permanent horizontal and vertical ground displacements at several points over a surface area of about $400 \times 150\,\mathrm{km}^2$ lying vertically above and around the estimated $280 \times 80\,\mathrm{km}^2$ buried rupture plane using the slip vaules shown in figure 2b. We note that while the estimated maximum borizontal and vertical displacements are 0·66 m and 0·14 m respectively along the survey line, they are about 4 m and 1·4 m respectively at points only a few kilometers to the northwest of this line (figures 4 and 5). Unfortunately, there are no geodetic observations to verify these calculations. But two reasons can be dvanced to suggest that these are lower bound estimates. Firstly, the minimum norm technique used for inversion yields a minimum estimate of the slip vector X (equation 2). Secondly, according to Middlemiss (1910) damage during the 1905 Kangra earthquake was less in the Dehradun region than that in the Kangra region. Furthermore, we surmise that: f more geodetic data with better geographic spread were available and inversion carried out for slip varying both along the strike and dip of the fault, the estimates of slip magnitudes in the Kangra region would be found to be significantly higher.

An implication of higher suspected slips of the fault under the Kangra region for hazard assessment is that the 1905 earthquake probably did not relieve as much strain in the Dehradun region as it did in the Kangra region. It may be that the strain in the Dehradun region had been released already by an earlier smaller magnitude earthquake which did not affect the Kangra region (Molnar 1987). Alternatively, the relative probability of occurrence of the next great earthquake would be more for the Dehradun region than for the Kangra region. But this probability is less than that for the region east of Dehradun lying in the central seismic gap (Khattri 1987).

We note in passing that the transient ground displacements during an earthquake exceed their permanent displacements as measured from the respective initial

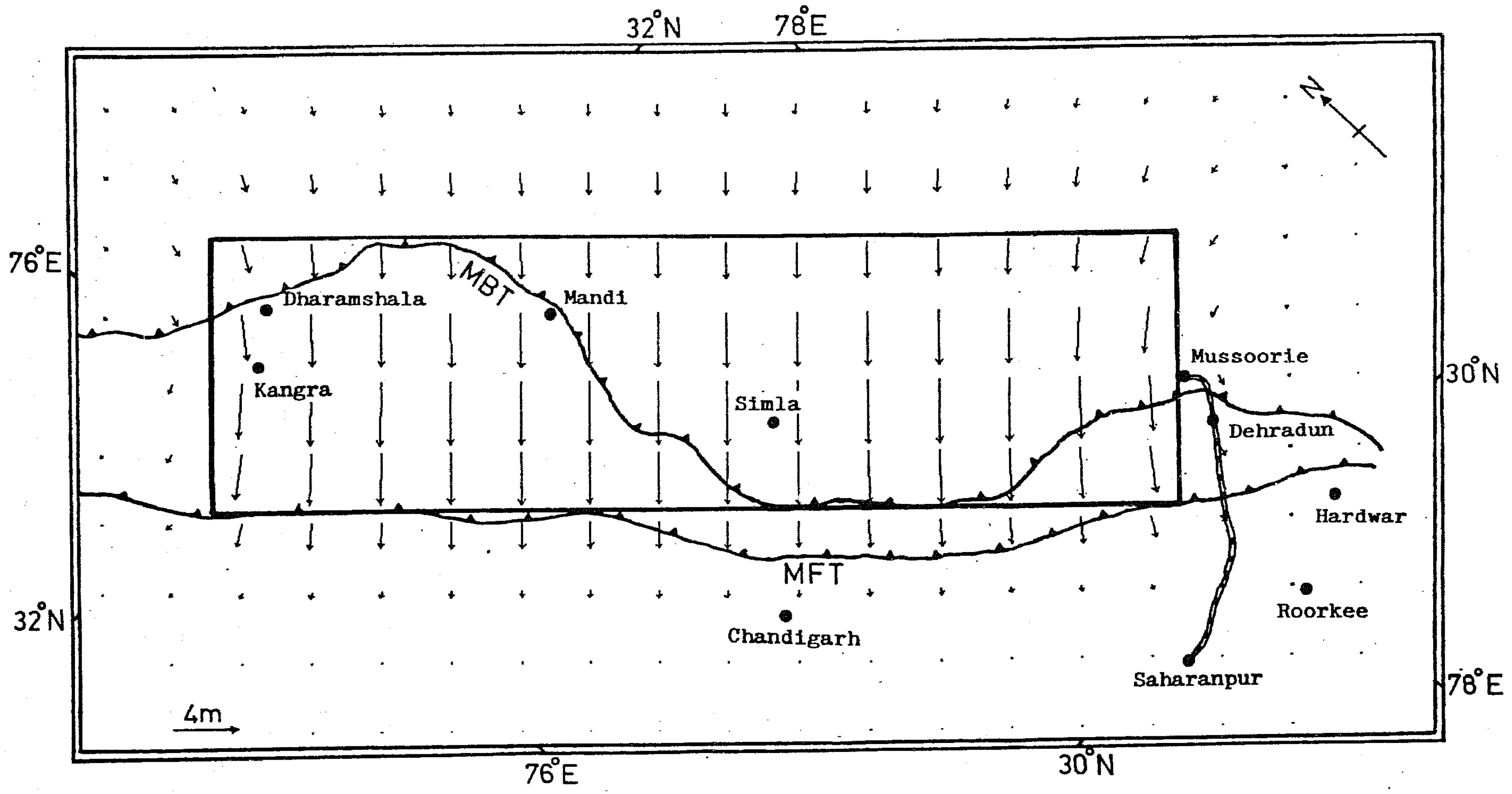

Figure 4. Predicted horizontal displacements on the ground surface due to slip distribution of figure 2b on the planar fault. The tail of an arrow indicates the geographic location of the point for which the arrow signifies the direction of horizontal movements. Important localities of the region are also shown for completeness. Tectonic information adopted from Raiverman *et al* (1983). Abbreviations are the same as in figure 1.

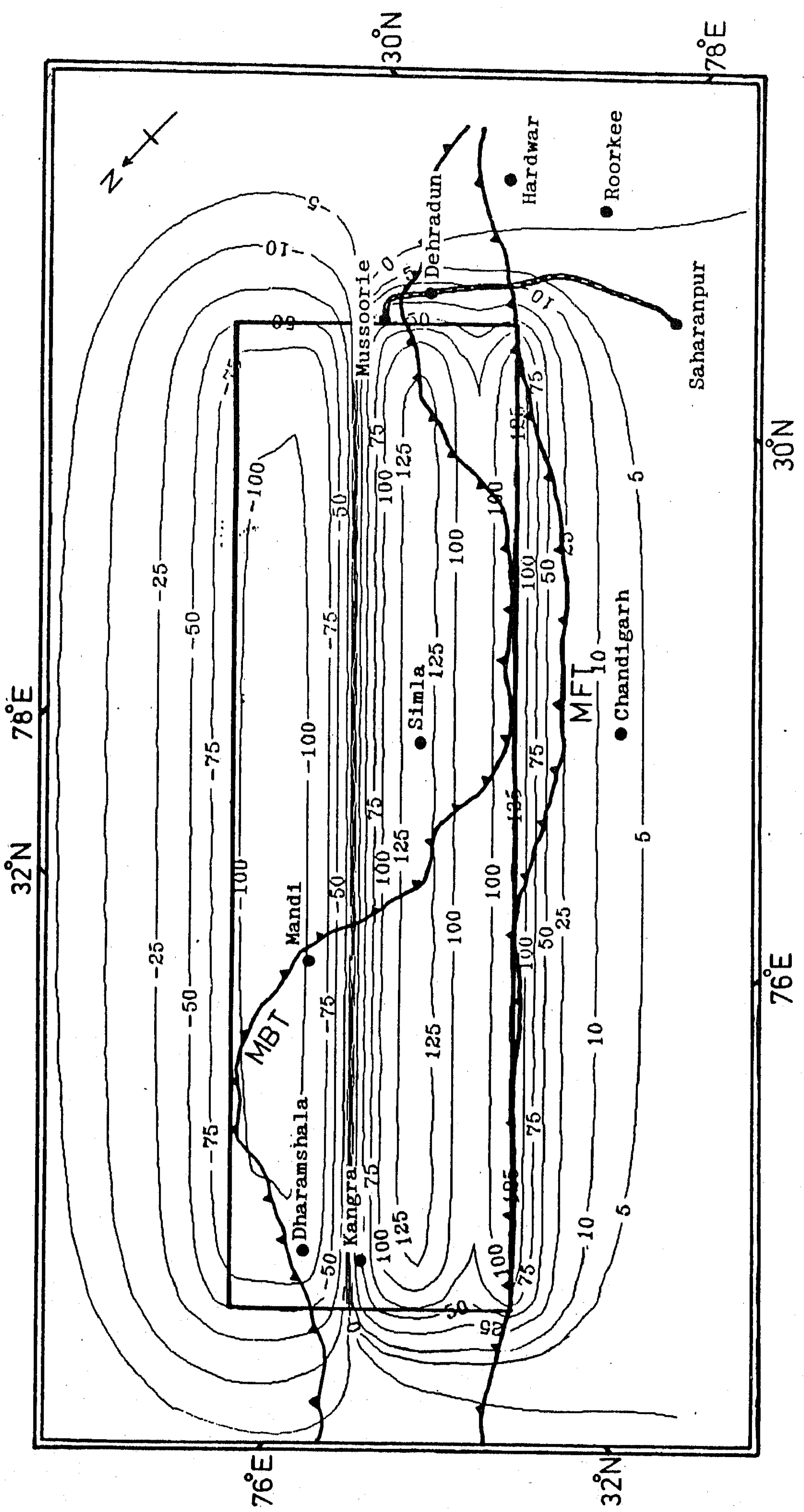

Figure 5. Analogous to figure 3, but showing contours of predicted vertical displacements on the ground surface.

equilibrium positions (see synthetic seismograms computed, for example, by Benmanahem and Singh 1981).

In the absence of such observations and calculations for other recent great earthquakes of the Himalaya the above estimates of slip on a great Himalayan rupture as well as of the associated maximum permanent ground displacements remain the only representative figures for the great Himalayan earthquakes, generally.

6. Action required

The present levelling line is situated in the region of rapid elevation change gradients at the eastern edge of the Kangra earthquake rupture zone (see figure 5). The segment of the Himalayan seismic belt immediately southeast of Dehradun is a seismic gap (Khattri 1987; Chander 1988, 1989). This levelling line would thus fall at the western margin of the rupture zone of the next great earthquake which will fill the western end of this seismic gap. Levelling along a grid of benchmarks established by spanning and subsuming the benchmarks of the earlier Saharanpur-Mussoorie line would thus constitute a most illuminating experiment for hazard assessment in the region as well as for advance detection of strain accumulation during the preparatory phase of the future great earthquake. It is also desirable that the reference benchmark at Saharanpur situated in the Indo-Gangetic alluvium, should be replaced by and tiedto the nearest bedrock site of the Indian shield.

7. Conclusions

The following conclusions may be drawn from inversion of the elevation measured before and after the 1905 Kangra earthquake, along the Saharanpur-Dehradun-Mussoorie levelling line.

- A reasonable lower bound estimate of the maximum slip on the causative fault of the Kangra earthquake is 7·5 m.
- The southeastern limit of Kangra earthquake rupture was about 10 km northwest of the Saharanpur-Dehradun-Mussoorie levelling line and the rupture extended southwestward upto the southern limit of the Outer Himalaya in Dehradun region.
- Correspondingly, the lower bound estimates of the permanent maximum vertical and horizontal ground displacements are of the order of 1·5 m and 4·0 m respectively.

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