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HELIOSEISMOLOGY

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Abstract

Many of the oscillations observed at the solar surface have been identified with resonant normal modes of the entire solar body. Accurate measurements of frequencies of these modes enable us to study the internal structure and dynamics of the Sun. Helioseismology has provided a handle to measure the depth of the convection zone and the primordial helium abundance, as well as the sound speed and the rotation velocity throughout much of the solar interior.

1. Introduction

Soon after the discovery of solar oscillations when the solar physicists were debating over the nature of observed oscillations, there was an interesting paper by Mark Kac (1966) on "Can we hear the shape of a drum ?". This question sounds absurd at first sight since one normally associates a shape with seeing rather than hearing. However, it is well known that the sounds of drum makes when it is struck are determined by its physical characteristics i.e. the material used, its tautness and the size and shape. Drums vibrate at certain distinct frequencies called normal modes. The problem which Kac posed is as follows .

Suppose a drum is being played in one room and a person with perfect pitch (i.e. one who can identify exactly all the normal modes of vibration) hears but cannot see the drum. Is it possible for her to deduce the precise shape of the drum just from hearing the normal modes of vibrations ?

Though this question still remains unanswered in the rigorous mathematical sense, it is found that a lot can be learnt from the frequency spectrum. This is essentially similar to the question the helioseismologists face today: What can be learnt about the internal structure and dynamics from the frequencies of the normal modes excited in the Sun ?

As a result of the interplanetary medium it is not possible to "hear" the Sun directly. But interestingly enough it is possible to "see" the sound waves. This was achieved by Leighton (1960) and Leighton et al. (1962) by observing the Doppler shift in the solar spectral lines. It is well known that for small velocities the shift in wavelength is proportional to the line of sight velocity of emitting medium. Thus if we find the spectral lines to oscillate back and forth, then we can conclude that the solar surface is oscillating. Initial studies revealed a roughly oscillatory pattern with velocity amplitudes of about 1 km/s and a period of roughly five minutes. These were therefore referred to as five minute oscillations. Now we know that the pattern is actually a superposition of some 10^7 different modes, each having a amplitude of the order of 10 cm/s. More information can be obtained if one takes a fourier transform of the velocity,

$$A(\omega) = \int v(t) \exp(i\omega t)\, dt,$$

in order to get the power or amplitude A as a function of frequency ω. This showed that most of the power is concentrated around a frequency of 3 mHz, although the individual modes could not be resolved properly.

As soon as this discovery was announced about a dozen theories were put forward to explain the oscillations. Each of these theories managed to get the period correct in such a simple manner that it was embarassing for the theorists to explain why these oscillations were not predicted earlier. These early theories have been reviewed by Stein and Leibacher (1974).

Ulrich (1970) and Leibacher and Stein (1971) independently proposed that these oscillations may be acoustic (or sound) waves trapped just below the solar surface. Ulrich further showed that the frequencies of such modes will depend on the horizontal wave-number k_H (or wavelength $2\pi/k_H$). For obtaining the spatial information in addition to the temporal data the observations have to be repeated over a series of points on the solar disk. using a slit which was perpendicular to the solar equator Deubner (1975) made Doppler velocity measurements spanning many hours over an equatorial strip on the solar disk. The Fourier transform in longitude and time resulted in power spectrum,

$$A(k,\omega) = \int\int v(x,t) \exp[i(kx + \omega t)]\, dxdt.$$

Figure 1 shows the more recent results of Deubner et al. (1979), also shown are the frequencies calculated from two different solar models. The general agreement between the observed ridges in the power spectrum and the theoretical frequencies essentially confirmed the hypothesis of Ulrich regarding the nature of five minute oscillations. This marked the beginning of the subject of helioseismology. The agreement between theory and observation was of course not perfect and it was realised that it can be improved if the thickness of the convection zone in the solar model is larger than what was supposed in contemporaneous models. Thus for the first time helioseismology provided us with a measure of thickness of the solar convection zone.

Comprehensive reviews of helioseismology are available and should be consulted for more details (cf. Brown et al. 1986, Christensen-Dalsgaard et al. 1985, Deubner and Gough 1984, Leibacher et al. 1985). here we will only consider a brief review of helioseismology in order to introduce some of the basic concepts and notation used in the subject.

2. Classification of Normal Modes

It is well known that every finite object like a tuning fork, a drum or a string stretched between two fixed points has a set of normal modes of vibration and when distrubed will oscillate in some combination of these normal modes. For example in the case of a string stretched between two fixed points a distance D apart, the normal modes correspond to wavelengths of $2D/n$, where n is any positive integer, and hence only the corresponding frequencies can be excited. The Sun being a three dimensional object three integers would be needed to specify a normal mode.

A gas sphere such as the Sun may be subject to many forces, for example gas pressure, gravity, electromagnetic forces and rotation. If such a gas is disturbed then these forces can act to restore the gas towards its initial state if it is in a stable equilibrium or to drive it even further from initial state if the equilibrium is unstable. If the initial configuration is stable, the gas will return to its initial state, but if dissipation is small it will overshoot the equilibrium state and will keep oscillating about its initial position. The normal modes of the Sun can be analysed by considering an infinitesimal disturbance from its equilibrium structure with the help of the usual equations of fluid mechanics. Since the perturbations are infinitesimal the equations may be linearized by

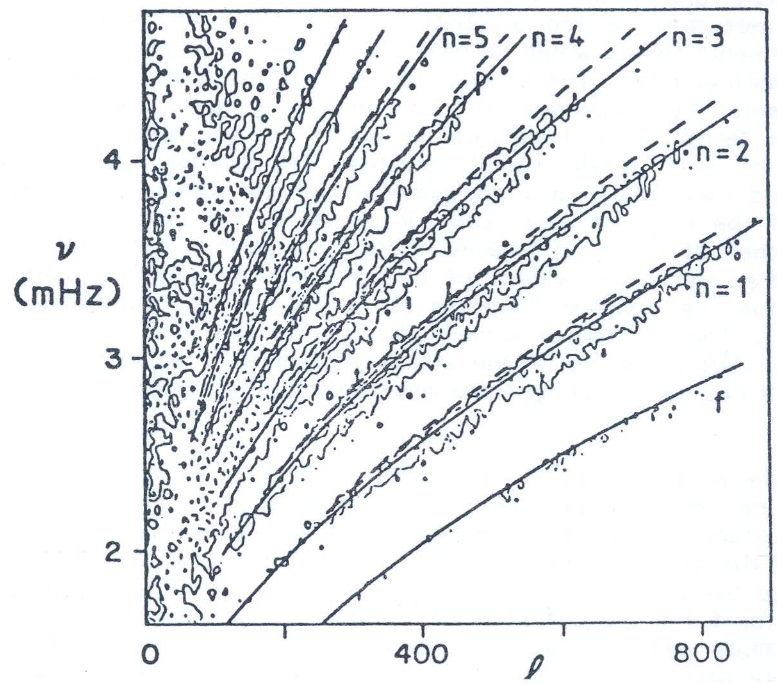

Fig.1. Contour of $\sqrt{\ell}$ times power in (ν,ℓ) plane from Dubner et al. (1979). The continuous curves are the predictions of a solar model with a helium abundance Y = 0.25, the dashed curves are for Y = 0.19. The Y = 0.19 model gives the correct observed neutrino flux.

neglecting all higher order terms in perturbations to get a system of linear homogeneous equations (cf. Unno et al. 1979) leading to an eigenvalue problem. Such equations can have nontrivial solutions only under special circumstances giving the eigensolutions or the normal modes.

For a spherically symmetric equilibrium stucture the normal modes can be expressed in terms of the spherical harmonics $Y_{\ell m}(\theta,\phi)$, to express perturbations in the form

$$p(r,\theta,\phi,t) = p_o(r) + p_{n\ell m}(r)\, Y_{\ell m}(\theta,\phi)\, \exp(i\omega_{n\ell m}t),$$

where p could be any physical quantity like pressure, temperature etc., $p_o(r)$ is the equilibrium value while the second term represents the infinitesimal perturbation. Further if the perturbations are adiabatic i.e. there is no heat exchange between different fluid elements then ω would be real and in that case ω can be identified with the frequency of the normal mode. The time period of oscillation is given by $P = 2\pi/\omega$. It is customary to measure the frequency in Hertz which is given by $\nu = 1/P = \omega/2\pi$. The integers ℓ and m detrmine the horizontal structure for the given mode, while n determines the number of nodes in the radial direction. The integer ℓ is referred to as the degree of mode and can assume any non-negative integer value, while m is called the azimuthal order of the mode and can take on values in the range of $-\ell$ to $+\ell$. For $m = 0$, ℓ gives the number of nodes as θ varies from 0 to π, while for $m = \pm\ell$ it gives the number of nodes in ϕ. For other values of m the structure is more complicated. For purpose of illustration the contour diagrams for a few of the spherical harmonics are shown in Figure 2. The horizontal wavenumber associated with the mode can be identified by $k_H^2 = \ell(\ell+1)/r^2$. Further if the equilibrium state is spherically symmetric then $p_{n\ell m}$ and $\omega_{n\ell m}$ will not depend on m since results should be independent of the choice of coordinate axes.

The simplest case is for $\ell = m = 0$ when the perturbations are independent of θ of ϕ and hence spherical symmetry is maintained even in perturbed state, and the entire solar surface oscillates in phase. Such oscillations are termed radial oscillations and most of the classical pulsating stars do execute radial oscillations. For $\ell > 0$ the peturbations are not spherically symmetric and are referred to as nonradial modes. Further the modes for $\ell \lesssim 3$ are referred to as low degree modes, while those for $4 \lesssim \ell \lesssim 100$ as intermediate degee modes and for $\ell \gtrsim 100$ as the high degree modes. This classification is mainly because different observational techniques are used to study these different classes of modes.

The frequency of oscillations is essentially determined by the restoring force and the inertia of the gas involved. Perhaps the simplest restoring force is compressibility or gas pressure. This gives rise to the sound waves or the acoustic modes. In the absence of other forces these follow the simple dispersion relation $\omega^2 = k^2 c_s^2$, where c_s is the sound speed.

Another important force in the Sun is gravity or buoyancy giving rise to the gravity modes, sometimes referred to as internal gravity modes to distinguish them from the surface graity waves. As compared to the pressure which is isotropic, gravity is a directed force and hence the properties of gravity modes are quite different from those of acoustic modes. Consider a small fluid element at radial distance r and displace it radially by a distance Δr. Assuming that the element moves adiabatically, i.e. without exchanging heat with the surrounding medium, and further as it reaches its new position it will achieve instantaneous pressure balance with the surrounding provided its speed is much less than the sound speed. Hence the density of this fluid element will be

$$\rho_e = \rho_o - \left(\frac{\partial \rho}{\partial P}\right)_{ad} \Delta P,$$

where ρ_o is the initial density and ΔP is the decrease in pressure between the two levels. The subscript ad denotes the adiabatic derivative. The density of the surrounding medium is

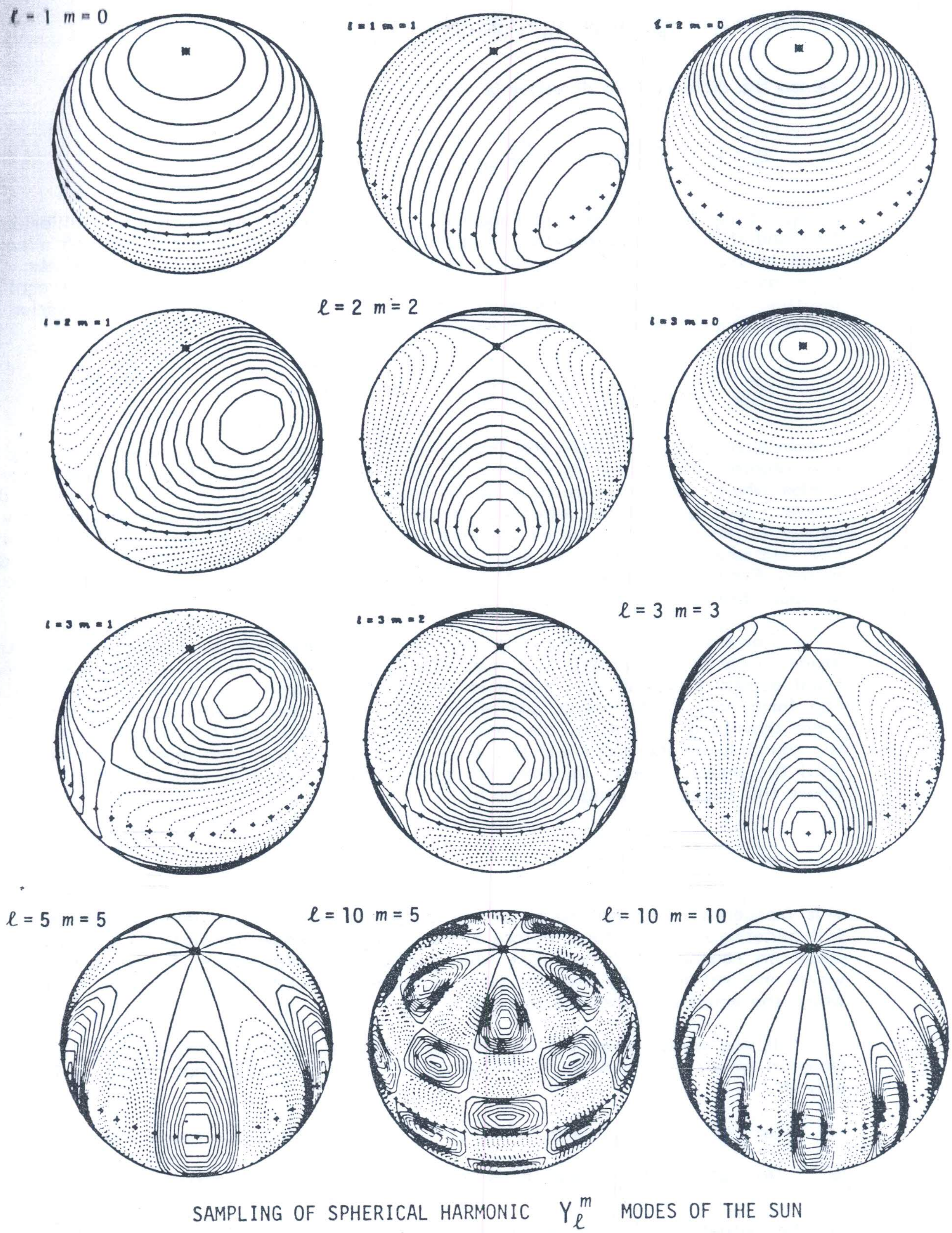

Fig.2. Contour plots of a sample of spherical harmonics $Y_{\ell m}$. Continuous curves represent positive values while dotted curves represent negative values.

$$\rho_s = \rho_o + \frac{d\rho}{dr}\Delta r = \rho_o - \left(\frac{d\rho}{dP}\right)_{Sun}\Delta P$$

Hence the difference in densities is

$$\rho_e - \rho_s = \left[\left(\frac{d\rho}{dP}\right)_{Sun} - \left(\frac{d\rho}{dP}\right)_{ad}\right]\Delta P \ ;$$

If $\rho_e < \rho_s$, the element will continue to move upwards away from its equilibrium position and the fluid configuration is regarded to be unstable and in fact such a condition leads to convection. This condition is realised in the stellar convection zones. On the other hand if $\rho_e > \rho_s$ then the element being heavier will come down to its equilibrium position. In this case the element will execute a simple harmonic oscillation about the equilibrium position with a frequency given by

$$N_{BV}^2 = \frac{g}{\rho}\left[\left(\frac{d\rho}{dP}\right)_{Sun} - \left(\frac{\partial\rho}{\partial P}\right)_{ad}\right]\left|\frac{dP}{dr}\right|$$

The frequency N_{BV} is called the Brunt-Väisälä frequency. It is obvious that gravity modes will require differential movement of fluid elements. Thus, if we consider the radial modes where the entire surface moves in phase there will be no difference in density between two neighbouring elements at the same radial distance and consequently there cannot be any buoyancy forces and as a result there cannot be any radial gravity modes. For nonradial modes we can have buoyancy forces and clearly there can arise gravity modes. Further the maximum of Brunt Väisälä frequency in the solar interior provides an upper limit to the frequency of gravity modes.

For the Sun to resonate with well defined frequencies these waves must be trapped inside a cavity within the Sun, just like the case of a string stretched between two fixed points. Obviously the Sun does not have any rigid boundaries and hence a cavity can form only if the waves are reflected by some mechanism. If we consider a sound wave travelling inwards in the Sun, then as it goes inside the temperature and hence the sound speed increases and the wave is refracted more and more away from vertical until the phase velocity becomes horizontal and the wave is reflected back. The radius where this happens is roughly given by

$$\omega^2 = \frac{\ell(\ell+1)}{r^2}c_s^2 = L^2$$

This frequency L is referred to as the Lamb frequency. Thus waves with high frequency will penetrate deeper down before being reflected, while increasisng the value of ℓ will result in raising the level from which reflection takes place. In the atmosphere the waves are reflected by steeply declining density and hence a cavity is formed within which the acoustic modes are trapped.

If one writes down the equations for adiabatic oscillations, ignoring the perturbations in gravity, it can be easily shown (cf. Eckart 1960) that waves will propagate only if

(a) $\omega^2 > N_{BV}^2$ and $\omega^2 > L^2$ (corresponding to acoustic waves)

or (b) $\omega^2 < N_{BV}^2$ and $\omega^2 < L^2$ (corresponding to gravity waves)

In other situations the waves cannot propagate and are said to be evanescent, in which case the amplitude generally falls off exponentially with distance. What this means in mathematical terms is that in the propagating case the eigenfunctions have spatially oscillatory character (i.e. of form Sin(kr) or Cos(kr)), while in the evanescent case the eigenfunctions have an exponential form (i.e.. exp(±kr)).

In the Sun the frequencies L and N_{BV} vary with depth and hence the nature of waves will also change with depth. Figure 3 shows the variation of L and N_{BV} for $\ell = 1$ with radial distance in the Sun. Note N_{BV} is independent of ℓ but L is roughly proportional to ℓ and so far higher ℓ the L curve can be raised up appropriately. The brunt Väisälä frequency has two peaks, one in the interior of about 470 μHz and one in the atmosphere of about 5 mHz, while in the convection zone N_{BV}^2 is negative. If we consider a typical g-mode (gravity mode) frequency ν_1 then for $r < r_1$, $\omega^2 > N_{BV}^2$ and $\omega^2 < L^2$ and the mode is evanescent, for $r_1 < r < r_2$, $\omega^2 < N_{BV}^2$ and $\omega^2 < L^2$ and the mode behaves like a gravity wave, for $r_2 < r < r_3$, $\omega^2 < N_{BV}^2$ and $\omega^2 > L^2$ and the mode is evanescent, for $r_3 < r < r_4$, $\omega^2 > N_{BV}^2$ and $\omega^2 > L^2$ and the mode behaves like an acoustic wave, for $r > r_4$, $\omega^2 < N_{BV}^2$ and $\omega^2 > L^2$ and the mode is evanescent. Thus the mode is trapped in two different cavities in one of which it behaves like a gravity mode, while in the outer cavity comprising mostly of the convection zone it acts like an acoustic mode.

At higher frequencies ($\nu \gtrsim$ 470 μHz) like ν_2 the situation is more clear as for $r < r_5$ and $r > r_6$ this mode is evanescent and hence it is an acoustic mode (or p-mode) trapped in the region $r_5 < r < r_6$. It is clear that for frequency $\nu \gtrsim 5$ mHz the modes will not be trapped in the Sun, and hence at such frequencies we do not expect sharp resonances from the Sun. Of course in solar corona the temperature once again rises to a million degrees Kelvin or more and hence the sound speed and so the Lamb frequency will once again rise and the high frequency modes can also be trapped in region including the chromosphere and corona. But since these regions are neither very stable in temporal terms nor are the horizontal variations negligible and as a result we cannot expect sharp resonant frequencies in such cases. It is interesting to note that the observed spectrum also does not extend beyond about 5 mHz thus supporting the general picture outlined above.

We have noted that there may not be a sharp distinction between gravity and acoustic modes in the Sun, as the same mode can act as acoustic mode in one region and gravity mode in another region. Nevertheless, a unique scheme has been devised for the nomenclature of the normal modes (cf. Unno et al. 1979). For a fixed value of ℓ the modes can be grouped in two series: the acoustic modes $p_1, p_2, p_3, \ldots$ form a series with increasing frequency, while the gravity modes $g_1, g_2, g_3, \ldots$ form a series with decreasing frequency and a limit point at $\nu = 0$. In addition we also have the so called f-mode or the fundamental mode with frequency in between the g_1 and p_1 modes, and in general its eigenfunction does not have any nodes. As $\ell \to 0$ the frequencies of acoustic modes tend to a finite onzero value while those of gravity modes tend to zero. Frequency of all the modes increases with ℓ and for the gravity modes it tends to a finite limit equal to the maximum of N_{BV} in the interior while for acoustic modes the frequency tends to infinity with ℓ.

A major difference between the normal modes excited in a string as compared to those in the Sun is that in the case of a string all the modes are trapped in the same region while in the Sun different modes are trapped in different regions. This is a very significant feature which gives the diagnostic power for helioseismology. Since each mode samples a different region, by analysing a large number of modes we can study the stratification inside the Sun. For example the frequency of acoustic modes is in some sense a measure of sound travel time across the cavity and hence by measuring the frequencies of acoustic modes for a range of ℓ and n values we can study the variation of sound speed with depth (cf. Gough 1986). Such a study shows that the sound speed agrees reasonably well with that predicted from the standard solar model.

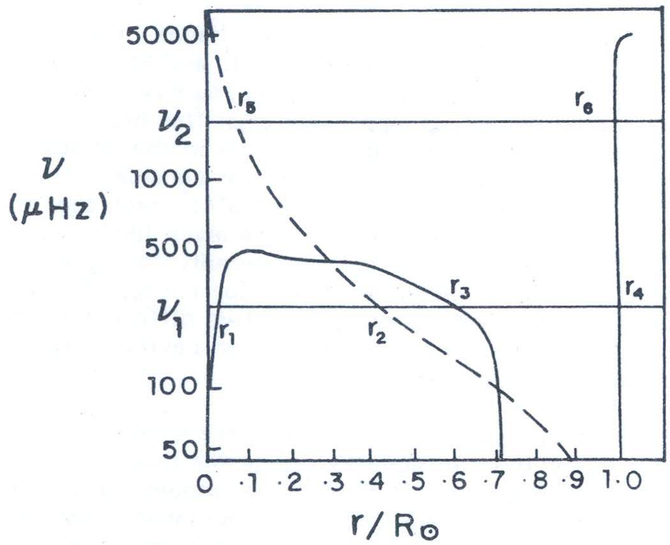

Fig.3 : The Lamb frequency for $\ell = 1$ (dashed curve) and the Brunt Väisälä frequency (continuous curve) as a function of radial distance in a solar model. The horizontal lines show the frequencies of a typical gravity mode (ν_1) and acoustic mode (ν_2).

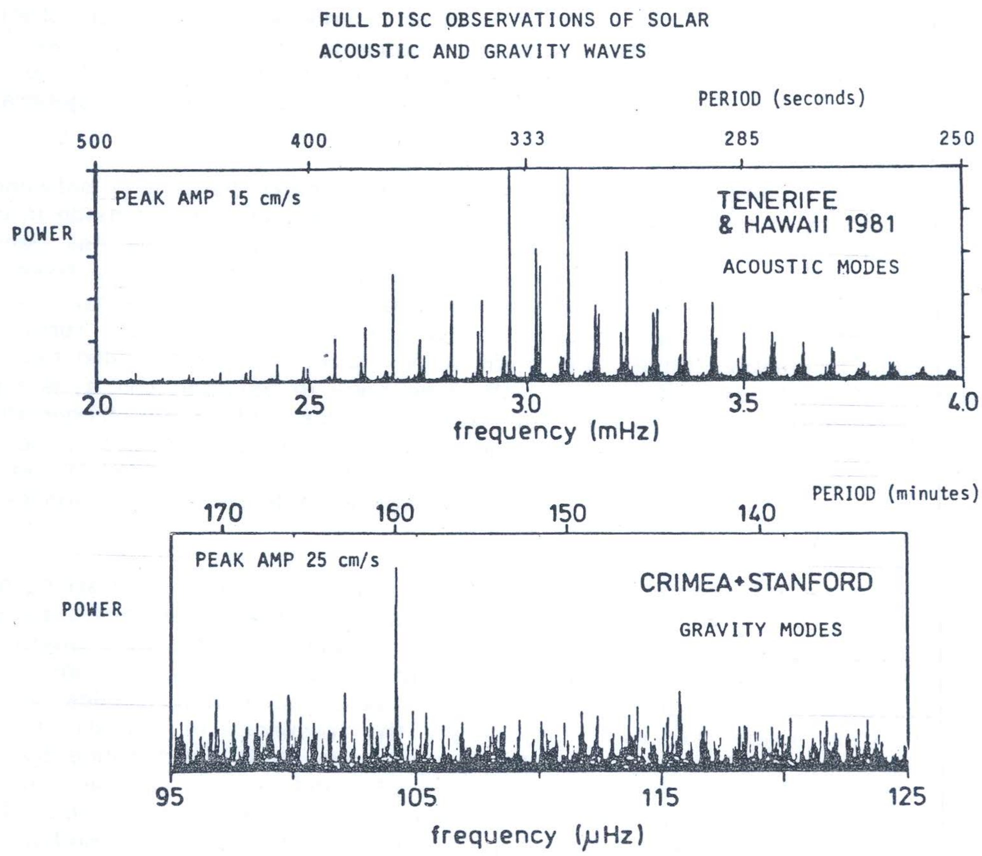

Fig.4. Power spectrum with frequency ν of low degree full-disk Doppler observations. The upper figure shows the observations of Claverie et al. (1984) for p-modes, carried out in 1981 over a period of 88 days from Tenerife and Hawaii providing upto 22 hours of coverage per day. The lower figure shows the results for g-modes from combined data of Stanford and crimea (Scherrer and Wilcox, 1983).