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SEISMIC ESTIMATE OF SOLAR HEAVY ELEMENT ABUNDANCES

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ABSTRACT

The recent downward revision of solar photospheric abundances of Oxygen and other abundant heavy elements has caused a serious discrepancy between standard solar model and the seismically determined solar structure. In order to obtain an independent estimate of heavy element abundances in the Sun, we use the dimensionless sound-speed derivative in the solar convection zone to determine the heavy element abundance using seismic data. This technique is similar to that successfully used to determine the helium abundance in the solar envelope. We find a heavy element abundance of $Z = 0.0175 \pm 0.002$.

Key words: Sun: oscillations; Sun: abundances; Sun: interior.

1. INTRODUCTION

Recent analyses of spectroscopic data using 3D atmospheric models have suggested that the solar abundance of oxygen and other abundant elements needs to be revised downwards (Asplund et al. 2004). Asplund et al. (2005; henceforth ASP) claim that the oxygen abundance should be reduced by a factor of about 1.48 from the earlier estimates of Grevesse & Sauval (1998; henceforth GS). The reduction in Z reduces opacity, in turn reducing the depth of the convection zone (CZ) in solar models computed using the new abundances (Bahcall & Pinsonneault 2004). The new abundances along with the current OPAL opacity tables (Iglesias & Rogers 1996) are not consistent with seismic constraints (Basu & Antia 2004; Antia & Basu 2005; Bahcall et al. 2004) and that the opacity needs to be increased by 11–20% to restore agreement. Independent calculation by the OP project (Seaton & Badnell 2004; Badnell et al. 2005) finds that the opacity values near the base of CZ are within 1–2% of OPAL values.

Antia & Basu (2005) suggested that it may be possible to get the correct convection zone depth if the Ne abundance is increased by a factor of 2.5 and the abundances of O and related elements are increased by their respective 1σ error estimates. This solution was supported by observations of Drake & Testa (2005) who found that most neighbouring stars seem to have a much higher Ne/O ratio as compared to the Sun. However, Schmelz et al. (2005) and Young (2005) reanalysed the solar data to find that the Ne/O ratio is consistent with the old lower value. Cunha et al. (2006) also find a higher Ne/O ratio in B-stars of orion association. Thus is it is still not clear if some adjustment is required in the solar Ne/O ratio. On the other hand, Ayres et al. (2006) using CO lines find O abundance close to the old GS value. Thus it is possible that there are significant uncertainties in spectroscopic determination of abundances (Pinsonneault & Delahaye 2006). Hence there is a need to explore independent techniques to determine solar heavy element abundances.

In this work we try to get an independent estimate of heavy element abundances in the Sun using seismic data in a manner similar to that used to determine Helium abundance (e.g., Däppen et al. 1991; Antia & Basu 1994).

2. THE TECHNIQUE

The adiabatic index and hence the sound speed is lowered in the ionization zone. This can be seen in the dimensionless gradient of sound speed, which can be used, for example, to measure the helium abundance (Gough 1984). We define the dimensionless sound-speed gradient

$$W(r) = \frac{1}{g}\frac{dc^2}{dr}, \qquad (1)$$

where, g is the acceleration due to gravity. In the adiabatically stratified region of the CZ, $W(r)$ is related to the adiabatic indices and $W(r) = -2/3$, if the material is fully ionized. The deviation of $W(r)$ from this value in the ionization zones of the heavy elements can in principle, be used to measure their abundances. Since different ionization stages of various heavy elements overlap with each other, it is difficult to isolate the effect of individual heavy element, but it should be possible to estimate the total heavy element abundance, Z from $W(r)$ in the lower part of the CZ.

To infer $W(r)$ in the Sun we use the Regularized Least Squares (RLS) inversion to infer the sound-speed in the solar interior (e.g., Antia 1996). The RLS technique is used here, since it is convenient to differentiate the resulting sound speed profile to compute $W(r)$. This inversion technique has been tested by using a variety of test models (Antia & Basu 2006). The inverted $W(r)$ can be calibrated for heavy element abundances using solar models with different abundances. To get a quantitative measure, we use the average value of $W(r)$,

$$\langle W(r) \rangle = \frac{\int_{r_1}^{r_2} W(r)\, dr}{r_2 - r_1}\,, \tag{2}$$

in different radius intervals to determine the heavy element abundance. In order to use this technique we need to construct solar models with different heavy element mixtures. Unfortunately, the standard EOS like OPAL (Rogers & Nayfonov 2002) and MHD use a fixed mixture of heavy elements and hence cannot be used for this purpose. Hence we use solar models constructed using the CEFF EOS (Eggleton et al. 1973; Guenther et al. 1992; Christensen-Dalsgaard & Däppen 1992). The $W(r)$ profile will also depend on the EOS and other parameters in the solar model. We study the sensitivity of $W(r)$ to various parameters, to estimate the systematic errors in this technique.

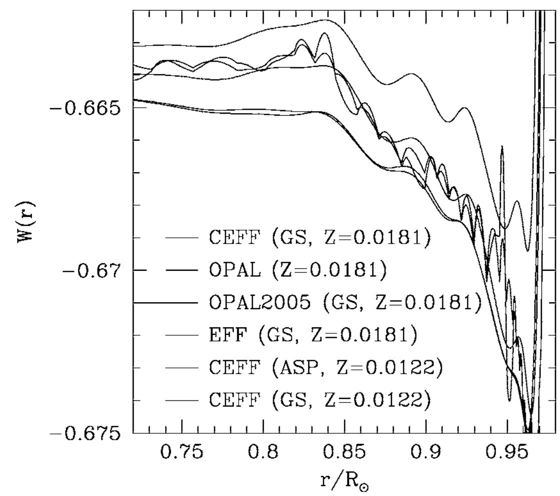

Figure 1. The function $W(r)$ for solar envelope models constructed with different equations of state, heavy-element mixtures, and abundances as marked in the figure. All these models have their CZ base at $r_b = 0.7133R_\odot$, and have a hydrogen abundance of $X = 0.739$.

3. RESULTS

To study the sensitivity of $W(r)$ to EOS and heavy element abundances, we construct solar models with different EOS and heavy element abundances and the results are shown in Fig. 1. This figure also shows the results using an updated versions of OPAL EOS (OPAL2005). It is clear that CEFF EOS yields results which are close to OPAL and hence can be used to determine Z. The difference between models with different Z is clear and it should be possible to use this to determine Z. To study the effect of different sources of systematic errors, we look at the difference in $\langle W(r) \rangle$ between solar models constructed with different parameters (and input physics) and a standard model constructed with CEFF equation of state. The results are listed in Table 1. All models have $Z = 0.0181$. Large differences indicate a source of large errors.

To get a quantitative estimate of Z we plot in Fig. 2 $\langle W(r) \rangle$ in three different radial intervals. The best fit lines in this figure are used to determine solar Z value using inverted $\langle W(r) \rangle$. The calibration curve using OPAL2005 models is very similar to that using the earlier version of OPAL EOS and hence is not shown.

To infer the heavy element abundance in the Sun we use the observed frequencies from GONG and MDI to infer $W(r)$. We use 105 sets of frequencies from GONG and 49 sets from MDI and for each set we use four different reference models using EFF, OPAL, OPAL2005 and CEFF EOS. The inverted $W(r)$ for the 420 inversions from GONG data and 196 from MDI data are shown in Fig. 3. Using the inverted $W(r)$ shown in Fig. 3, and the

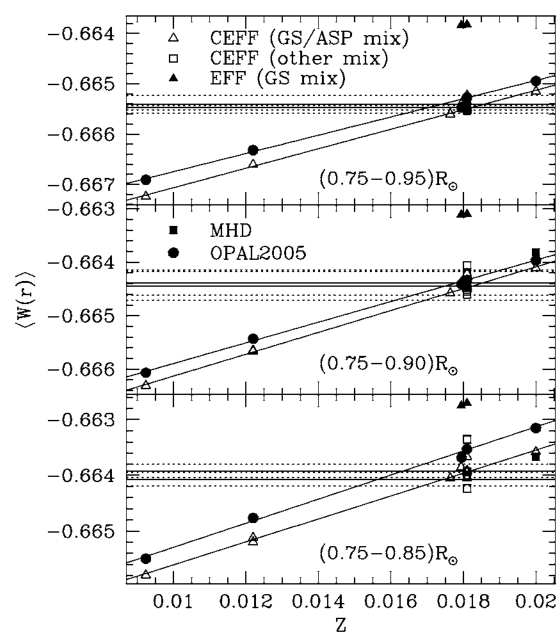

Figure 2. The average value of $W(r)$ in three different radius ranges plotted as a function of Z for solar models with different equations of state and mixtures of heavy elements as marked in the figure. The CEFF models with 'other mixtures' include models constructed with the abundance of some individual elements increased by a factor of 2 as compared to GS value. The reference and test models used in this study are also included in this figure. The blue line is the best-fit line through CEFF models of different Z, the green line is that through OPAL models. The horizontal lines show the average observational results, cyan for GONG data and magenta for MDI. The dotted lines show 1σ errors.

Table 1. Differences in $\langle W(r)\rangle$ due to various sources of systematic errors in the sense (test model) − (standard model with CEFF equation of state). All models have $Z = 0.0181$

Model difference	$10^4\delta\langle W(r)\rangle$ $(.75,.85)R_\odot$	$(.75,.90)R_\odot$	$(.75,.95)R_\odot$
OPAL EOS	3.859	1.274	2.076
CEFF EOS with OPAL mixture	3.187	0.101	−0.672
CEFF EOS with ASP mixture	−1.318	−0.342	−0.013
Mix. with C increased by factor 2	−3.218	−0.859	−0.228
Mix. with N increased by factor 2	−0.158	0.319	0.085
Mix. with O increased by factor 2	5.576	2.434	0.351
Mix. with Ne increased by factor 2	−0.734	−1.398	−0.455
Mix. with Mg increased by factor 2	−1.099	−0.627	−0.219
Mix. with Si increased by factor 2	−0.873	−0.357	−0.063
Mix. with S increased by factor 2	0.123	0.405	0.406
Mix. with Fe increased by factor 2	−1.067	−0.476	−0.058
CZ depth reduced by $0.01R_\odot$	2.522	2.481	2.517
With X reduced by 0.019	−0.291	−0.270	−0.310
With different low temperature opacity	0.050	−0.024	−0.057
With different treatment of convection	−0.052	0.027	0.030
With radius increased by 200 km	−0.024	0.044	0.030

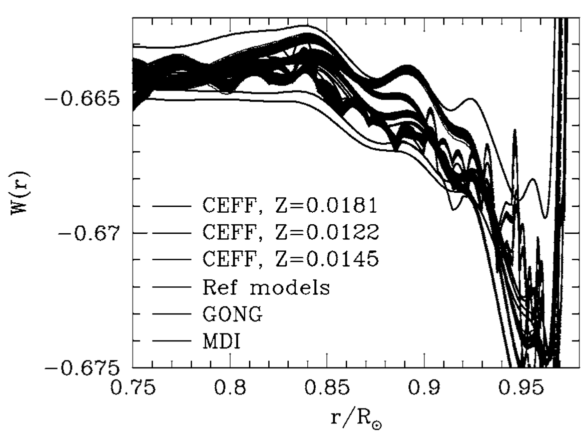

Figure 3. The function $W(r)$ for the Sun obtained using four different reference models for the different sets of solar oscillations frequencies from GONG and MDI are compared with that in solar models with different heavy element abundances. The model with $Z = 0.0145$ uses ASP mixture with Ne abundance increased by a factor of 2 (Antia & Basu 2005).

Table 2. The heavy element abundance, Z (in percentages) as inferred using average values of $W(r)$ in different radius ranges.

Data	Calib. Model	$(.75,.85)R_\odot$	$(.75,.90)R_\odot$	$(.75,.95)R_\odot$
GONG	CEFF	$1.82 \pm .06$	$1.86 \pm .12$	$1.82 \pm .02$
GONG	OPAL	$1.66 \pm .06$	$1.78 \pm .12$	$1.69 \pm .03$
MDI	CEFF	$1.75 \pm .06$	$1.82 \pm .13$	$1.86 \pm .09$
MDI	OPAL	$1.59 \pm .06$	$1.75 \pm .14$	$1.73 \pm .10$
		Mean value $Z = 1.75 \pm 0.09$		
Rhodes	CEFF	$1.79 \pm .06$	$1.86 \pm .12$	$1.90 \pm .07$
Rhodes	OPAL	$1.63 \pm .06$	$1.79 \pm .12$	$1.78 \pm .07$
		Mean value $Z = 1.77 \pm 0.09$		

calibration curves shown in Fig. 2 we infer the heavy element abundance in the Sun. The results are shown in Table 2. All abundances are in terms of percentages. The errorbars quoted in the table are the respective standard deviation of all estimates and do not include other systematic errors. The mean value for the observed frequencies is an average over all estimates listed here and the error bar is the standard deviation of these values. We have also used the set of observed frequencies with high degree modes from MDI (Rhodes et al. 1998), which is marked as 'Rhodes' in the Table. These frequencies were obtained from data collected for 61 days beginning in May 1996. These results obtained from these data are not included in the global average since we only have one set, unlike the rest of the MDI or the GONG data.

Table 3. The heavy element abundance, Z (in percentage) as inferred using average values of $W(r)$ in different radius ranges for different models and for the Sun.

Model difference	$(.75,.85)R_\odot$ Z_1	$(.75,.90)R_\odot$ Z_2	$(.75,.95)R_\odot$ Z_3	Z_1-Z_3	Z_2-Z_3
GS Mix. with C incr. by factor 2	1.664	1.775	1.805	−0.141	−0.031
GS Mix. with N incr. by factor 2	1.813	1.832	1.821	−0.009	0.011
GS Mix. with O incr. by factor 2	2.083	1.937	1.836	0.247	0.101
GS Mix. with Ne incr. by factor 2	1.785	1.748	1.794	−0.009	−0.045
GS Mix. with Mg incr. by factor 2	1.765	1.788	1.807	−0.042	−0.019
GS Mix. with Si incr. by factor 2	1.776	1.801	1.821	−0.045	−0.021
GS Mix. with S incr. by factor 2	1.824	1.838	1.839	−0.014	−0.001
GS Mix. with Fe incr. by factor 2	1.769	1.794	1.814	−0.046	−0.021
ASP mix. with $Z=1.26$	1.200	1.228	1.238	−0.038	−0.011
ASP mix. with Ne incr. by factor 4	1.463	1.417	1.518	−0.055	−0.100
ASP mix. with Ne incr. by factor 2+	1.419	1.416	1.454	−0.035	−0.038
GONG + MDI data sets	$1.796\pm.067$	$1.845\pm.123$	$1.832\pm.057$	$-.036\pm.088$	$.013\pm.135$

To study the possibility of determining the abundances of individual heavy elements we construct solar models by varying the abundance of one element at a time. Results are shown in Fig. 4. From this figure one can identify some peaks in $W(r)$ due to C, O and Ne around $r=0.89R_\odot, 0.82R_\odot$ and $0.92R_\odot$ respectively. In principle, these can be used to determine the abundances of these elements. Since the contribution of different elements is not the same at all depths, the average of $W(r)$ in different depth ranges will be affected differently by each element. This can be converted to difference in inferred Z value using the calibration curves and the results are shown in Table 3. Since the abundance using the entire region $(0.75,0.95)R_\odot$ is rather insensitive to these changes, we take the differences w.r.t. these values for all models. For models with the ASP mixture, the differences are small as may be expected from the fact that the abundances of the most important elements are decreased by similar factors. For observed frequencies, the differences are smaller than the errorbars and it is difficult to infer anything about abundances of individual elements from these.

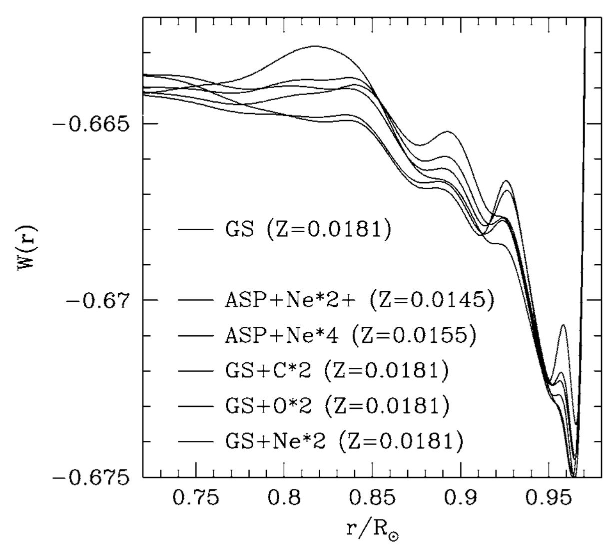

*Figure 4. The function $W(r)$ for solar envelope models constructed with the CEFF equation of state. The models have heavy-element mixtures and Z as marked in the figure. These mixtures include cases where the relative abundance of C, N or O are increased by a factor of 2 over the GS or ASP value. The mixture ASP+Ne*2+ is with Ne abundance increased by a factor of 2 and CNO abundances increased by 1σ as considered by Antia & Basu (2005). All these models have the CZ base at $r_b = 0.7133R_\odot$, and have a hydrogen abundance $X=0.739$.*

The differences Z_1-Z_3, Z_2-Z_3, along with total Z can in principle, be used to estimate the abundances of some individual elements. For example, if we assume that $W(r)$ is mainly determined by C, O, Ne, then one can write (using constants from Table 3):

$$-.141\frac{\delta Z_C}{Z_C}+.247\frac{\delta Z_O}{Z_O}-.009\frac{\delta Z_{Ne}}{Z_{Ne}} = Z_1-Z_3 \quad (3)$$

$$-.031\frac{\delta Z_C}{Z_C}+.101\frac{\delta Z_O}{Z_O}-.045\frac{\delta Z_{Ne}}{Z_{Ne}} = Z_2-Z_3 \quad (4)$$

$$\delta Z_C+\delta Z_O+\delta Z_{Ne} = \delta Z = 0 \quad (5)$$

where Z_C, Z_O, Z_{Ne} are the abundances by mass of C, O and Ne. Here we have set $\delta Z=0$ as we assume that mean Z is determined as above. Using the differences for observed frequencies we get:

$$\frac{\delta Z_C}{Z_C} = 0.37 \pm 0.86 \quad (6)$$

$$\frac{\delta Z_O}{Z_O} = 0.04 \pm 0.31 \quad (7)$$

$$\frac{\delta Z_{Ne}}{Z_{Ne}} = -0.62 \pm 2.75 \quad (8)$$

Thus it is clear that the error estimates on individual abundances are too large. Most of the error in Table 3, is due to differences between inversions using different reference models. The random error (e.g., using same reference model) is an order of magnitude smaller. Thus if we can get better understanding of systematic errors, possibly with improved EOS it may be possible to reduce the errorbars and get a reasonable estimate for the abundances of some individual elements. Even with current errorbars the oxygen abundance can be inferred to an accuracy of about 30% and appears to favour the higher GS value, though errorbars are too large to firmly rule out the recent reduction in O abundance.

4. SUMMARY AND DISCUSSION

Using observed frequencies with 4 different reference models for inversion and 2 sets of calibration models we get $Z = 0.0175 \pm 0.002$ where the errorbars include systematic errors from different sources (Antia & Basu 2006). This value is consistent with GS but much larger than the revised estimate of Asplund et al. (2005). In principle, it is possible to determine the abundance of individual heavy elements using this technique, but that requires more reliable EOS and better understanding of systematic errors. The solar models using ASP composition with Ne abundance enhanced (Antia & Basu 2005) do not appear to match the $W(r)$ value for the Sun (Fig. 3) in the region $0.8R_\odot < r < 0.9R_\odot$.

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