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Aerodynamic noise from wave-turbulence interaction

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Abstract. In order to estimate the acoustic energy scattered when a unit volume of free turbulence, such as in free jets, interacts with a plane steady sound wave, theoretical expressions are derived for two simple models of turbulence: eddy model and isotropic model. The effect of convection by mean motion of the energy-bearing eddies on the incident sound wave and on the sound generated from wave-turbulence interaction is taken into account. Finally, by means of a representative calculation, the directionality pattern and Mach number dependence of the noise so generated is discussed.

Keywords. Aeroacoustics; aerodynamic noise; wave-turbulence interaction.

1. Introduction

Aerodynamic noise is a problem of statistical fluid dynamics where statistical functions (e.g., velocity correlation) rather than basic disturbances (such as velocity) play significant role. The present investigation aims at a type of aerodynamic noise scattered from wave-turbulence interaction. As pointed out by Kraichnan [10], the problem of the scattering of sound by a turbulent flow is a most conspicuous acoustical phenomenon associated with low Mach number turbulence. In the past, the problem has been discussed in some detail by Lighthill [12], Kraichnan [10], Ford and Meecham [6] and Howe [9], among others. The analysis carried out in these references is based on a flow model in which, in the words of Lilley *et al* [13], the sound sources may move but not the flow. Therefore, the effect of convection is not recognized explicitly. In the present analysis the convection of pressure disturbance by mean motion of energy-bearing eddies is explicitly taken into account. Also, unlike the implicit form of solutions obtained in these references for the intensity of scattered energy, the present analysis provides a quantitative information so as to draw a more realistic picture of scattered energy as a function of exit Mach number, emission angle and frequency and amplitude of sound wave incident upon the flow.

2. Formulation of the problem

The analysis to be followed here is essentially a problem of interaction where two distinct fundamental oscillatory disturbances in the small-amplitude fluctuating motion of a fluid, are involved. The two disturbances are: turbulence (or, irregular vorticity diffusion showing random variations with space and time) and sound (or acoustic wave motion). Let $u_i'(X, t)$ be the fluctuating component of the total flow velocity of the turbulent flow, and V_i be the particle velocity of the plane sound wave incident upon the flow. Then the mathematical description of pressure p of the sound generated solely by nonlinear interaction of turbulence with incident sound wave may be given in terms of an approximate version of the generalized aerodynamic noise equation as (Chandraker and Munjal [1], Munjal and Chandraker [14]).

$$\left(\frac{\partial^2}{\partial t^2} + 2U_0 \frac{\partial^2}{\partial x_1 \partial t} + U_0^2 \frac{\partial^2}{\partial x_1^2} - a_0^2 \frac{\partial^2}{\partial x_i^2}\right) \hat{p}\,(X, t) = 2\gamma \frac{\partial u_i'}{\partial x_j} \frac{\partial V_j}{\partial x_i}, \tag{1}$$

where $\quad \hat{p} = p/p_0 \qquad (2)$

The subscripts i and j are tensor notations. The coefficient γ is the ratio of specific heats defined by the relation

$$a_0^2 = \frac{\gamma p_0}{\rho_0} \tag{3}$$

The quantities U_0, a_0, ρ_0 and p_0 are the average values of convection velocity, ambient sound speed, ambient density and ambient pressure, respectively. X and t are, respectively, space and time co-ordinates. The x_1-axis is assumed to be aligned with the axis of the jet. Equation (1) neglects the effects of the gradients of mean flow. Further, the mean flow has been assumed to extend even beyond the source region. These approximations have been made for mathematical convenience.

The solution of (1) in an unbounded medium may be written by the method of Kirchoff's retarded potential solution technique as (Ribner [15])

$$\hat{p}\,(X, t) = \frac{\gamma}{2\pi\,\beta_{av} a_0^2} \int \left(\frac{\partial u_i'}{\partial y_j} \frac{\partial V_j}{\partial y_i}\right)_{(Y, t'')} \frac{d^3\,Y}{|X' - Y'|} \tag{4}$$

where

$$|X' - Y'| = \left\{\left(\frac{x_1 - y_1}{\beta_{av}}\right)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2\right\}^{1/2}$$

$$t'' = \beta_{av}\, t - \frac{|X' - Y'| - M_0 x_1'}{a_0}$$

$$\beta_{av} = (1 - M_0^2)^{1/2}, \; M_0 = U_0/a_0,$$

$$X' = \left(\frac{x_1}{\beta_{av}}, x_2, x_3\right), \quad Y' = \left(\frac{y_1}{\beta_{av}}, y_2, y_3\right). \tag{5}$$

X and Y are respectively observer and source co-ordinate systems. $d^3 Y$ is an infinitesimal volume element of source region. Integration is over all possible values of Y. The quantity with subscript (Y, t'') is to be evaluated at position Y and time t''.

Assuming the turbulence to be in compressible (Goldstein and Rosenbaum [8]), i.e.,

$$\partial u'_i/\partial y_i = 0, \tag{6}$$

one obtains

$$\frac{\partial u_i'}{\partial y_j}\frac{\partial V_j}{\partial y_i} = \frac{\partial^2 (u_i' V_j)}{\partial y_i\, \partial y_j} - u_i' \frac{\partial^2 V_j}{\partial y_j\, \partial y_i}. \tag{7}$$

Following Lighthill [11] and Goldstein [7] one may easily show for $|X| \gg |Y|$

$$\int \left[\frac{\partial^2 (u_i' V_j)}{\partial y_i\, \partial y_j}\right]_{(Y,\, t'')} \frac{d^3 Y}{|X' - Y'|}$$

$$\simeq \int a_i\, a_j\, \frac{\partial^2\, [u_i' V_j]}{\partial t^2} (Y, t'') \frac{d^3 Y}{|X' - Y'|}, \tag{8}$$

where

$$a_i = \left(\frac{1}{a_0\, \beta_{\mathrm{av}}\, \beta^{(i)}}\right) \left(M_0\, \delta_{i1} - \frac{x_i' - y_i'}{|X' - Y'|}\right), \tag{9}$$

$$\begin{aligned} \beta^{(i)} &= \beta_{\mathrm{av}} \text{ if } i = 1, \\ &= 1 \quad \text{if } i \neq 1, \end{aligned} \tag{10}$$

δ_{ij} is Kronecker delta. The proof of (8) is straightforward if one makes use of the divergence theorem twice in left-hand-side and then neglect resulting terms of order $(|X' - Y'|^{-2})$ and $(|X' - Y'|^{-1})$ in favour of those of the order $|X' - Y'|$.

Noting that the typical frequencies of flow-fluctuations u_i' at a point are small compared with those of the incident sound wave V_i, one obtains (Kraichnan [10], Lighthill [12])

$$\frac{\partial^2\, (u_i' V_j)}{\partial t^2} \simeq u_i' \frac{\partial^2\, V_j}{\partial t^2}. \tag{11}$$

An expression for particle velocity due to plane steady sound wave incident upon flow may be written as (Lighthill [12])

$$V_j = \epsilon\, a_0 \cos\, (k_c\, x_1 - \omega t)\, \delta_{j1}, \tag{12}$$

where

$$1 \gg \epsilon > 0, \tag{13}$$

from the requirement of small amplitude motion. ω is the radian frequency of incident sound wave. k_c is convected wave number defined by

$$k_c \equiv \omega/(a_0 + U_0) \tag{14}$$

A far-field approximation common in jet noise literature is (Lighthill [11], Goldstein [7]) (for $|X| \gg |Y|$)

$$x_i - y_i \simeq x_i, \; i = 1, 2, 3. \tag{15}$$

Generally, the approximation is valid only in the middle of emission angle range $80° > \theta > 10°$ (where θ is the emission angle measured from the jet axis in a co-ordinate system, which has its origin at the jet exit centre). The approximation $x_1 - y_1 \simeq x_1$ is not valid for $\theta > 82{\cdot}5°$. Similarly the approximation $x_2 - y_2 \simeq x_2$ (or $x_3 - y_3 \simeq x_3$) is not valid for $\theta < 7{\cdot}5°$.

A modified form of the approximation proposed by the authors in their earlier work is (Munjal and Chandraker [14], Chandraker and Munjal [3]),

$$x_i - y_i \simeq x_i - y_i^*, \tag{16}$$

where y_i^* is some hypothetical constant value of y_i chosen judiciously so as to represent the location of most intense sound sources and may be interpreted as a 'characteristic location' of all sound sources distributed in whole or part of the jet field.

Substitution of (7), (8), (11), (12) and (16) in (4) yields

$$\hat{p}\,(X, t) = \gamma\, A_i \int (u_i'\; V_1)_{(Y,\, t'')}\; d^3Y, \tag{17}$$

where

$$A_i = \frac{-\,\omega^2\, a_i^*\; a_1^*}{2\pi\, \beta_{av}\, a_0^2\, |X' - Y'^*|^2} + \frac{k_c^2\, \delta_{i1}}{2\pi\, \beta_{av}\, a_0^2\, |X' - Y'^*|}, \tag{18}$$

$$a_i^* = \left(\frac{1}{a_0\, \beta_{av}\, \beta^{(i)}}\right) \left(M_0\, \delta_{i1} - \frac{x_i' - y_i'^*}{|X' - Y'^*|}\right). \tag{19}$$

3. Expression for scattered energy

Mean rate of scattered energy crossing unit area at X may be defined as (Lighthill [12], Goldstein [7]),

$$I(X) = \frac{p_0^2}{\rho_0\, a_0}\, \overline{\hat{p}^2}, \tag{20}$$

where symbol 'overbar' denotes time averaging. From (17) and (20)

$$I(X) = \frac{p_0^2\, \gamma^2\, A_0\, A_j}{\rho_0\, a_0} \int\!\!\int \overline{(V_1)_1\, (V_1)_2\, (u_i')_1\, (u_j')_2}\; d^3Y_1\, d^3Y_2, \tag{21}$$

where the subscripts 1 and 2 outside the parentheses refer to (Y_1, t_1'') and (Y_2, t_2''), respectively. The times t_1'' and t_2'' are defined as

$$t_1'' = \beta_{av} t + \{M_0 x_1' - |X - Y_1'|\}/a_0,$$

$$t_2'' = \beta_{av} t + \{M_0 x_1' - |X - Y_2'|\}/a_0. \tag{22}$$

On defining

$$Y_1 = Y, \; Y_2 = Y + \zeta \; (\text{or } \zeta = Y_2 - Y_1), \tag{23}$$

one may write expression for mean rate of scattered energy per unit volume of turbulence and crossing unit area at X as

$$I(X/Y) = \frac{p_0^2 \gamma^2 A_i A_j}{\rho_0 a_0} \int \int \overline{(V_1)_1 (V_1)_2} \cdot \overline{(u_i')_1 (u_j')_2} \, d^3\zeta, \tag{24}$$

where

$$I(X) \equiv \int I(X/Y) \, d^3 Y.$$

The integrand in (24) is in a form of product of two time-averaged terms. The equivalence of two integrands (one of (21) and the other of (24)) is natural if one notes that the fluctuations in particle velocity due to sound wave incident upon the flow are statistically independent of the flow fluctuations (Lighthill [12]).

For the observer in the far field ($|X| \gg |Y|$) and making use of (12) and the notion of 'characteristic location' (Munjal and Chandraker [12]) one may write

$$\overline{(V_1)_1 (V_2)_2} = \tfrac{1}{2} \epsilon^2 a_0^2 \cos (\tilde{k}_c' \cdot \zeta), \tag{25}$$

where

$$\tilde{k}_c' = (\tilde{k}_{c_1}/\beta_{av}, \tilde{k}_{c_2}, \tilde{k}_{c_3}),$$

$$\tilde{k}_{c_i} = \frac{\omega}{a_0} \frac{(x_i' - y_i'^*)}{|X' - Y'^*|}. \tag{26}$$

From the assumption of 'acoustically compact sources', Lighthill [11, 12] shows that

$$\overline{(u_i')_1 (u_j')_2} = Q_{i,j}(Y, \zeta) \tag{27}$$

where the correlation function $Q_{i,j}$ is independent of time.

Making use of (25) and (27) in (24) one obtains

$$I(X/Y) = \frac{1}{2} \left(\frac{\epsilon^2 a_0 p_0^2 \gamma^2 A_i A_j}{\rho_0} \right) \int \cos (\tilde{k}_c', \zeta) \, Q_{i,j}(Y, \zeta) \, d^3\zeta. \tag{28}$$

The integral in (28) is evaluated here for two simple hypothetical structures of turbulence.

3.1. *Eddy model of turbulence*

It is a variant of the eddy model extensively used by Ffowcs Williams [5] and Ribner [15] among others. In this model a simple Gaussian distribution is assumed for the correlation $Q_{i,j}$ as

$$Q_{i,j}(Y, \zeta) = \overline{u_0^2}(Y)\, \alpha_{ij} \exp\left\{-\frac{1}{2}\left(\frac{\zeta_1^2}{l_1^2} + \frac{\zeta_2^2}{l_2^2} + \frac{\zeta_3^2}{l_3^2}\right)\right\}, \tag{29}$$

where $\alpha_{ij} = 1$ for all values of i and j.
$(\overline{u_0^2})^{1/2}$ is some characteristic turbulence velocity. l_1, l_2 and l_3 are scales of turbulence in three principal directions.

Following the standard integration formulae and making use of (3) and (29) in (25), one obtains

$$I(X/Y) = 2^{1/2}\, \pi^{3/2}\, \alpha_{ij}\, \epsilon^2\, \rho_0\, a_0^5\, \overline{u_0^2}(Y)\, A_i\, A_j$$
$$l_1 \cdot l_2 \cdot l_3 \cdot \exp\left\{-\tfrac{1}{2}\left(\tilde{k}'^2_{c1}\, l_1^2 + \tilde{k}'^2_{c2}\, l_2^2 + \tilde{k}^2_{c3}\, l_3^2\right)\right\}. \tag{30}$$

3.2. *Isotropic model of turbulence*

Another widely used turbulence structure is the isotropic model. This model is assumed to be locally valid for small volume of jet. The unknown statistical average values $(\overline{u_0^2})^{1/2}$ and l have been measured by Davies *et al* [4] for free jets. The analytical description of isotropic model is usually stated as follows (Goldstein [7])

$$Q_{i,j}(Y, \zeta) = \frac{\overline{u_0^2}}{2l^2}\left[\exp\left\{-\frac{1}{2l^2}(\zeta_1^2 + \zeta_2^2 + \zeta_3^2)\right\}\right.$$
$$\left.\left(\{2l^2 - (\zeta_1^2 + \zeta_2^2 + \zeta_3^2)\}\,\delta_{ij} - \zeta_i\,\zeta_j\right)\right], \tag{31}$$

where l is the longitudinal macroscale of turbulence.

Substitution of (3) and (31) in (28) and then integration, yields

$$I(X/Y) = 2^{-1/2}\, \pi^{3/2}\, \epsilon^2\, \rho_0\, a_0^5\, \overline{u_0^2}(Y)\, A_i\, A_j\, l^5$$
$$\left\{\left(\tilde{k}'^2_{c1} + \tilde{k}'^2_{c2} + \tilde{k}'^2_{c3}\right)\delta_{ij} - \tilde{k}'_{ci}\,\tilde{k}'_{cj}\right\}$$
$$\exp\left\{-\frac{l^2}{2}\left(\tilde{k}'^2_{c1} + \tilde{k}'^2_{c2} + \tilde{k}'^2_{c3}\right)\right\}. \tag{32}$$

3.3. *Comparison with existing theories*

A short comparison with the existing theories may be in order here. Kraichnan [10] derived an expression in implicit form for the power spectrum $\langle p^+(n, \omega_s)\rangle$ of the scattered energy flux for isotropic turbulence structure as

$$\langle p^+(n, \omega_s)\rangle = \frac{\pi\,\omega_s^4}{8\,a_0^8}\left(\frac{\sin^2\theta}{4\sin^2\frac{\theta}{2} + \delta^2}\right) \cdot F. \tag{33}$$

The implicitness appears through unknown function F, the four-dimensional spectrum density of turbulence. The factor δ is defined as

$$\delta = (\,|\,\omega_s\,| - \omega_0)/\omega_0,$$

ω_s and ω_0 are respectively the frequency of scattered noise and the sound wave component. Vector n_i is defined as

$$n_i = x_i \,/\, |\, X\,|\,.$$

Lighthill [12] derived an expression for $I(X/Y)$ as

$$I(X,\ Y) \sim \rho_0\, a_0^{-1}\, \epsilon^2\, \omega^2 \cos^2\theta \cot^2\left(\frac{\theta}{2}\right) |\, X\,|^{-2} \cdot E, \qquad (34)$$

where function E, the unknown turbulence energy spectrum, is unknown.
When one compares results (30) and (32) of the present text with (33) and (34), one finds that the present results are in explicit form enabling one to easily draw qualitative and quantitative information relevant to particular wave-scattering problem.

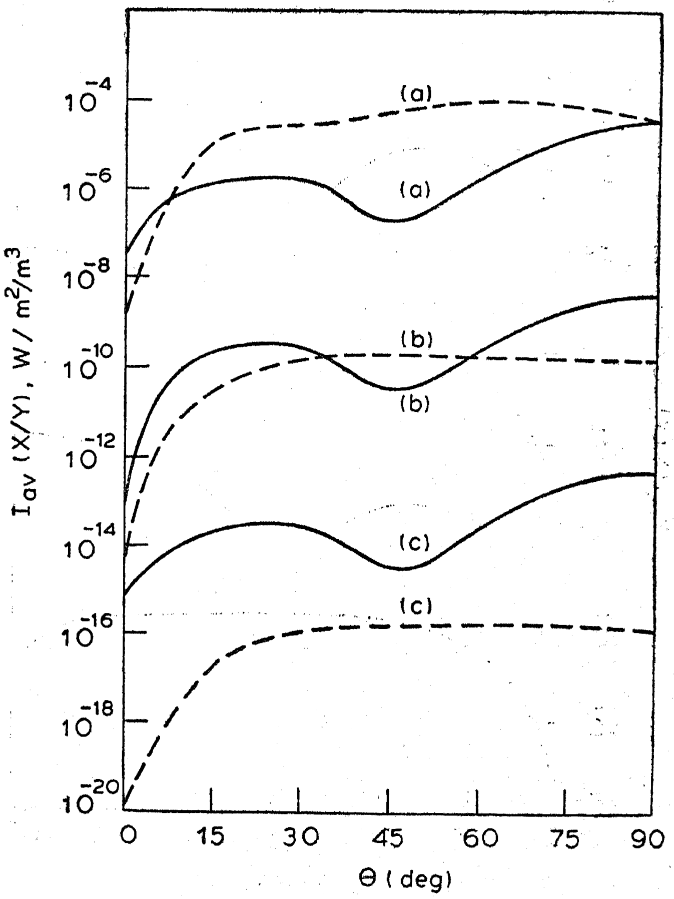

Figure 1. Directivity of noise scattered. $M_e = 0{\cdot}5$ and $\epsilon = 2{\cdot}0 \times 10^{-5}$. ——, eddy model; - - - -, isotropic model. (a) $=10^4$ Hz, (b) $f{=}10^3$ Hz, (c) $f{=}10^2$ Hz.

4. A representative evaluation for a round jet

In this section an evaluation is carried out for a unit volume of turbulence located at the mid-point of the mixing region of a turbulent round jet. The central region of annular mixing part of the jet is the most intense noise producing region. Evaluation is carried out for the following set of conditions:

M_e : 0·3, 0·4, 0·5, 0·6, 0·7, 0·8, 0·9

ϵ : 2×10^{-6}, 2×10^{-5}, 2×10^{-4}

f : 10^2, 10^3, 10^4 Hz (circular frequency)

θ : 0, 15, 30, 45, 60, 75, 90 degrees

where M_e is Mach number at jet exit based on ambient sound speed and $f=\omega/2\pi$. θ is emission angle measured from the jet axis. The results for $I(X/Y)$ are expressed in watts/m²/m³. The input data needed to evaluate $I(X/Y)$ are taken from various sources (Goldstein [7], Davies *et al* [4], Ribner [15])

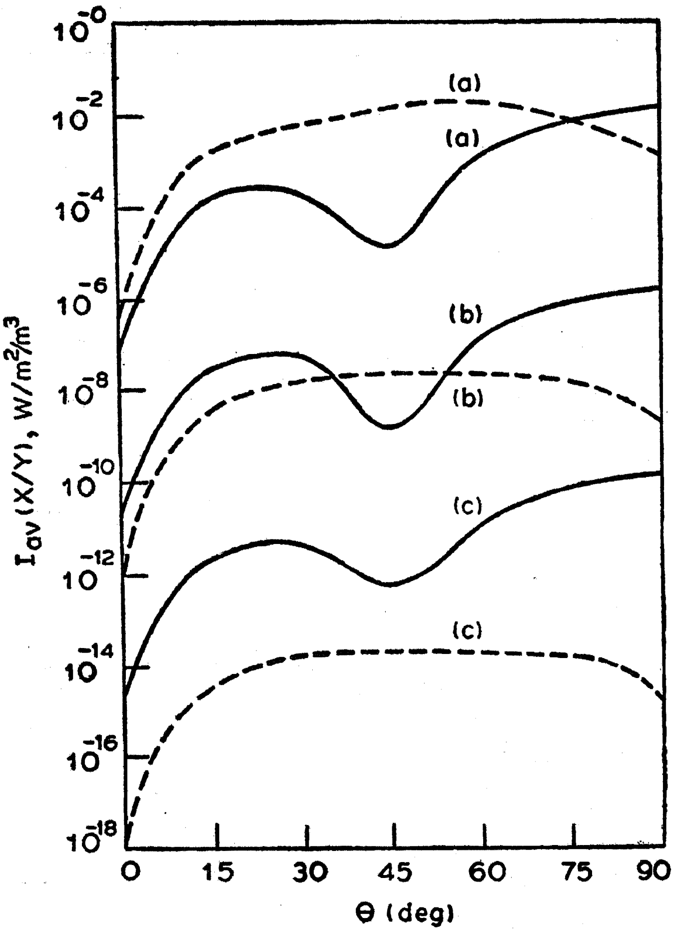

Figure 2. Directivity of noise scattered. M_e=0·7 and $\epsilon = 2{\cdot}0\times10^{-5}$, ——, eddy model; - - - -, isotropic model. (a) f=10⁴ Hz, (b) f=10³ Hz, (c) f=10² Hz.

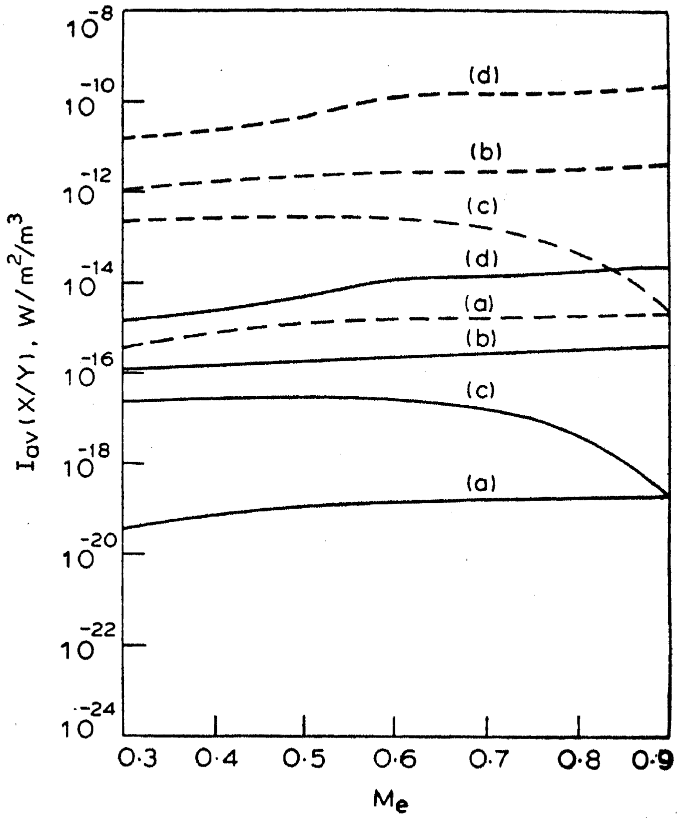

Figure 3. Mach number dependence of noise scattered. Eddy model, $\epsilon=2{\cdot}0\times10^{-6}$. ———, $f=10^2$ Hz; - - - - $f=10^3$ Hz. (a) $\theta=0°$, (b) $\theta=15°$, (c) $\theta=45°$, (d) $\theta=90°$.

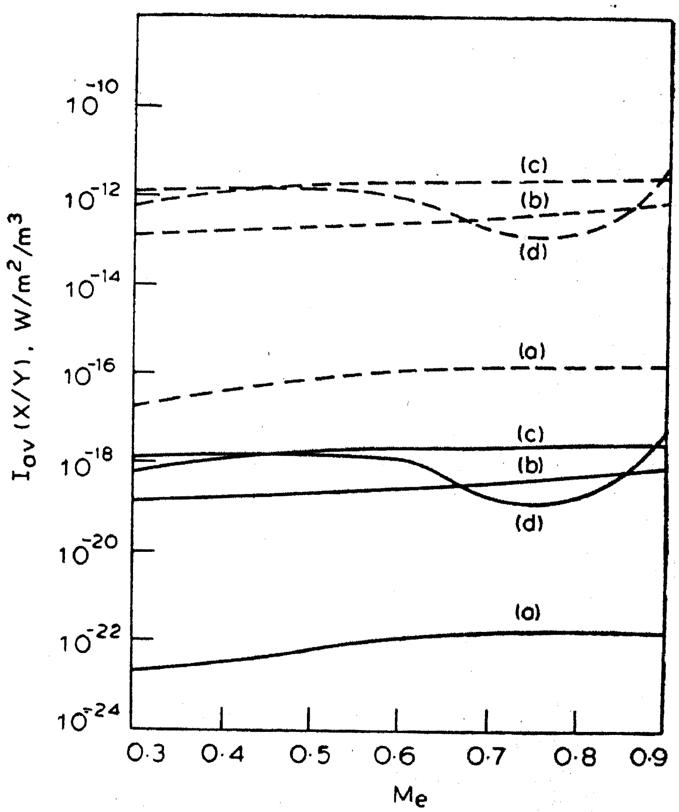

Figure 4. Mach number dependence of noice scattered. Isotropic mc $2{\cdot}0\times10^{-6}$, ———, $f=10^2$ Hz; - - - - $f=10^3$ Hz. (a) $\theta=0°$, (b) $\theta=15°$, (c (d) $\theta=90°$.

as
$$\begin{aligned} y_1^* &= 2D,\ (y_2^{*2}+y_3^{*2})^{1/2}=0{\cdot}5\ D,\\ D &= 25\ \text{mm},\ R=3\ \text{m}\\ \rho_0 &= 1{\cdot}177\ \text{kg/m}^3,\ a_0=347{\cdot}6\ \text{m/s}\\ (\overline{u_0^2})^{1/2} &= 0{\cdot}14\ M_e a_0,\ l_1=0{\cdot}65\times10^{-2}\ \text{m},\\ l_2 &= l_3=0{\cdot}2\times10^{-2}\text{m},\ l=0{\cdot}709\times10^{-2}\ \text{m},\\ d^3Y &= (85{\cdot}12\times10^{-6}\,y_1^3)\ \text{m}^3. \end{aligned}$$

Computation is carried out for $\phi=45°$ where ϕ is circumferential or azimuthal angle made by projection of the observer vector X in x_2—x_3 plane with the x_2-axis. It is assumed that the characteristic location of source, observer and origin, all lie in a single straight line.

Figures 1, 2, 3 and 4 show the pattern of results for the above sample data.

5. Results and conclusions

The following points emerge from the preceding sections:

(a) The present analysis is an improvement over the earlier work in that it yields explicit expressions for scattered energy.

(b) It also provides more qualitative and quantitative information on wave-scattering problem by taking into account the effect of convection by mean motion of energy-bearing eddies upon the sound incident upon the flow as well as on the sound scattered.

(c) A representative evaluation shows:

(d) (i) For isotropic model of turbulence, the energy scattered is minimum at low angles of emission ($\theta<20°$), but remains, more or less, constant at other angles. The directivity pattern for eddy model of turbulence has a secondary minimum at or around $\theta=45°$.

(ii) Scattered energy remains more or less constant with the change in M_e for various emission angles and sound frequencies.

Another point to be noted here concerns with flow-acoustic interaction effects in free jets. It is commonly believed that 'refraction dimple' i.e. a minimum sound level near the jet axis ($\theta<15°$) arises due to flow-acoustic interaction effects (Goldstein [7]). As θ increases 'dimple' almost flattens out. This particular feature is observable in figures 1 and 2 where sound radiation appears to be minimum near $\theta=0°$.

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