HOW CORRELATED ARE F-MODE FREQUENCIES WITH SOLAR ACTIVITY?

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ABSTRACT

Temporal variations of solar f-mode frequencies are studied using data from the Global Oscillation Network Group (GONG) and the Michelson Doppler Imager (MDI) covering the period from 1995 to 2000. The frequencies show an increase with activity. There appears to be one component in the time varying part of the frequencies which is correlated with the solar activity indices. Superposed on this is an oscillatory variation with a period of 1 year, whose origin is not clear. The amplitudes of both the oscillatory and non-oscillatory component increases with the degree (and hence frequency) of the mode.

Key words: Sun: oscillations; Sun: activity; Sun: interior.

1. INTRODUCTION

The f-mode is believed to be a surface gravity mode and its frequency is essentially independent of the stratification in the solar interior. Hence, these modes provide a diagnostic of flows and magnetic fields in the near surface regions. The frequencies of f-modes also provide an accurate measure of solar radius (Schou et al. 1997; Antia 1998). Since these frequencies have now been measured very accurately, any possible temporal variation in solar radius by as little as 1 km can be studied by looking for variations in the f-mode frequencies (Tripathy & Antia 1999).

There have been many reports about possible variation of solar radius with time (Delache et al. 1985; Wittmann et al. 1993; Fiala et al. 1994; Laclare et al. 1996; Noeel 1997). The reported change in measured angular semi-diameter varies from $0.1''$ to $1''$, which implies a change of 70 to 700 km in radius. However, there is no agreement among observers about these variations. If these variations are real, we should expect a variation in f-mode frequencies by 0.01% to 0.1%, which is much larger than the estimated errors in these frequencies. It is therefore of interest to look for corresponding variations in the f-mode frequencies. Preliminary studies of temporal variations in f-mode frequencies have also given conflicting results about the variation of the solar radius. Dziembowski et al. (1998) using Michelson Doppler Imager (MDI) data found that the solar radius reached a minimum around the minimum activity period and was larger by about 5 km, 6 months before and after the minimum. But later results using longer time interval did not find any systematic changes in solar radius from the f-mode frequencies (Dziembowski et al. 2000). On the other hand, Antia et al. (2000a) using Global Oscillation Network Group (GONG) data found that solar radius has decreased by about 5 km between 1995 and 1998 and this variation appeared to be correlated to level of solar activity. However, these results were based on only a few data sets.

In this work we try to use more extensive data from both GONG and MDI covering the period from 1995 to 2000 to study possible variation in f-mode frequencies and attempt to understand the discrepancy between earlier results. We also attempt to test if the observed variations in f-mode frequencies can arise from a change in solar radius. Any variation in solar radius will yield relative f-mode frequency differences, which are independent of degree, ℓ. On the other hand frequency differences due to magnetic field variation would yield relative frequency differences which vary significantly with ℓ (Campbell & Roberts 1989). Thus we need a reasonably large ℓ range to distinguish between these possibilities.

The GONG data are obtained from the rotation corrected m-averaged spectra (Pohl & Anderson 1998) obtained from 108 days of observations. These spectra cover all degrees with $\ell \leq 200$. We have used 18 spectra covering the period from 1995 May 7 to 1999 December 23. There is some temporal overlap between a few of these data sets. These spectra have been fitted to calculate the f-mode frequencies as described by Antia et al. (2000a). The MDI data (Schou 1999) consist of 19 non-overlapping data sets each covering a period of 72 days, starting from 1996 May 1 and ending on 2000 June 20. The MDI data sets contain f-mode frequencies up to $\ell = 300$. Since f-modes have rather low power and the power decreases rapidly with frequency, the frequencies have been measured re-

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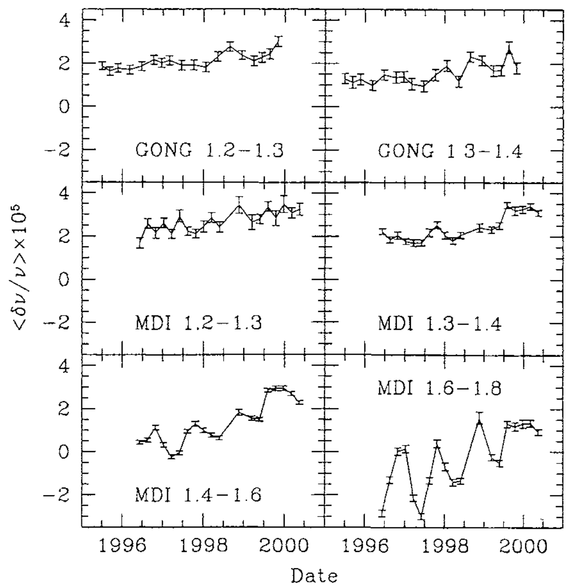

Figure 1. The mean relative frequency differences for f-modes between the observed values and those for a solar model are shown as a function of time for different frequency intervals as marked in various panels. The frequency intervals are in mHz.

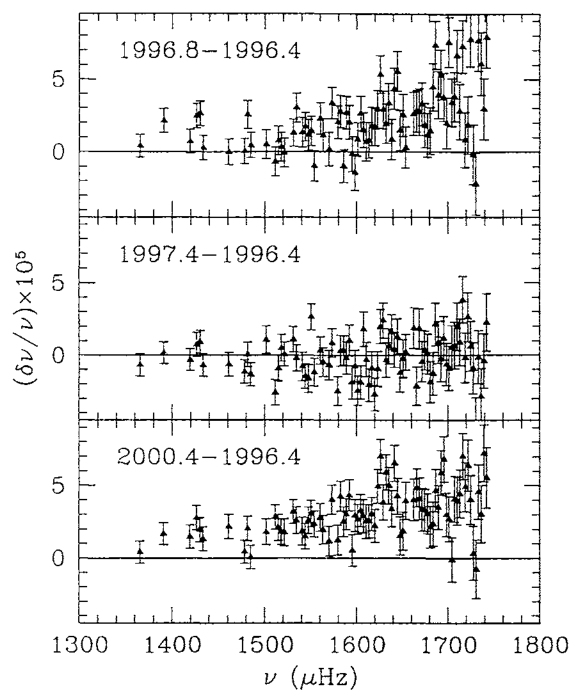

Figure 2. Relative frequency difference for f-modes between a few MDI data sets. The dates are marked in the panels.

liably only for $\ell > 140$ ($\nu > 1200$ μHz). We do not include lower frequency modes in this study.

2. RESULTS

Since the errors in individual f-mode frequencies are a bit too large to see small variations, we take average of relative frequency differences over modes in different frequency ranges. For all data sets we take frequency differences between the observed frequencies and those computed for a solar model with a radius of 695.78 Mm. Fig. 1 shows the mean relative frequency differences for f-modes in different frequency intervals for both GONG and MDI data as a function of time. It is clear that there is a significant variation in the f-mode frequencies in all frequency intervals. Furthermore, it appears that the variation has two components, one a oscillatory component with a period around 1 year and a non-oscillatory component which appears to increase with time. There is also a significant difference between various frequency intervals. Clearly, the behaviour of frequency differences is more complicated than what has been found earlier from a smaller sample of data sets. This may explain the apparent discrepancy between different results. In particular there is a strong frequency dependence in temporal variation, which may account for the differences between different studies which considered averages over different frequency intervals. In view of the strong frequency dependence we need to consider averages over limited frequency intervals.

Fig. 2 shows the relative frequency difference between a few sets of observed frequencies from MDI. The upper panel shows the difference between two sets taken just 0.4 year apart, during a period when the solar activity had not changed significantly. The time difference is roughly half of the oscillation period and that appears to be giving very large frequency difference, particularly at high frequencies. The middle panel shows the frequency difference between two sets separated by almost exactly 1 year, which is one oscillation period. During this time the change in solar activity was small and as a result we see very little frequency difference. The bottom panel shows the frequency difference between two sets separated by 4 years which should again suppress the oscillatory part. The frequency difference is likely to be due to significant increase in solar activity. Comparing the top and bottom panel it may appear that the frequency differences are similar, although a closer look reveals that the variation with frequency is steeper in top panel.

It is clear that a comparison of the frequency differences between a few selected epochs can lead to misleading conclusions. For example, Dziembowski et al. (1998) looked at five data sets from MDI covering the first oscillation period starting from epoch with minimum frequency and concluded that the estimated solar radius reached a minimum (or the frequency reached a maximum) in between this interval. The time of minimum radius happened to coincide with the minimum in solar activity. If the reported change were to be correlated with solar activity we would have expected a much larger change in the solar radius during the full solar cycle, which has not been seen in subsequent observations.

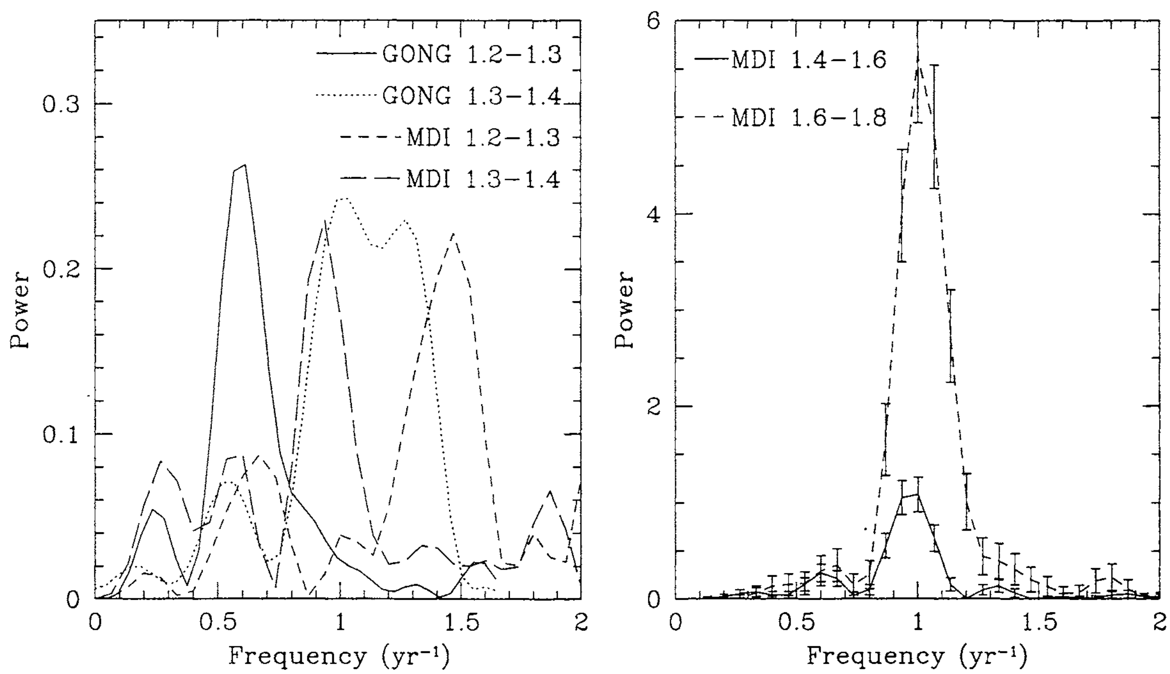

Figure 3. The power spectra of residual frequency difference after subtracting a linear trend for various frequency intervals. In lower frequency range the peaks in power spectra are comparable to the error estimate and hence the errors are not shown. Note the difference in y-axis scale between the two panels.

On the other hand, Antia et al. (2000a) also used only 5 data sets each of 108 days duration from GONG covering a period of 3 years from 1995 to 1998. This data set was too sparse to reveal 1 year period in frequencies. Further the GONG data sets are restricted to $\ell \leq 200$ and hence the oscillatory part is quite small as can be seen from Fig. 1. Dziembowski et al. (2000) considered more data sets from MDI and their results actually show the oscillations, but with limited time interval considered in their study the periodic nature of these oscillations was not identified. Thus by considering both oscillatory and non-oscillatory components in frequency differences, we can understand all earlier results.

In order to separate out the oscillatory and non-oscillatory components of the frequency differences, we first remove the smooth trend by a least squares fit to a linear function in solar activity. We use the 10.7 cm radio flux as a measure of solar activity. The mean 10.7 cm flux is obtained from the Solar Geophysical data web page (www.ngdc.noaa.gov/stp/stp.html) of the US National Geophysical Data Center. We fit a linear function of form

$$\left\langle \frac{\delta\nu}{\nu} \right\rangle = aF_{10} + b, \qquad (1)$$

to obtain the constants a, b. Here F_{10} is the mean 10.7 cm radio flux during the time interval covered by the helioseismic data sets. This linear trend is then subtracted from $\langle \delta\nu/\nu \rangle$ to obtain the residuals which should contain the oscillatory part. We take the Fourier transform of these residuals to obtain the power spectrum. This exercise is repeated for different frequency intervals. The resulting power spectra are shown in Fig. 3. At low frequencies there is no significant peak in the power spectra as all the peaks are comparable to the error estimates. But at high frequencies there is a dominant peak around a period of 1 year. For the frequency interval 1600–1800 μHz in MDI data this peak is about 8σ and is statistically significant. Except for the lowest frequency range of 1200-1300 μHz in both GONG and MDI data all other spectra show some peak at this frequency, the height of the peak increases with frequency. Thus it appears that the oscillatory trend is present at all frequencies. The period is found to be exactly one year within the resolution of spectra. Considering the fact that both GONG and MDI data sets span a period of about 4 years, the frequency resolution in power spectra would be about 0.25 yr^{-1}. The half-width of peak in the spectrum is comparable to the frequency resolution and hence the actual width may even be smaller.

From the Fourier transform we can calculate the amplitude and phase of the oscillatory component and subtract it from the total frequency difference and compare the residual with the linear trend that was removed earlier. The results are shown in Fig. 4. The residuals are consistent with the linear trend and are well correlated with solar activity. The correlation coefficient between the residual and the mean 10.7 cm radio flux is found to be between 0.75–0.95 for different frequency ranges. Data obtained during the coming year or two when solar activity starts decreasing should be able to confirm whether the variation is indeed correlated to solar activity. We also subtract the linear trend from the frequency difference and compare the residuals with the oscillatory

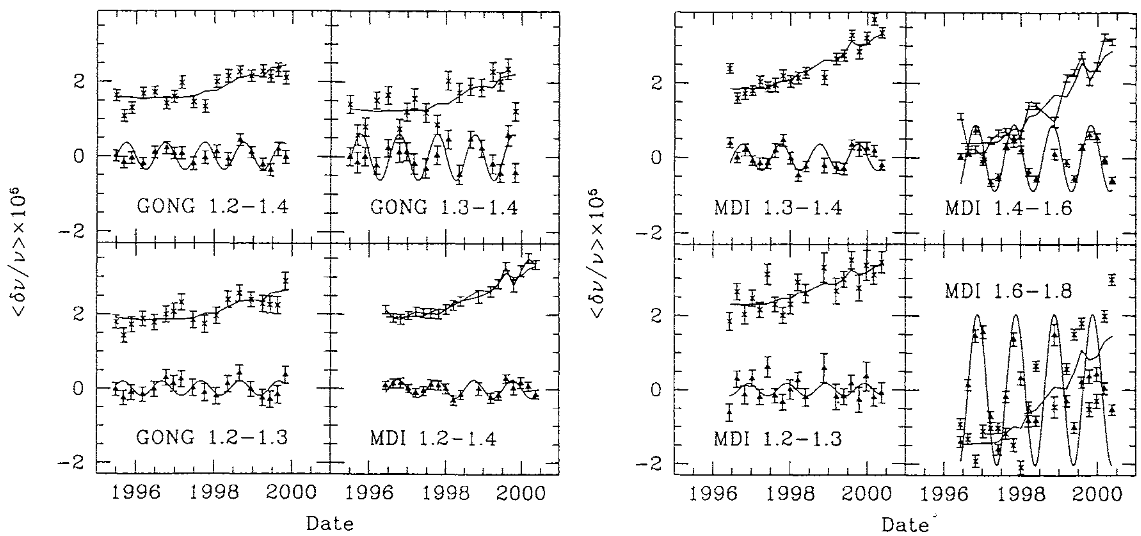

Figure 4. The oscillatory part and the non-oscillatory part of the frequency difference are compared with residuals obtained after subtracting the non-oscillatory and oscillatory parts, respectively. The frequency interval (in mHz) and data sets are marked in each panel.

part. These points are also shown in Fig. 4. It is clear that the amplitude of both the oscillatory part as well as the non-oscillatory part increases with increasing frequency. To study the frequency dependence of these amplitudes we use smaller frequency intervals of 50μHz and the resulting amplitudes are shown in Fig. 5. The amplitude appears to be varying as ν^p, and the power law index p can be calculated from the slope of linear fit to be 8.6 and 4.1 for the oscillatory and non-oscillatory part, respectively. This would translate to a variation as $\ell^{4.3}$ and ℓ^2 for the amplitude of the two components. Much of this variation can be accounted for by the mode inertia. If the frequency differences are scaled by the mode inertia then the slope is reduced as seen by thick lines in Fig. 5. For the oscillatory part the slope is reduced, while for the non-oscillatory part the slope has reversed.

The oscillatory part for various frequency intervals appears to be in phase, as can be seen from Fig. 6, which shows the oscillatory part in different frequency ranges. The lowest frequency range of 1200–1300 μHz is not included as there is no peak in the corresponding spectra around 1 year period. There is very little difference in phase. Thus at all frequencies, the frequency difference appears to have an oscillatory component with a period of 1 year and the oscillations are in phase, with only the amplitude increasing with frequency. The non-oscillatory part in the frequency differences is also shown in Fig. 6.

Although in this work we are primarily concerned with f-modes, it is of interest to check if similar oscillations are present in p-mode frequencies. To check this we have repeated the exercise for $n = 1, 2, 3$ using MDI data for frequency difference with respect to MDI data from the first 360 days of observations. We find a marginally significant peak in the Fourier transform for $n = 1, 2$. Fig. 7 shows the power spectra for p_1-modes. The peak in this case is much less significant but it may be noted

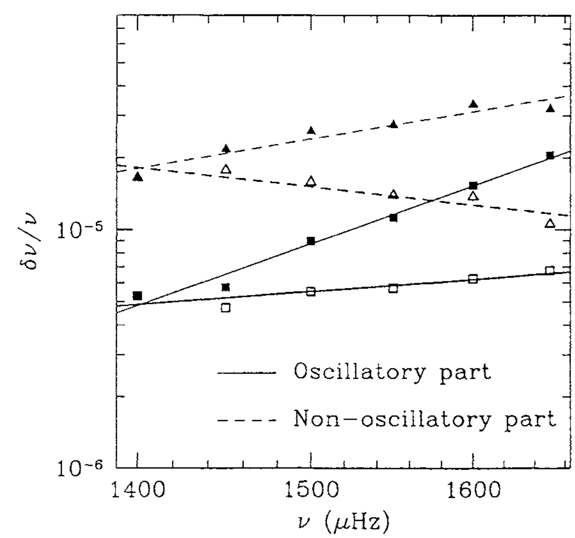

Figure 5. The amplitude of oscillatory and non-oscillatory part as a function of frequency. The straight lines show a linear fit to the points. The heavy lines with open symbols show the results after scaling by the mode inertia.

that highest degree, ℓ for p_1-modes is only about 200 in MDI data. It is quite possible that at higher degree the peak becomes more significant. So far we have only considered the mean frequencies, but we may also expect the corresponding splitting coefficients to show similar behaviour. We have tried similar exercise for frequency splitting coefficients and results for coefficient a_1 for f-modes are shown in Fig. 7. It can be seen that only the highest frequency bin shows a significant peak. Similar results are seen for coefficient a_3, while other coefficients

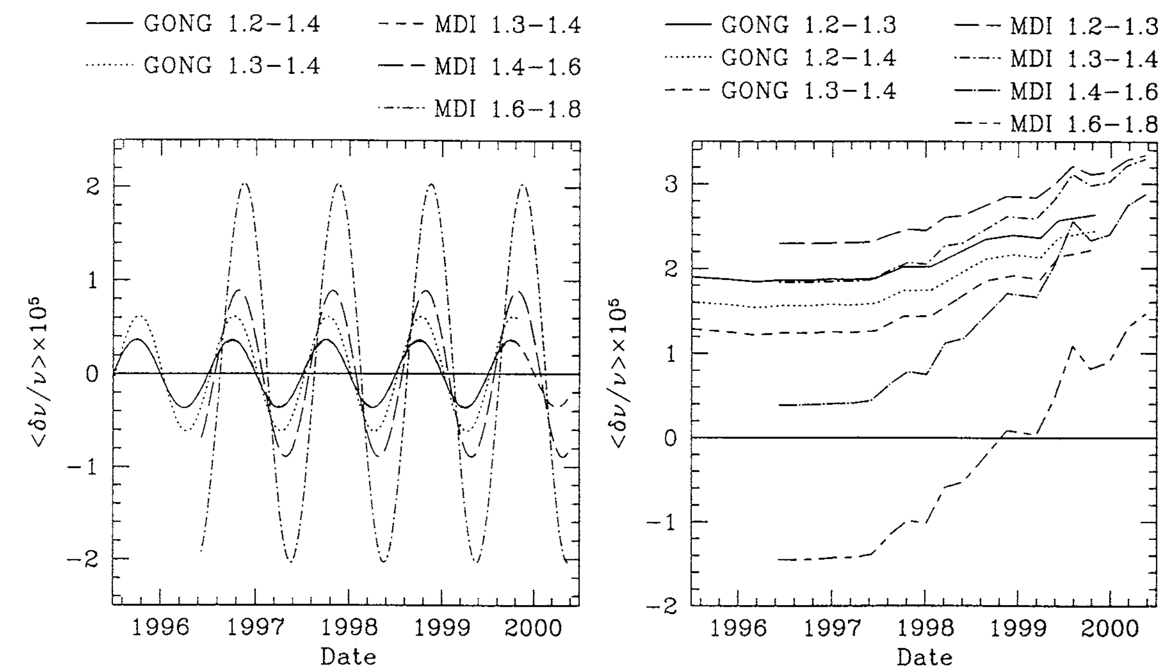

Figure 6. The oscillatory part (left panel) and non-oscillatory part (right panel) of frequency variation as a function of time for various frequency ranges.

do not show any significant peak. For $n = 1$ modes too, the coefficients a_1 and a_3 show a marginally significant peak. Thus it appears that 1 year oscillation is prevalent in low radial order modes.

3. DISCUSSION

We have studied the temporal variation in f-mode frequencies in various frequency intervals using both GONG and MDI data over the period 1995–2000. The variation consists of two components, an oscillatory part with a period of 1 year and a non-oscillatory part which appears to be correlated with solar activity. This decomposition explains apparent discrepancies in earlier results using f-mode frequencies (Dziembowski et al. 1998, 2000; Antia et al. 2000a). Since the period of the oscillatory component matches with the orbital period of Earth, it is likely to be an artifact. Apart from the yearly variation in orbital velocity and Sun-Earth distance, the inclination of Sun's rotation axis as seen from the Earth also varies periodically on the same time scale. Consequently, the spatial window function used for spherical harmonic decomposition and the associated spatial leaks have a 1-yr period. It is the seasonal variation of these spatial leaks that may result in the 1-yr period in frequency shifts. If that were true then we would expect the effect to increase with ℓ because the blending of peaks and leaks increases with ℓ and consequently the ability of the peak-finding algorithm to clearly identify peaks decreases. This would result in a seasonally-varying bias (that increases with ℓ) in the results from the peak-finding algorithm. It is possible that the p-modes are less affected by this, because the ve-

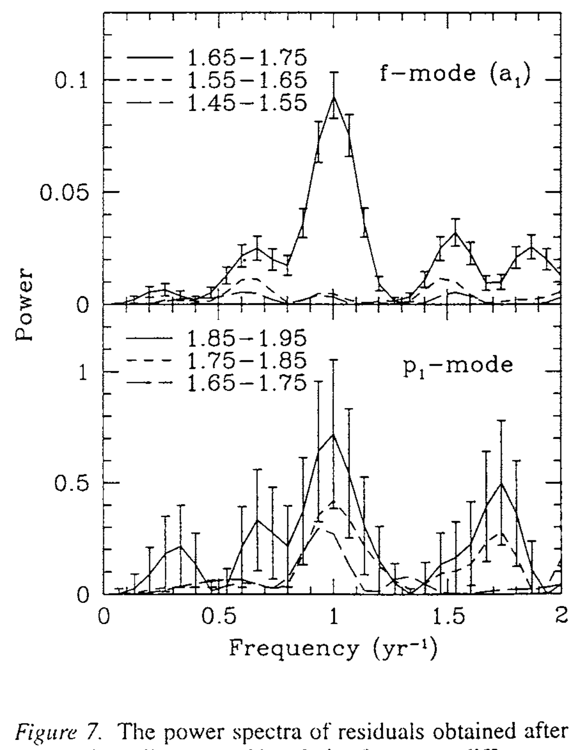

Figure 7. The power spectra of residuals obtained after subtracting a linear trend in relative frequency differences for p_1-mode are shown in lower panel. The upper panel shows the power spectra of residuals in frequency splitting coefficient a_1 for f-modes. The frequency ranges in mHz are labelled in the panels.

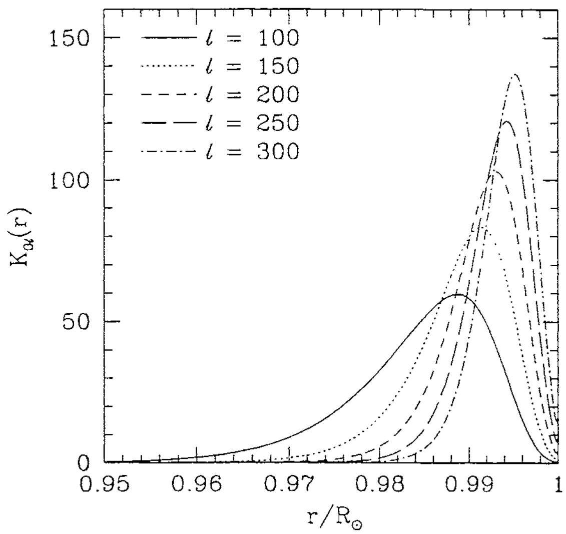

Figure 8. The kinetic energy density in f-modes as a function of radial distance for various ℓ values.

locity due to p-mode oscillations is predominantly radial, while that due to f-modes is horizontal.

A solar origin of this periodicity will be more difficult to reconcile with our results, but if that were to be true the oscillations can only arise from some periodic variation in outermost layers of the Sun. To identify the depth resolution of f-modes we show in Fig. 8 the kinetic energy density in f-modes as a function of radial distance for various degree. The peak occurs just below the surface with the depth of the peak decreasing with ℓ. Since the amplitude of oscillatory part increases very steeply with ℓ the disturbance has to be localized in outermost layers ($r \gtrsim 0.996R_\odot$). Only in that case we can expect to find such steep variation with ℓ. If the perturbation is located in deeper layers then all modes will feel it to similar extent. Use of even higher ℓ modes may be able to localize the perturbation more accurately.

In addition to the oscillatory part there is also a non-oscillatory trend which appears to be correlated with solar activity. This variation cannot be due to the variation in solar radius because in that case the relative frequency difference would be independent of frequency. Since this part also has relatively steep dependence on ℓ, it can be caused by temporal variation in solar magnetic field. In order to get the steep variation with ℓ the time-varying component of magnetic field should be concentrated in outermost region (Campbell & Roberts 1989). This inference is also consistent with the fact that no significant temporal variation in internal structure has been seen in either the spherically symmetric part (Basu & Antia 2000) or in the asphericity (Antia et al. 2000b). This would suggest that major part of solar cycle variations are located in a thin layer near the surface.

Considering all these results we do not find any evidence for temporal variation in solar radius during the current solar cycle. Any variation in solar radius during this period should be less than 1 km. It may be noted that if the solar radius actually varies by 1 km in 11 years then the rate of gravitational energy released or absorbed in that process would be much more than the solar luminosity. So any possible change in solar radius is likely to be a superficial phenomenon restricted to a thin layer just below the surface. The evolutionary change is solar radius is only about 2 cm per year, which is too small to be detected.

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