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Test of CP Violating Neutral Gauge Boson Vertices in $e^+e^- \rightarrow \gamma Z$

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Abstract

Forward-backward asymmetry in the process $e^+e^- \rightarrow \gamma Z$ is proposed as a test for CP -violating anomalous $\gamma\gamma Z$ and γZZ couplings. Longitudinally polarized electron beams may be used to disentangle the effects of these couplings. We estimate possible limits that can be obtained from e^+e^- colliders at centre-of-mass energies of 200 GeV and 500 GeV.

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The standard model (SM) has been found to be in extremely good agreement with all experimental data to date. However, a direct precise measurement of the form and magnitude of the gauge-boson self-couplings has yet to be done, and has to wait for e^+e^- experiments above the W^+W^- threshold proposed at LEP200 in the near future. While there has been considerable literature on measurement of possible anomalous γW^+W^- and ZW^+W^- couplings at both hadronic [1] and leptonic colliders [2], not much attention has been paid to possible trilinear couplings among the neutral gauge bosons γ and Z , which are absent in SM to the lowest order in the gauge coupling constants. These would be equally interesting to look for at e^+e^- colliders, possibly even at the existing ones. Examples of processes where anomalous trilinear couplings of these neutral gauge bosons would contribute are $e^+e^- \rightarrow \gamma\gamma, \gamma Z, ZZ$. These processes also have SM contributions to them through t - and u -channel electron exchange diagrams. Experiments could therefore compare the SM predictions with data and put bounds on the anomalous couplings.

Of the processes mentioned above the process $e^+e^- \rightarrow \gamma Z$ is all the more interesting for an additional reason. If CP is conserved, the photon observed in the reaction should be produced symmetrically in the forward and backward directions [3]. Thus the observation of a simple forward-backward asymmetry of the photon emission direction would be a signal of a CP -violating coupling. Such an asymmetry is far simpler to observe accurately than more complicated energy and azimuthal asymmetries of decay products needed to observe CP violation in the case of charged gauge bosons in the final state [4].

The above considerations have motivated us to look at tests of CP -odd anomalous $\gamma\gamma Z$ and γZZ couplings in the reaction

$$e^-(p_-) + e^+(p_+) \rightarrow \gamma(k_1) + Z(k_2). \quad (1)$$

We calculate the forward-backward asymmetry of the photon (or the Z) in the centre-of-mass (c.m.) frame in the presence of CP -violating couplings, as well as the contribution of these couplings to the total cross section. We then estimate possible limits on the couplings which can come from future experiments at LEP200 and also at a future Next Linear Collider

(NLC) operating at a c.m. energy of 500 GeV¹.

A general effective Lagrangian CP -violating for $\gamma\gamma Z$ and γZZ interactions, restricted to operators of dimension 6, can be written as

$$\begin{aligned} \mathcal{L} = & e \frac{\lambda_1}{2m_Z^2} F_{\mu\nu} \left(\partial^\mu Z^\lambda \partial_\lambda Z^\nu - \partial^\nu Z^\lambda \partial_\lambda Z^\mu \right) \\ & + \frac{e}{16c_W s_W} \frac{\lambda_2}{m_Z^2} F_{\mu\nu} F^{\nu\lambda} \left(\partial^\mu Z_\lambda + \partial_\lambda Z^\mu \right), \end{aligned} \quad (2)$$

where $c_W = \cos \theta_W$ and $s_W = \sin \theta_W$, θ_W being the weak mixing angle.

Equation (2) represents the most general CP -violating Lagrangian invariant under electromagnetic gauge transformations. Terms involving divergences of the vector fields have been dropped from the Lagrangian as they would not contribute when the corresponding particle is on the mass shell, or is virtual, but coupled to a fermionic current which is conserved. Since we will neglect the electron mass, the corresponding current can be assumed to conserved.

The Lagrangian in eq.(2) must be understood in the sense of representing effective interactions arising from some underlying fundamental theory, and therefore, the couplings $\lambda_{1,2}$, or rather their momentum space counterparts, would represent form factors, and are not really constants. Furthermore, $\lambda_{1,2}$ could, in general, be complex quantities. In SM, of course, these CP -violating effective interactions do not arise even at the one-loop level, and are therefore expected to be extremely small.

The CP -violating couplings of the Lagrangian in eq.(2) can induce both electric and weak dipole moments of fermions at the one-loop level. A look at the chirality structure shows that, to the leading order, only λ_1 contributes to the the weak dipole moment. On the other hand, the electric dipole moment receives contributions from both λ_2 and λ_1 , but is much

¹Forward-backward asymmetry in this process has been considered in ref.[5]. However, the interactions they consider do not obey electromagnetic gauge invariance and Bose symmetry. Moreover, they were mainly concerned with results at LEP1 at c.m. energy of 100 GeV. While they did point out the need for beam polarization in discriminating between various contributions to the asymmetry, they did not discuss the results of polarization quantitatively, as we have done here.

more sensitive to the former. With a suitable procedure [6] for regulating the divergences accompanying such a calculation with non-renormalizable interactions, the contributions to the dipole moment are expected to be

$$d_f \sim \frac{\alpha}{16\pi s_W^2 c_W^2} \frac{m_f}{m_Z^2} e \lambda \ln \frac{\Lambda^2}{m_Z^2} \quad (3)$$

where λ is either λ_1 or λ_2 and Λ is the cutoff denoting the onset scale for new physics. Assuming $\Lambda \sim 1$ TeV, eqn.(3) gives for the electron dipole moment a contribution of about $\lambda \times 10^{-24} e$ cm. This then translates to a limit of about $|\lambda_2| \lesssim 10^{-3}$ from the bound on the electric dipole moment of the electron[7], known to be less than about $10^{-27} e$ cm. Similarly $|\lambda_1|$ can be constrained from the weak dipole moment ². However, these are only indirect limits, and a correct estimate crucially depends on assumptions about the structure of the theory. Thus a direct measurement of the couplings is more desirable.

The Standard Model diagrams contributing to the process (1) are of course those with a t - and a u -channel electron exchange, while the extra piece in the Lagrangian (2) introduces two s -channel diagrams with γ - and Z -exchange respectively. The corresponding matrix element is then given by

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4, \quad (4)$$

where

$$\begin{aligned} \mathcal{M}_1 &= \frac{e^2}{4c_W s_W} \bar{v}(p_+) \not{\epsilon}(k_2) (g_V - g_A \gamma_5) \frac{1}{\not{p}_- - \not{k}_1} \not{\epsilon}(k_1) u(p_-), \\ \mathcal{M}_2 &= \frac{e^2}{4c_W s_W} \bar{v}(p_+) \not{\epsilon}(k_1) \frac{1}{\not{p}_- - \not{k}_2} \not{\epsilon}(k_2) (g_V - g_A \gamma_5) u(p_-), \\ \mathcal{M}_3 &= \frac{ie^2 \lambda_1}{4c_W s_W m_Z^2} \bar{v}(p_+) \gamma_\mu (g_V - g_A \gamma_5) u(p_-) \frac{(-g^{\mu\nu} + q^\mu q^\nu / m_Z^2)}{q^2 - m_Z^2} V_{\alpha\nu\beta}^{(1)}(k_1, q, k_2) \epsilon^\alpha(k_1) \epsilon^\beta(k_2), \\ \mathcal{M}_4 &= \frac{ie^2 \lambda_2}{4c_W s_W m_Z^2} \bar{v}(p_+) \gamma_\mu u(p_-) \frac{(-g^{\mu\nu})}{q^2} V_{\alpha\nu\beta}^{(2)}(k_1, q, k_2) \epsilon^\alpha(k_1) \epsilon^\beta(k_2). \end{aligned} \quad (5)$$

²The authors of ref.[8] have also considered the contribution of one of the form factors to the electric dipole moment of fermions. Our limits though are much stronger.

We have used $q = k_1 + k_2$, and the tensors $V^{(1)}$ and $V^{(2)}$ corresponding to the three-vector vertices are given by

$$\begin{aligned}
V_{\alpha\nu\beta}^{(1)}(k_1, q, k_2) &= k_1 \cdot q g_{\alpha\beta} k_{2\nu} + k_1 \cdot k_2 g_{\alpha\nu} q_\beta - k_{1\beta} q_\alpha k_{2\nu} - k_{1\nu} q_\beta k_{2\alpha} \\
V_{\alpha\nu\beta}^{(2)}(k_1, q, k_2) &= \frac{1}{2} [g_{\alpha\beta} (k_2 \cdot q k_{1\nu} - k_1 \cdot q k_{2\nu}) - g_{\nu\alpha} (k_2 \cdot q k_{1\beta} + k_1 \cdot k_2 q_\beta) \\
&\quad + g_{\nu\beta} (k_1 \cdot k_2 q_\alpha - k_1 \cdot q k_{2\alpha}) + q_\alpha k_{2\nu} k_{1\beta} + q_\beta k_{1\nu} k_{2\alpha}].
\end{aligned} \tag{6}$$

In the above equations, the vector and axial vector Z couplings of the electron are given by

$$g_V = -1 + 4 \sin^2 \theta_W; \quad g_A = -1. \tag{7}$$

For compactness, we introduce the notation :

$$\begin{aligned}
\bar{s} &\equiv \frac{s}{m_Z^2}, \\
\mathcal{B} &= \frac{\pi\alpha^2}{8s_W^2 m_W^2 \bar{s}} \left(1 - \frac{1}{\bar{s}}\right) (g_V^2 + g_A^2), \\
C_A &= \frac{\bar{s} - 1}{4} \text{Im} \left(\lambda_1 - \lambda_2 \frac{g_V}{g_V^2 + g_A^2} \right), \\
C_{2L} &= \frac{(\bar{s} - 1)^2}{128} \left\{ |\lambda_1|^2 - 2\text{Re}(\lambda_1 \lambda_2^*) \frac{g_V}{g_V^2 + g_A^2} + \frac{|\lambda_2|^2}{g_V^2 + g_A^2} \right\}.
\end{aligned} \tag{8}$$

Using eqns. (4 - 8), we obtain the differential cross section for the process (1) to be

$$\frac{d\sigma}{d\cos\theta} = \mathcal{B} \left[\frac{1}{\sin^2\theta} \left(1 + \cos^2\theta + \frac{4\bar{s}}{(\bar{s} - 1)^2} \right) + C_A \cos\theta + C_{2L} (\bar{s} + 2 + (\bar{s} - 2) \cos^2\theta) \right], \tag{9}$$

where θ is the angle between photon and the e^- directions. The total cross section corresponding to the cut $\theta_{\min} < \theta < \pi - \theta_{\min}$ can then be easily obtained by integrating the differential cross section above and has been listed in Table 1 for two different values of c.m. energy and beam polarization (see later).

The presence of a term proportional to $\cos\theta$ in eqn. (9) reflects the CP violation in the

interaction. It leads to a forward-backward asymmetry given by the expression

$$\begin{aligned}
A_{FB} &\equiv \frac{F - B}{F + B} \\
&= \frac{C_A}{2} \cos^2 \theta_{\min} \left[\left\{ \frac{\bar{s}^2 + 1}{(\bar{s} - 1)^2} \ln \left(\frac{1 + \cos \theta_{\min}}{1 - \cos \theta_{\min}} \right) - \cos \theta_{\min} \right\} \right. \\
&\quad \left. + C_{2L} \left\{ (\bar{s} + 2) \cos \theta_{\min} + \frac{\bar{s} - 2}{3} \cos^3 \theta_{\min} \right\} \right]^{-1}, \tag{10}
\end{aligned}$$

where F (B) represents the number of events with the photon in a forward (backward) direction, with photon events lying within an angle of θ_{\min} from the beam axis excluded.

We have used A_{FB} obtained above to estimate what limits can be put on the imaginary parts of the anomalous couplings λ_1 and λ_2 from the future experiments at LEP200 with $\sqrt{s} = 200$ GeV and at a possible future linear collider with $\sqrt{s} = 500$ GeV. For simplicity, we neglect the real parts of $\lambda_{1,2}$, which appear in the total cross sections, and henceforth $\lambda_{1,2}$ will refer to the imaginary parts of the respective quantities. To get a 90% confidence limit on the anomalous couplings, we require that $F - B$ must exceed $2.15\sqrt{N}$, where $N = F + B$ is the total number of γZ events that can be observed with a certain integrated luminosity. We assume an integrated luminosity of 500 pb^{-1} for LEP200 and 10 fb^{-1} for the NLC. Our results are shown in Figs. 1 and 2, respectively for LEP200 and NLC. We have taken θ_{\min} to be 25° , as that gives the largest sensitivity.

It is obvious from an examination of the expression for the forward-backward asymmetry A_{FB} that it is very insensitive to the value of λ_2 , since the latter comes multiplied by the vector coupling g_V of the electron, which is very small (≈ -0.08). This in turn is a reflection of the fact that the CP violating anomalous couplings do not violate parity and therefore the interference of the photon exchange term with the SM contributions can only contain g_V . This insensitivity is clearly seen in the figures.

It is possible to enhance the relative contribution of the λ_2 term if the electron (and/or positron) beams can be longitudinally polarized³.

³It should be remembered, though, that polarization of the electron beam alone results in the initial state not being an eigenstate of CP . However, since only opposite e^+ and e^- helicities contribute in the limit of

This would give contributions which are proportional to g_A , which is not small. It can be easily checked that the result of including longitudinal polarizations P_e and $P_{\bar{e}}$ of the electron and positrons beams, respectively, is obtained from the differential cross section of eq.(9) by the following replacements:

$$\begin{aligned} g_V^2 + g_A^2 &\rightarrow g_V^2 + g_A^2 - 2Pg_Vg_A, \\ g_V &\rightarrow g_V - Pg_A, \end{aligned} \tag{11}$$

where $P = (P_e - P_{\bar{e}})/(1 - P_eP_{\bar{e}})$, and multiplying the differential cross section by an overall factor of $1 - P_eP_{\bar{e}}$.

We have also shown in Figs. 1 and 2 the effect of electron polarization $P_e = \pm 0.5$ assuming the positron to be unpolarized ⁴. As expected, the slope of the bands change with P_e . Experiments with polarized beams thus afford us complementary studies and hence help unravel the effects due to the two couplings in eqn. (2). In case of a higher polarization, the sensitivity of the experiments to λ_2 would be much better.

It is seen that in general, limits of order 0.4–0.6 can be put on individual couplings at LEP200, and of order 0.02–0.04 at NLC. That the sensitivity increases with energy is easily understood from the fact that the standard model contributions decrease with energy, whereas the contribution of the anomalous couplings are constant with energy for small values of the coupling, and eventually increase with energy.

Though our main aim was to study the CP -violating forward-backward asymmetry, we have also looked at the limits obtained on the anomalous couplings from measuring the total number of events with a cut of 25° for the angle between the photon and beam directions. The 90% C.L. limits on the couplings are shown in Figs. 3 and 4, allowing for a systematic vanishing electron mass, the initial state is effectively CP even. It can easily be seen that $O(\alpha)$ corrections with helicity flip collinear photon emission, which survive even for vanishing electron mass, do not contribute to A_{FB} .

⁴ This seems to us to be a conservative assumption as the Stanford Linear Collider (SLC) has already obtained a polarization of 62%. At least at linear colliders, it should certainly be possible to attain a higher polarization.

uncertainty of 2% in cross section determination. The limits obtained are similar to those from the measurement of forward-backward asymmetry⁵. It should however be borne in mind that the cross section could in principle receive contributions from several new interactions, as for example, several CP conserving γZZ and $\gamma\gamma Z$ couplings not included in our Lagrangian. Thus the limits obtained are with the assumption that these couplings are absent. The forward-backward asymmetry, on the other hand can receive contributions only from the CP -violating interactions. Moreover, the cross sections would also receive contributions from the real parts of $\lambda_{1,2}$, which we have taken to be absent. Finally, systematic uncertainties in normalization get cancelled out when one considers a forward-backward asymmetry. Thus limits from the forward backward asymmetry are more significant.

We should, of course, compare these limits from the bounds that can already be inferred from limits on the radiative decays of the Z . The simplest (naive) possibility namely $Z \rightarrow \gamma\gamma$ of course does not exist on account of Yang's theorem. Upper bounds on $Br(Z \rightarrow f\bar{f}\gamma)$ however do exist [7] and are of the order of 5×10^{-4} . The relevant matrix element is obtained from eqn(4) on reversing the sign of the photon momentum, and the extra contribution to the decay width given by

$$\begin{aligned} \delta\Gamma(Z \rightarrow f\bar{f}\gamma) &= \frac{\alpha^2 m_Z}{15360\pi c_W^2 s_W^2} \left\{ (g_V^2 + g_A^2)|\lambda_1|^2 - 2Qg_V \text{Re}(\lambda_1\lambda_2^*) + Q^2|\lambda_2|^2 \right\} \\ &\approx 6.5 \times 10^{-7} \text{ GeV} \left\{ (g_V^2 + g_A^2)|\lambda_1|^2 - 2Qg_V \text{Re}(\lambda_1\lambda_2^*) + Q^2|\lambda_2|^2 \right\} \end{aligned} \quad (12)$$

where $g_{V,A}$ are the corresponding couplings of f to Z as in eqn(7) and Q is its charge. (For hadronic final states, the right hand side in the expression above has to be multiplied by 3.)⁶.

As is easily seen, the limits imposed by such decays are at best one (two) orders of magnitude weaker than the bounds obtainable at LEP200 (NLC). In fact, the cleanest existing bound (on λ_1) would come from the cross-section measurement for the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ at

⁵In fact, the limits derived from total cross section are somewhat better for λ_2 , but this feature would disappear for higher beam polarization.

⁶Note that $\delta\Gamma$ includes the contribution from the full available phase space unlike the experimental limits which have kinematical cuts associated with them.

LEP1 [9]. Translating the error bar in neutrino counting to a limit on such couplings, we have $\lambda_1 \lesssim 4$.

Unitarity would constrain the high energy behaviour of the amplitude for the process considered here. We find that the constraint on the anomalous couplings obtained by demanding that unitarity not be violated until a scale Λ is weaker than that obtainable even from LEP200 so long as Λ is not smaller than a few TeV.

It should be noted that in principle a CP -violating dipole type of coupling of the electron to the photon or Z could give rise to the asymmetries we discuss. However, the corresponding contribution would be proportional to the electron mass, and would be negligible.

To summarize, we have pointed out an extremely simple CP -violating effect that can be studied quite easily at forthcoming accelerators. We have shown how anomalous trilinear neutral gauge boson couplings can give rise to this effect. We have shown that limits of the order of 0.2–0.3 can be put on the imaginary parts of the corresponding form factors at LEP200 with an integrated luminosity of 500 pb^{-1} and of the order of 0.02–0.03 at NLC with c.m. energy 500 GeV and integrated luminosity of 10 fb^{-1} . Longitudinal polarization of the electron beam can improve the sensitivity to the $\gamma\gamma Z$ coupling, which is otherwise small.

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Table Caption:

1. Standard Model $e^+e^- \rightarrow \gamma Z$ cross-sections for various electron polarization P_e (see eqn.11). The positron is unpolarized.

Figure Captions:

1. 90% C.L. bounds on the $\lambda_1 - \lambda_2$ plane ($\lambda_i \equiv \text{Im}\lambda_i$) obtainable from A_{FB} at LEP200 with integrated luminosity of 500 pb^{-1} . The solid, dashed and dot-dashed lines are for electron polarizations of 0, +0.5 and -0.5 respectively (positron is unpolarized).
2. As in Fig. 1 but for NLC ($\sqrt{s} = 500 \text{ GeV}$ and integrated luminosity of 10 fb^{-1}).
3. As in Fig 1, but for bounds from total cross-section alone.
4. As in Fig. 3 but for NLC.

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Energy	CrossSection(pb)		
	$P_e = -0.5$	$P_e = 0$	$P_e = 0.5$
$\sqrt{s} = 200$ GeV	8.49	7.93	7.38
$\sqrt{s} = 500$ GeV	0.938	0.876	0.815

Table 1:

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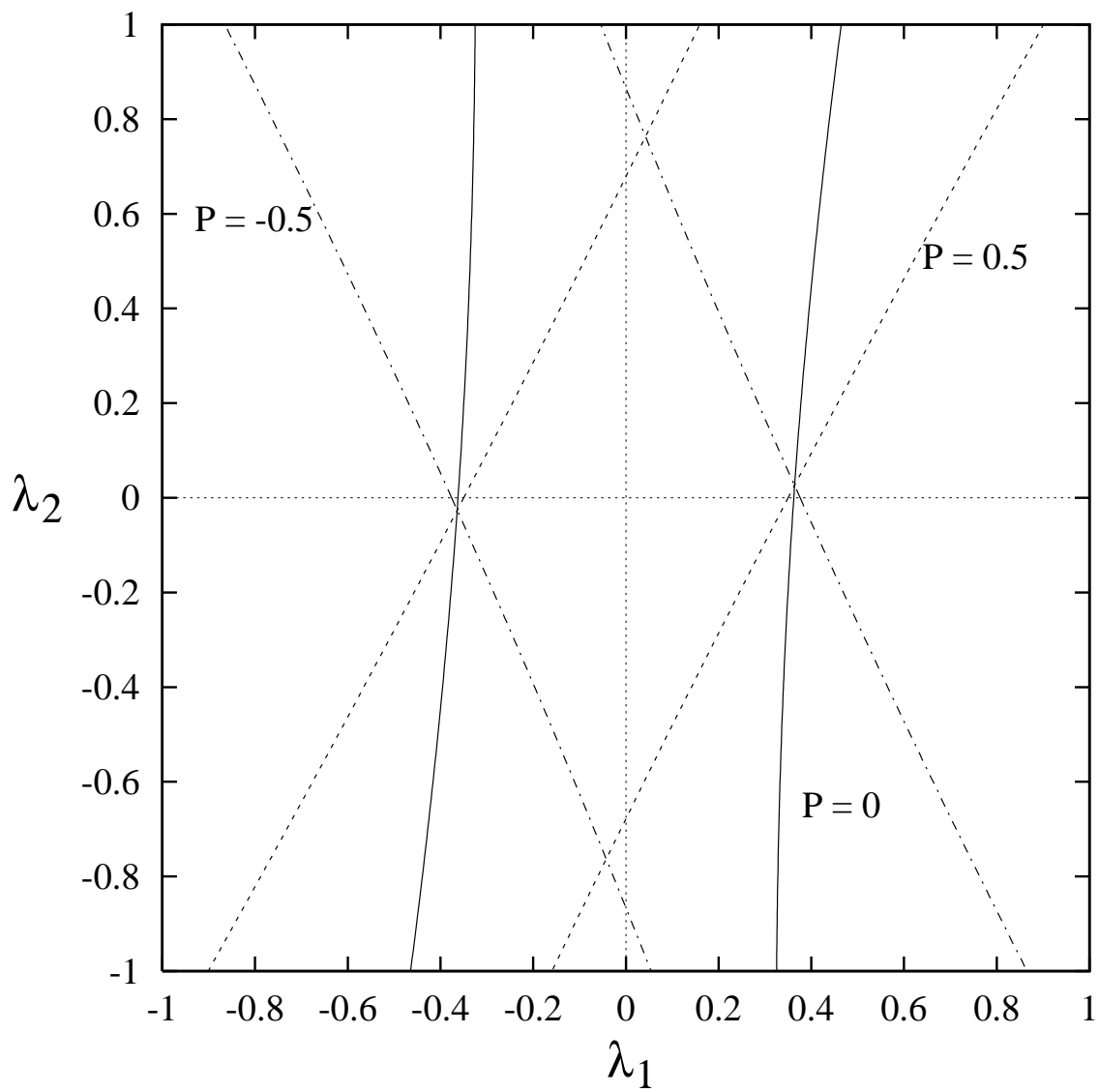


Fig. 1

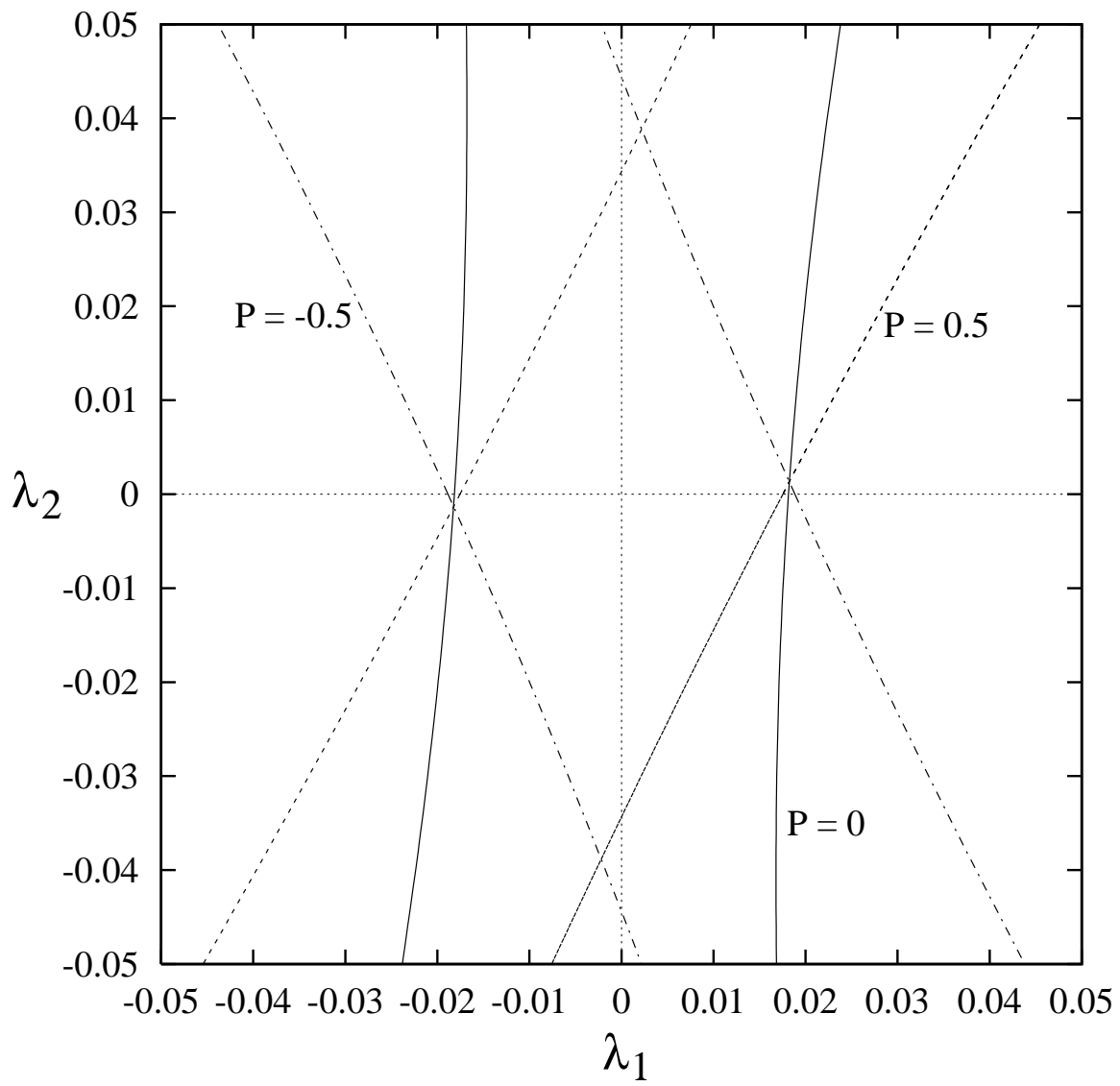


Fig. 2

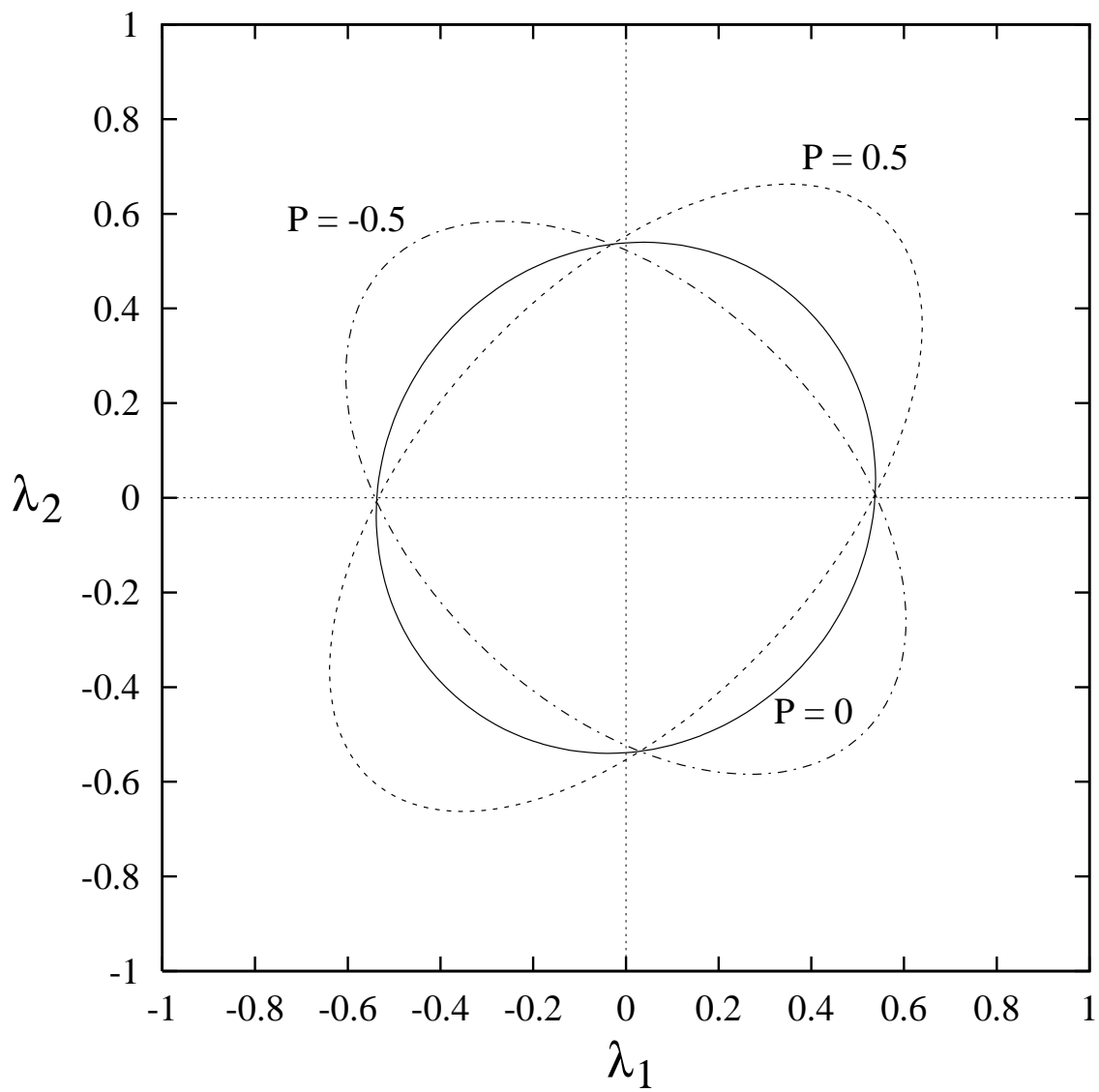


Fig. 3

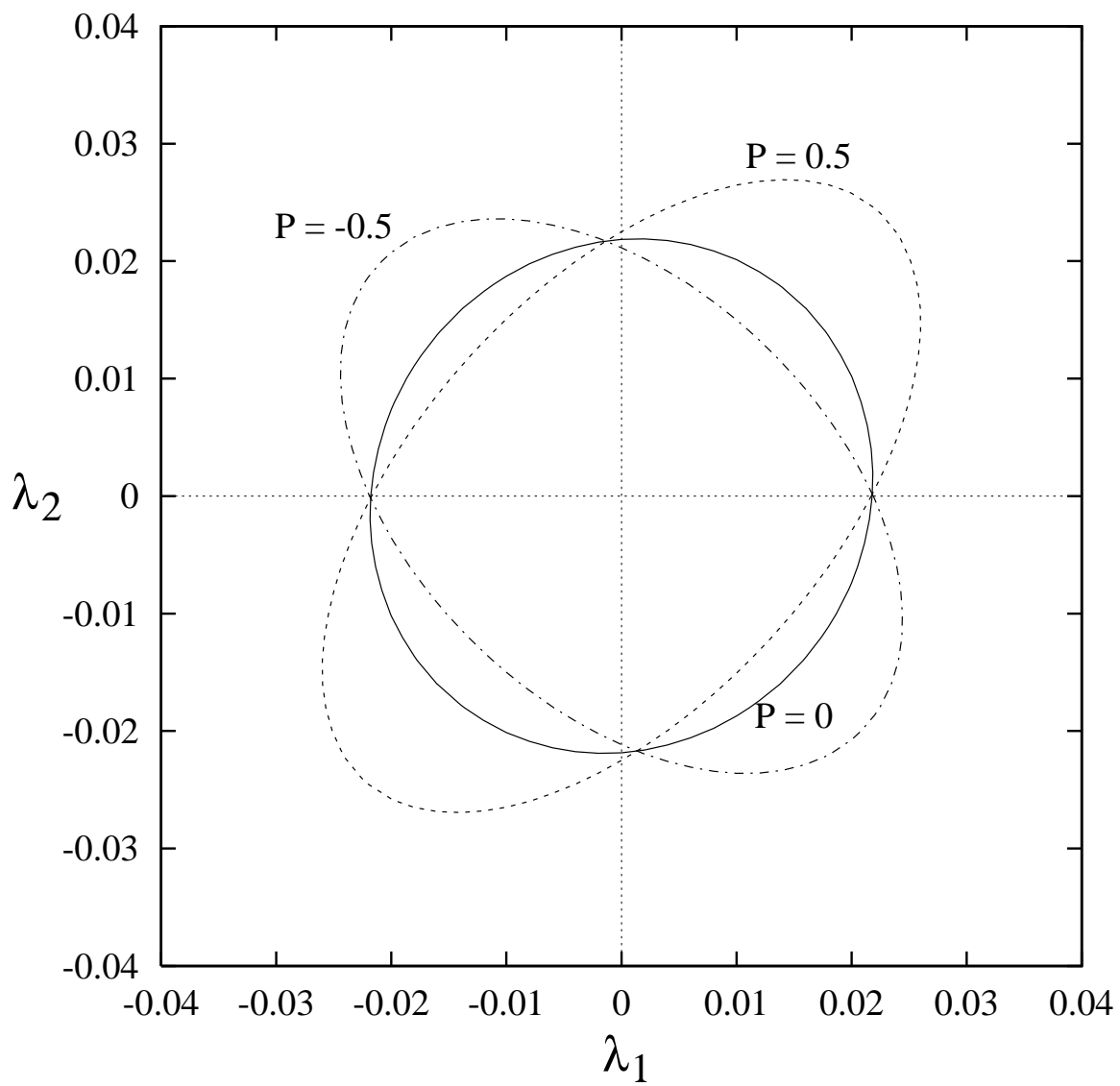


Fig. 4