A supersymmetric solution to the KARMEN time anomaly

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We interpret the KARMEN time anomaly as being due to the production of a (dominantly bino) neutralino with mass 33.9 MeV, which is the lightest supersymmetric particle but decays into 3 leptons through the violation of R-parity. For independent gaugino masses M_1 and M_2 we find regions in the (M_1, M_2, μ, $\tan\beta$) parameter space where such a light neutralino is consistent with all experiments. Future tests of this hypothesis are outlined.

I. INTRODUCTION

In 1995, the KARMEN experiment at the Rutherford Appleton Laboratory reported an anomaly in the time distribution of the charged and neutral current events induced by neutrinos from π^+ and μ^+ decays at rest [1]. This was ascribed to the production of a new particle, denoted x, in the anomalous pion decay

$$\pi^+ \to \mu^+ + x, \tag{1}$$

with a small branching ratio in the range $\sim 10^{-16} - 10^{-8}$ depending on the lifetime of x. The particle must be neutral since it passes through over 7 m of steel shielding. The time-of-flight to the detector is $3.6 \pm 0.25\,\mu$s, implying that the particle moves non-relativistically with velocity $v_x = 5.2^{+2.2}_{-1.4} \times 10^6$ m s^{-1}. This requires its mass to be 33.9 MeV and its kinetic energy to be $T_x \approx 5$ keV.[1] The observed energy in the detector is $\sim 11 - 35$ MeV, which must therefore come from the decay of x. Since 1995 the KARMEN experiment has been upgraded to significantly reduce the cosmic ray background. It has recently been reported that the time anomaly persists in the new data [2]. A time-of-flight likelihood analysis adopting the hypothesis that it is due to a decaying particle as described above has a negative natural log-likelihood ratio of 9, i.e. less than 1 in 10^4 chance of being a statistical fluctuation. The significance is thus sufficiently high that we are motivated to reexamine its physical origin.

There have already been several proposals to explain the KARMEN anomaly [3–5]. In Ref. [3], the authors considered in detail the possibility that x is a neutrino and concluded that a $SU(2)_{\rm L}$ doublet neutrino was excluded by existing data. This was further reinforced by the subsequent improvement [6] in the experimental upper limit on the branching ratio,

$$\mathrm{BR}(\pi^+ \to \mu^+ + x) < 1.2 \times 10^{-8} \quad (95\%\ \mathrm{C.L.}), \tag{2}$$

versus the minimum value of $\sim 2 \times 10^{-8}$ required in the doublet neutrino interpretation. However a sterile neutrino interpretation was found to be consistent, within strict limits on the mixing parameters (see also Ref. [7]), although this may still be in conflict with astrophysical and cosmological constraints [3].

In Ref. [5] a solution was proposed based on the anomalous muon decay $\mu^+ \to e^+ + x$, where x is taken to be a scalar boson of mass 103.9 MeV. However this implies too large a value for the energy released in the x decay and the required branching ratio is also constrained by the recent bound $\mathrm{BR}(\mu^+ \to e^+ + x) < 5.7 \times 10^{-4}$ (90% C.L.) [8]. Thus it is necessary to add to the model 2 other scalar bosons into which x can cascade decay in order to dilute the energy [5]. This model appears viable but is somewhat baroque.

In Ref. [4], a supersymmetric solution was considered. The x particle was interpreted as a photino (or zino) and the anomalous pion decay

$$\pi^+ \to \mu^+ + \tilde{\gamma} \tag{3}$$

[1] The required mass is within 0.02% of the pion–muon mass difference.

was assumed to proceed via the R-parity violating operator $L_2 Q_1 D_1^c$. [2] The same operator then enables the photino to decay radiatively as

$$\tilde{\gamma} \to \gamma + \nu_\mu \tag{4}$$

via a one-loop diagram with a d quark and $\tilde{d}$ squark in the loop. However the expected peak at 17 MeV has not been reported in the new data [2] on the energy spectrum of the anomalous events, so a 2-body decay for the x particle seems disfavoured. Therefore this model [4] may not be viable in its present form. We present below the necessary extension to produce a 3-body decay for such a light neutralino. [3]

II. THE MODEL

We consider the lightest neutralino in supersymmetry, $\tilde{\chi}_1^0$, to be the hypothetical x particle, with mass $m_{\tilde{\chi}_1^0} = 33.9$ MeV. This will also be the lightest supersymmetric particle (LSP) in our model. Since x is effectively stable on collider time-scales (this is quantified below) our model will experimentally look very similar to the MSSM. Now in a GUT-inspired MSSM, $M_1 = (5/3)\tan^2\theta_W M_2$, and assuming this relation requires $m_{\tilde{\chi}_1^0} > 32.3$ GeV from current LEP data [13]. Thus in order to obtain a very light neutralino we must consider M_1 and M_2 to be independent parameters. A small M_2 implies at least one light chargino which is excluded by experiment, while a small M_1 implies that the LSP will be dominantly bino. We will quantify this below and determine regions in the $(M_1, M_2, \mu, \tan\beta)$ parameter space consistent with all experimental limits. The solutions indeed turn out to be dominantly bino with a small higgsino contribution.

Furthermore we invoke 2 non-zero R-parity violating operators. The pion decay

$$\pi^+ \to \mu^+ \tilde{\chi}_1^0 \tag{5}$$

proceeds through the operator $\lambda'_{211} L_2 Q_1 D_1^c$ and the leading order Feynman diagrams are shown in Fig. 1. The neutralino is assumed to decay as

$$\tilde{\chi}_1^0 \to e^+ e^- \nu_{\mu,\tau} \tag{6}$$

through either $\lambda_{121} L_e L_\mu E_e^c$ or $\lambda_{131} L_e L_\tau E_e^c$. Note that this is the only kinematically accessible tree-level 3-body visible decay for such a light LSP.

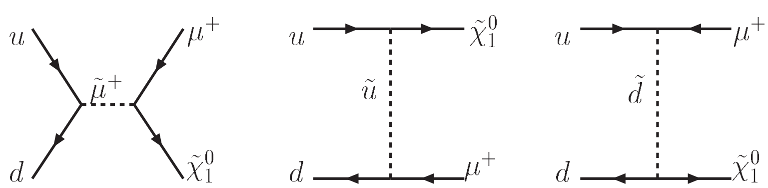

FIG. 1. Tree-level Feynman diagrams for pion decay via the operator $L_2 Q_1 D_1^c$.

In Fig. 2 we show the values of the branching ratio for $\pi^+ \to \mu^+ + \tilde{\chi}_1^0$ and lifetimes $\tau_{\tilde{\chi}_1^0}$ which are compatible with the KARMEN data [1,2]. In order to determine the required range of the couplings λ'_{211}, $\lambda_{1\{2,3\}1}$ in our model which are consistent with the solutions in Fig. 2, we must first determine the pion branching ratio in terms of the supersymmetric parameters. The partial width as computed from the diagrams in Fig. 1 is

$$\Gamma(\pi \to \mu \tilde{\chi}_1^0) = \frac{\lambda'^2_{211} f_\pi^2 m_\pi^2 p_{\rm cm}}{8\pi (m_d + m_u)^2} \left(\frac{A_e}{M_{\tilde{\mu}}^2} - \frac{A_u}{2M_{\tilde{u}}^2} - \frac{A_d}{2M_{\tilde{d}}^2} \right)^2 \left(m_\pi^2 - m_\mu^2 - m_{\tilde{\chi}_1^0}^2 \right), \tag{7}$$

[2] For reviews on R-parity violation see Ref. [9].

[3] Such a decay was also proposed in Ref. [10] which invoked possible mixing between neutrinos and gauginos/higgsinos as the reason for neutralino instability rather than R-parity violating vertices. However to explain the KARMEN anomaly then requires the Higgs mixing term $\mu H_1 H_2$ in the superpotential to be unnaturally small, $\mu \leq 30$ MeV. Moreover this scenario implies a MeV mass ν_τ which is definitively ruled out by cosmological and astrophysical arguments [11,12].

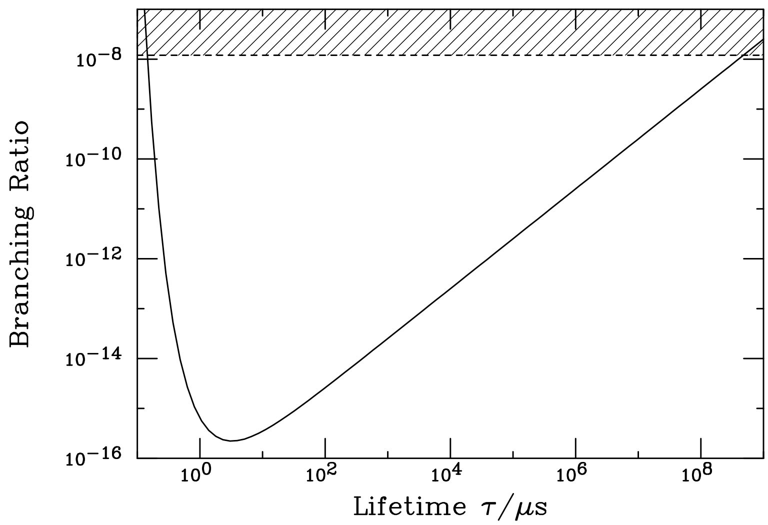

FIG. 2. The solutions to the KARMEN anomaly in terms of the anomalous pion branching ratio and the x-particle lifetime. The hashed area denotes the experimental upper bound (2).

where m_π and m_μ denote the pion and the muon masses, $M_{\tilde{\mu},\tilde{u},\tilde{d}}$, denote the corresponding scalar fermion masses, m_u, m_d are the first generation *current* quark masses, and f_π is the charged pion decay constant. The constants $A_{e,u,d}$ refer to the neutralino coupling and are given in Table I both for the general case and for the limiting cases of either a pure bino or photino neutralino. The phase space factor is given by $p_{\rm cm} = ([m_\pi^2 - (m_\mu + m_{\tilde{\chi}_1^0})^2][m_\pi^2 - (m_\mu - m_{\tilde{\chi}_1^0})^2])^{1/2}/(2m_\pi)$. In the Appendix we give some details of how this result is obtained.

The branching ratio for the anomalous pion decay, assuming the decay $\pi^+ \to \mu^+\nu_\mu$ to be dominant, is given by [4]

$$\mathrm{BR}(\pi \to \mu\tilde{\chi}_1^0) = \frac{\lambda'^2_{211} m_\pi^5 p_{\rm cm}}{2G_{\rm F}^2 m_\mu^2 (m_d + m_u)^2} \left(\frac{A_e}{M_{\tilde{\mu}}^2} - \frac{A_u}{2M_{\tilde{u}}^2} - \frac{A_d}{2M_{\tilde{d}}^2} \right)^2 \frac{\left(m_\pi^2 - m_\mu^2 - m_{\tilde{\chi}_1^0}^2\right)}{(m_\pi^2 - m_\mu^2)^2} \tag{8}$$

$$\approx 2.6 \times 10^{-8} \left(\frac{\lambda'_{211}}{10^{-4}}\right)^2 \left(\frac{150\ \mathrm{GeV}}{M_{\tilde{f}}}\right)^4$$

$$< 1.2 \times 10^{-8}. \tag{9}$$

To obtain a numerical estimate, we have assumed in Eq. (9) that the scalar fermions are mass degenerate, $M_{\tilde{\mu},\tilde{u},\tilde{d}} = M_{\tilde{f}}$, and that the neutralino is pure bino. In the last line we have quoted the experimental bound (2), shown as a hashed area in Fig. 2. This bound can be satisfied by a small coupling and/or a large sfermion mass. It can also be satisfied by a fine-tuned cancellation between different diagrams for distinct sfermion masses, but we disregard this possibility. The last inequality (9) can be translated into an upper bound on λ'_{211}:

[4] This corrects the result given earlier [4].

TABLE I. Neutralino coupling coefficients for the pion decay.

Coefficient	General formula	Pure photino	Pure bino
A_e	$eN'_{l1} + \frac{gN'_{l2}}{\cos\theta_{\rm W}}\left(\frac{1}{2} - \sin^2\theta_{\rm W}\right)$	e	$-g'Y_{e\rm L}$
A_u	$-ee_uN'_{l1} - \frac{gN'_{l2}}{\cos\theta_{\rm W}}\left(\frac{1}{2} - e_u\sin^2\theta_{\rm W}\right)$	$-ee_u$	$-g'Y_{u\rm L}$
A_d	$ee_dN'_{l1} - \frac{ge_d\sin^2\theta_{\rm W}N'_{l2}}{\cos\theta_{\rm W}}$	ee_d	$g'Y_{d_{\rm R}}$

TABLE II. Coefficients for the neutralino decay

Coefficient	General formula	Pure photino	Pure bino
B_1	$-\left(eN'_{l1} + \frac{gN'_{l2}}{\cos\theta_{\mathrm{W}}}\left[\frac{1}{2} - \sin^2\theta_{\mathrm{W}}\right]\right)$	$-e$	$g'Y_{e\mathrm{L}}$
B_2	$\frac{gN'_{l2}}{2\cos\theta_{\mathrm{W}}}$	0	$g'Y_{e\mathrm{L}}$
B_3	$\left(eN'_{l1} - \frac{gN'_{l2}\sin^2\theta_{\mathrm{W}}}{\cos\theta_{\mathrm{W}}}\right)$	e	$-g'Y_{e_{\mathrm{R}}}$

$$\lambda'_{211} < 6.8 \times 10^{-5}\left(\frac{M_{\tilde{f}}}{150\ \mathrm{GeV}}\right)^2 . \tag{10}$$

In the limit where $M_{\tilde{f}} \gg m_{\tilde{\chi}_1^0}$, the neutralino decay rate for the operator $\lambda_{1j1}L_1L_jE_1^c$, $j = 2, 3$, is given by [14]

$$\begin{aligned}\Gamma(\tilde{\chi}_1^0 \to e^+\bar{\nu}_j e^-) &= \frac{\lambda_{1j1}^2 m_{\tilde{\chi}_1^0}^5}{3072\pi^3}\left(\frac{B_1^2}{M_{\tilde{e}\mathrm{L}}^4} + \frac{B_2^2}{M_{\tilde{\nu}^j\mathrm{L}}^4} + \frac{B_3^2}{M_{\tilde{e}_{\mathrm{R}}}^4} - \frac{B_1B_2}{M_{\tilde{e}\mathrm{L}}^2 M_{\tilde{\nu}^j\mathrm{L}}^2} - \frac{B_1B_3}{M_{\tilde{e}\mathrm{L}}^2 M_{\tilde{e}_{\mathrm{R}}}^2} - \frac{B_2B_3}{M_{\tilde{\nu}^j\mathrm{L}}^2 M_{\tilde{e}_{\mathrm{R}}}^2}\right)\\ &= \frac{3\alpha\lambda_{1j1}^2 m_{\tilde{\chi}_1^0}^5}{1024\pi^2\cos^2\theta_{\mathrm{W}} M_{\tilde{f}}^4},\end{aligned} \tag{11}$$

where $M_{\tilde{e}\mathrm{L},\tilde{e}_{\mathrm{R}},\tilde{\nu}^j\mathrm{L}}$ denote the scalar lepton masses, and $B_{1,2,3}$ are the relevant $\tilde{\chi}_1^0 f\tilde{f}$ couplings given in Table II. In the second equation we have again assumed a pure bino LSP and degenerate scalar fermions. As a numerical estimate for the lifetime of the bino LSP with $m_{\tilde{\chi}_1^0} = 33.9$ MeV we obtain

$$\tau_{\mathrm{bino}} = 13.2\,\mathrm{s}\left(\frac{0.01}{\lambda_{1\{2,3\}1}}\right)^2\left(\frac{M_{\tilde{f}}}{150\ \mathrm{GeV}}\right)^4 \tag{12}$$

$$< 4.78 \times 10^2\ \mathrm{s}, \tag{13}$$

where the last inequality is obtained by using the bound (2) and the set of solutions shown in Fig. 2. The resulting bound on the coupling is

$$\lambda_{1\{2,3\}1} > 1.66 \times 10^{-3}\left(\frac{M_{\tilde{f}}}{150\ \mathrm{GeV}}\right)^2 . \tag{14}$$

Given the perturbative upper bound, $\lambda_{ijk} < \sqrt{4\pi}$, and the lower bound on the sfermion mass from LEP 2, $M_{\tilde{f}} > 100$ GeV, we also have a lower limit on the lifetime of $\tau_{\mathrm{bino}} > 2.6 \times 10^{-4}$ s. Thus there is a solution range of 6 orders of magnitude in lifetime or 3 orders of magnitude in coupling. For these lifetimes the LSP is stable on collider physics time scales.

We now have all the ingredients to fix the model parameters. In our model, each point along the curves in Fig. 2 corresponds to a specific anomalous pion branching ratio (9) and a specific neutralino lifetime (12). If we assume the scalar fermions are mass degenerate, we can translate this into specific values of λ'_{211} and $\lambda_{1\{2,3\}1}$ for a fixed sfermion mass. This set of solutions in the R-parity violating parameter space is shown in Fig. 3 for $M_{\tilde{f}} = 150$ GeV (solid line), $M_{\tilde{f}} = 300$ GeV (dashed line), and $M_{\tilde{f}} = 1000$ GeV (dot-dashed line). The hashed lines at $\lambda, \lambda' = \sqrt{4\pi}$ denote the perturbative limit. For large scalar fermion masses (> 1 TeV) we quickly run out of room for perturbative solutions. The solutions above and to the left of the stars are excluded by the inequalities (10, 14).

III. CONSTRAINTS ON THE R-PARITY VIOLATING COUPLINGS

The R-parity violating couplings we have introduced violate lepton number and are thus constrained by laboratory experiments. The best bounds at the 2σ level have been been summarized as [15]

$$\begin{aligned}&\lambda'_{211} < 0.059\left(\frac{M_{\tilde{d}_{\mathrm{R}}}}{100\ \mathrm{GeV}}\right),\\ &\lambda_{121} < 0.049\left(\frac{M_{\tilde{e}_{\mathrm{R}}}}{100\ \mathrm{GeV}}\right) \quad \Rightarrow \tau_{\mathrm{bino}} > 0.24\ \mathrm{s},\\ &\lambda_{131} < 0.062\left(\frac{M_{\tilde{e}_{\mathrm{R}}}}{100\ \mathrm{GeV}}\right) \quad \Rightarrow \tau_{\mathrm{bino}} > 0.15\ \mathrm{s}.\end{aligned} \tag{15}$$

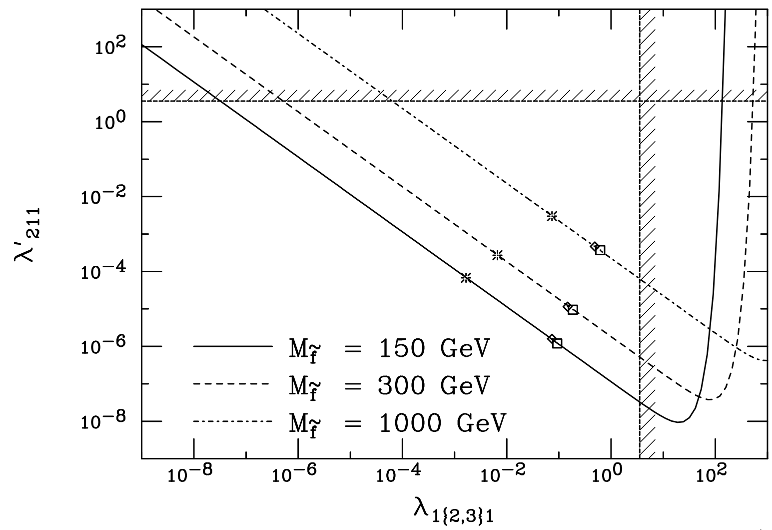

FIG. 3. Solutions to the KARMEN anomaly in terms of the R-parity violating couplings $\lambda'_{211}L_2Q_1D_1^c$ and $\lambda_{1\{2,3\}1}$, for different (assumed degenerate) sfermion masses. The hashed lines indicate upper limits on the couplings from perturbativity. The stars and diamonds (squares) give the upper limits on the couplings λ'_{211} and λ_{121} (λ_{131}), respectively. Solutions above and to the left of the stars are excluded, as are solutions below and to the right of the squares (diamonds).

The bound on λ'_{211} is from measurements of $R_\pi = \Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$ [16], the bound on λ_{121} is from charged-current universality [16], while the bound on λ_{131} is from a measurement of $R_\tau = \Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$ [16]. The above bound on λ'_{211} is weaker than the bound (10), and we do not consider it further. In Fig. 3 the above bounds on the coupling λ_{121} and λ_{131} forbid solutions to the right of the diamonds and squares, respectively. We are thus left with a range of solutions of about 2 orders of magnitude in λ'_{211} and $\lambda_{1\{2,3\}1}$. This corresponds to 4 orders of magnitude in the pion branching ratio and in the LSP lifetime, respectively. In the last 2 equations we have translated the upper bound on $\lambda_{1\{2,3\}1}$ into a lower bound on the lifetime using Eq. (12), to be compared with the upper bound (13). Note that these bounds are independent of the sfermion mass.

Besides bounds on individual couplings, we must also consider bounds on the product of the couplings $\lambda'_{211}\lambda_{121}$ or $\lambda'_{211}\lambda_{131}$. In the first case, we can get an additional contribution to pion decay $\pi^+ \to \tilde{\mu}^+ \to e^+\nu_e$ which changes the prediction for R_π

$$R_\pi = R_\pi^{\rm SM}\left[1 - \frac{m_\pi^2 \lambda'_{211}\lambda_{121}}{2\sqrt{2}G_{\rm F}M^2_{\tilde{\mu}{\rm L}}m_e(m_u+m_d)}\right]^2. \tag{16}$$

As the corresponding Feynman diagram has a different structure from the t-channel squark exchange which gives the bound on λ'_{211} in Eq. (15) we get a much stricter bound on the product of the couplings than on either of the couplings individually. This leads to the following bound at the 2σ level

$$\lambda'_{211}\lambda_{121} < 4.6\times 10^{-7}\ \left(\frac{m_{\tilde{\mu}{\rm L}}}{100\ {\rm GeV}}\right)^2. \tag{17}$$

This means that in the case of λ_{121} (as opposed to λ_{131}) the maximum scalar fermion mass which will solve the KARMEN anomaly in our model is 450 GeV.

The couplings λ'_{211} and λ_{131} violate muon and tau lepton number, respectively, and can thus lead to the decay $\tau \to \mu\gamma$. The experimental bound has recently been improved [17],

$$\mathrm{BR}(\tau \to \mu\gamma) < 1.0\times 10^{-6} \quad (90\%\ \text{C.L.}), \tag{18}$$

but is still 4 orders of magnitude weaker than the experimental upper bound on BR($\mu \to e\gamma$). Therefore a bound on a product of couplings which yield the decay $\tau \to \mu\gamma$ via a one-loop penguin diagram, e.g. $\lambda_{121}\lambda_{131}$, must be 2 orders of magnitude weaker than the corresponding bound on the couplings which give $\mu \to e\gamma$, i.e. one would expect $\lambda_{121}\lambda_{131} < \mathcal{O}(10^{-2})$ [18]. In our model, the couplings $\lambda'_{211}\lambda_{131}$ only contribute to the decay $\tau \to \mu\gamma$ at the 2-loop level and the bound is thus significantly weaker than $\mathcal{O}(10^{-2})$. (The decay $\tau \to \mu ee$ is similarly suppressed.) This is significantly weaker than the bound (15) so we have no new bounds on the product $\lambda'_{211}\lambda_{131}$. Furthermore since the bound (10) on λ'_{211} is so restrictive in our model, we need not worry about the model dependent bounds from flavour changing neutral currents [19].

In Refs. [20–22] severe cosmological bounds were derived on all R-parity violating couplings from considerations of GUT-scale lepto/baryogenesis in the early universe:

$$\lambda,\ \lambda',\ \lambda'' < 5 \times 10^{-7} \left(\frac{m_{\tilde{f}}}{1\ \text{TeV}} \right). \tag{19}$$

Subsequently it was shown that it is sufficient for just one lepton-flavour to satisfy this bound [21,22]. In Fig. 3 we can see that for our model both couplings violate the bound (19). For the case $(\lambda'_{211}, \lambda_{121})$ we must therefore demand that *either* all electron number violating couplings or all tau number violating couplings satisfy Eq. (19), while for the case $(\lambda'_{211}, \lambda_{131})$, we must demand that *all* electron number violating couplings satisfy Eq. (19). Alternatively, baryogenesis could plausibly occur at the electroweak scale, in which case the bounds (19) do not apply.

IV. EXPERIMENTAL CONSTRAINTS ON A LIGHT NEUTRALINO

We now summarize relevant experimental constraints on a light neutralino LSP and show that these are satisfied in regions of $(M_1, M_2, \mu, \tan\beta)$ parameter space for a dominantly bino $\tilde{\chi}_1^0$ with a small higgsino contribution. In our model, M_1 and M_2 are *not* related by the supersymmetric grand unified relation and we treat them as separate free parameters.

A. Bounds from $e^+e^- \to \nu\bar{\nu}\gamma$

The process $e^+e^- \to \nu\bar{\nu}\gamma$ can be measured in electron-positron collisions by detecting the photon and the missing energy due to the neutrinos [23]. As the lightest neutralino in our model is long-lived on the time scale of collider experiments, the process $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ will give the same experimental signature.

The cross section for this latter process has been calculated [24] for the case of a pure photino and can be easily extended to the pure bino case we are considering here. The cross section is shown as a function of the centre-of-mass energy in Fig. 4 and is rather low. The expected number of events for a number of different experiments are given in Table III assuming a scalar fermion mass of $M_{\tilde{f}} = 150$GeV. We have used the same cuts on the energy and angle of the photon as in Ref. [24].

As can be seen from Table III, *no* limits on this process can be set by LEP, as the expected number of events is much less than one. The recent results from OPAL [25] give 138 observed events (with a statistical error of ± 11.9) against the standard model expectation of 141.1 ± 1.1 events from $e^+e^- \to \nu\bar{\nu}\gamma$. There is also an expected non-physics background of 2.3 ± 1.1 events. Thus there is no evidence for any excess.

With the higher luminosities expected at the B-factories KEK-B and BaBar, a few events may be expected. The Standard Model (SM) cross section at this energy is 2.3 fb, corresponding to 230 ± 15 events at KEK-B and 70 ± 8

TABLE III. Cross-sections for the production of $\tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ at e^+e^- colliders for (an assumed common) sfermion mass $M_{\tilde{f}} = 150$ GeV and general expectations of the integrated luminosity.

Experiment	Integrated luminosity (pb^{-1})	Energy	Cross-section (fb)	Number of events
LEP	6.65	130	5.87	0.04
	5.96	136	6.14	0.04
	9.89	161	7.11	0.07
	10.28	172	7.44	0.08
	54.5	183	7.72	0.42
	75.	200	8.05	0.60
KEK-B	1×10^5	10.5	6.74×10^{-2}	6.7
BaBar	3×10^4	10.5	6.74×10^{-2}	2.0
NLC	3×10^5	500	6.19	1857

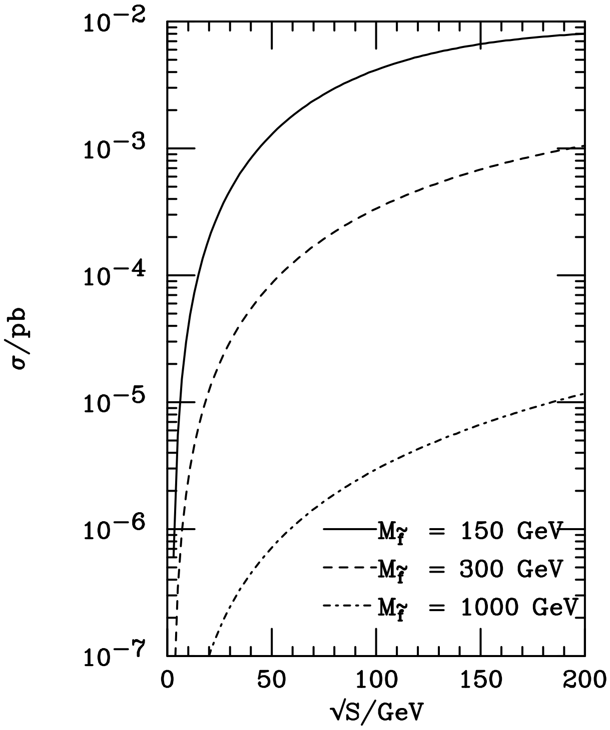

FIG. 4. Cross-section for for the production of a purely bino neutralino with mass 33.9 MeV through $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$.

events at BaBar. [5] The statistical uncertainty still exceeds the signal rate so we do not expect any sensitivity to a light neutralino.

At the NLC we expect a substantially higher number of events. The SM cross section for the same cuts is about 0.35 pb for 3 neutrinos [27] corresponding to 1.1×10^5 events, with a small statistical error of 330 events. Thus this can provide a test of our model.

B. Bounds from the invisible decay of the Z^0

In our model, $m_{\tilde{\chi}_1^0} \ll M_{Z^0}/2$, therefore the decay $Z^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0$ is kinematically accessible, and the $\tilde{\chi}_1^0$ can be considered to be effectively massless, just like a neutrino. The LSP decays outside the LEP detectors, thus the process $Z^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0$ will contribute to the invisible width of the Z^0. The current measurement of the invisible Z^0 width translated into the number of light neutrino species is [26]

$$N_\nu = 3.00 \pm 0.08, \tag{20}$$

so we must require that $\Gamma(Z^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0) < 0.08\,\Gamma(Z^0 \to \nu\bar{\nu})$, where the rhs refers to one neutrino species only.

A pure bino LSP does not couple to the Z^0 at tree-level. The dominant contribution will thus come from the Higgsino admixtures of the LSP, $N_{13},\ N_{14}$, in the notation of Ref. [28]. This enters with the fourth power in the decay rate $Z^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0$. The Higgsino has equal strength coupling to the Z^0 compared to a neutrino, thus yielding the constraint

$$\sqrt{|N_{13}|^2 + |N_{14}|^2} < 0.5 \approx (0.08)^{1/4}. \tag{21}$$

We shall see below that it is straightforward to find regions which satisfy this in $(M_1, M_2, \mu, \tan\beta)$ parameter space.

[5] This corresponds to one year of running based on the luminosities given in Ref. [26].

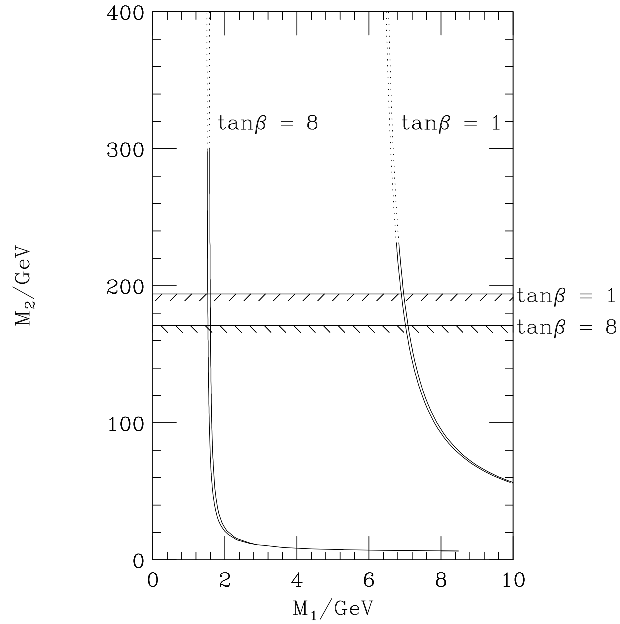

FIG. 5. Solutions in $(M_1, M_2, \mu, \tan\beta)$ parameter space giving a $m_{\chi_1^0} = 33.9$ MeV neutralino for $\mu = 300$ GeV and 2 representative values of $\tan\beta$. The width of the lines is 0.01 MeV. Below the hashed lines the chargino mass is less than 150 GeV. The dotted lines have $\Delta\rho_{\rm SUSY} < 10^{-4}$ and the solid lines have $\Delta\rho_{\rm SUSY} < 5\times10^{-4}$.

C. Solutions in MSSM parameter space

It is important to establish whether it is indeed possible to have a neutralino LSP with $m_{\chi_1^0} = 33.9$ MeV within the MSSM. To this end we have scanned the MSSM parameter space $(M_1, M_2, \mu, \tan\beta)$ with independent M_1, M_2, for a neutralino in the mass range

$$33.89 \text{ MeV} < m_{\chi_1^0} < 33.91 \text{ MeV}. \tag{22}$$

This leads to the neutralino iso-mass curves shown in Fig. 5, taking $\mu = 300$ GeV and 2 representative values of $\tan\beta$. We have not been able to find any solutions with $\mu < 0$. In order to obtain such a light neutralino some fine-tuning is required in the MSSM parameters, of about a few parts in 10^3 for $\tan\beta = 1$ and a few parts in 10^2 for $\tan\beta = 8$ [29]. The fine-tuning is reduced for larger M_2 and μ and small M_1 because a light neutralino can then be generated by the see-saw mechanism; it is reduced for large values of $\tan\beta$ because in the limit $\beta = \pi/2$ there is a zero mass eigenvalue for $M_1 \approx 0$ [29].

We have checked that the Higgsino contribution always satisfies the bound (21). In order to avoid an observable light chargino we require $m_{\chi_1^\pm} > 150$ GeV, which eliminates the region below the hashed lines in Fig. 5 for the specified values of $\tan\beta$.

The LSP is indeed dominantly bino along the solution curves in Fig. 5. The second lightest neutralino, $\tilde{\chi}_2^0$, is dominantly wino for $M_2 < 300$ GeV, while for larger values it is mainly higgsino. For $M_2 \gtrsim 110$ GeV, $m_{\tilde{\chi}_2^0} \gtrsim 100$ GeV, and for $M_2 \gtrsim 235$ GeV, $m_{\tilde{\chi}_2^0} \gtrsim 200$ GeV.

D. Bounds from oblique electroweak radiative corrections

Any new physics which couples to the SM can give contributions to the electroweak precision observables via radiative corrections. The effect of the new physics on vacuum polarization diagrams, the so called oblique radiative corrections, is usually parameterised using either the S, T, and U parameters of Ref. [30] or the ϵ_1, ϵ_2 and ϵ_3 parameters of Ref. [31]. The calculation of these parameters is based on an expansion in $q^2/M^2_{\rm new}$, where q^2 is the momentum scale of the gauge boson propagator, typically M_Z^2 or smaller and $M^2_{\rm new}$ is the scale of the new physics, assumed to be well above M_Z^2. If however there are new *light* particles in the spectrum, as in the present case, these approximations are typically insufficient and one must in general also calculate the box or vertex corrections [32]. This full calculation is however beyond the scope of our analysis and is not attempted here.

There is one exception however and that is the ratio of the charged to neutral current neutrino–electron/muon scattering events — the ρ-parameter. This is defined at $q^2 = 0$ and the expansion is thus trivial. In the following we limit ourselves to a calculation of the contribution to the ρ parameter from the full set of charginos and neutralinos. The radiative correction to the ρ parameter, $\Delta\rho$, is given by the W and Z self energies with zero momentum flow [6]

$$\Delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}. \tag{23}$$

The dominant SM contributions to $\Delta\rho$ arise from the (heavy) top quark and the Higgs boson. Assuming the mass of the latter to be $M_H = M_Z$, and subtracting the SM contributions we are left with the experimental limit on new physics at the 2σ level of [26]

$$-3.7 \times 10^{-3} < \Delta\rho_{\rm new} < 1.1 \times 10^{-3}. \tag{24}$$

We have calculated the contribution to $\Delta\rho$, which we denote $\Delta\rho_{\rm SUSY}$, from all charginos and neutralinos for given parameter points $(M_1, M_2, \mu, \tan\beta)$. We find full agreement with the results given in Ref. [33]. We then determine $\Delta\rho_{\rm SUSY}$ along the solution curves given in Fig. 5. The dotted lines indicate solutions for which $\Delta\rho_{\rm SUSY} < 10^{-4}$, while the solid lines indicate solutions for which $\Delta\rho_{\rm SUSY} < 5\times10^{-4}$. Thus there is no conflict at least with the experimental constraint on the ρ parameter.

E. Cosmological and astrophysical constraints

Massive particles are expected to come into thermal equilibrium in the early universe and their relic abundance is essentially the equilibrium value at 'freeze-out' when their self-annihilation rate drops below the Hubble expansion rate. For the neutralino under consideration, the self-annihilation cross-section is (s-wave) suppressed [34] so the surviving abundance is rather high:

$$m_{\tilde{\chi}_1^0}\left(\frac{n_{\tilde{\chi}_1^0}}{n_\gamma}\right) \approx 1.2 \times 10^{-2}\ \text{GeV}\left(\frac{m_{\tilde{\chi}_1^0}}{33.9\ \text{MeV}}\right)^{-2}\left(\frac{m_{\tilde{f}}}{150\ \text{GeV}}\right)^{4}, \tag{25}$$

This energy density will be subsequently released when the neutralinos decay and this has the potential to disrupt standard cosmology, in particular primordial nucleosynthesis [11]. Specifically, since the neutralinos will be non-relativistic during nucleosynthesis, the Hubble expansion rate will be speeded up, while the electromagnetic energy generated through the subsequent decays will dilute the nucleon-to-photon ratio, resulting in an increased helium-4 abundance [35]. The decay electrons will also Compton scatter the thermal background photons to energies high enough to directly alter the abundance of e.g. deuterium through photodissociation [36]. The observationally inferred primordial abundances thus enable stringent bounds to be placed on the relic abundance of the decaying particle as a function of its lifetime. For the above abundance (25), the decay lifetime is required to be less than a few thousand seconds in order that the primordial D/H ratio is not reduced below its conservative lower limit of 10^{-5}, and further required to be less than a few hundred seconds in order that the primordial ^{4}He mass fraction not exceed its conservative upper limit of 25% [35]. Thus the cosmological lifetime bound is essentially the same as the one derived earlier (13) from experimental considerations.

Very light neutralinos can also be produced through nucleon bremsstrahlung and e^+e^- annihilation in supernovae such as SN 1987a. Since the squark/selectron masses are now restricted to be above m_W [26], the neutralinos cannot be trapped in the supernova core by scatterings on nuclei or electrons, so will escape freely. The energy lost through this process can be comparable to the neutrino luminosity so may result in significant shortening of the $\bar{\nu}_e$ burst. The neutralino luminosity can be decreased by increasing the sfermion mass but it has been shown that consistency with observations of SN 1987A is not possible for any sfermion mass less than $\mathcal{O}(1)$ TeV [37]. This constraint is evaded if the neutralino is unstable due to R-parity violation, as in the present case. However there are then further constraints on the energy released in the decays. Given the experimental upper bound (13) as well as the cosmological upper bound on the lifetime, the decays would have occurred *within* the progenitor star. Moreover the lower bound (15) on the lifetime implies that the neutralinos cannot have decayed within the supernova core. The electromagnetic energy released in the decays would have been thermalised leading to distortions of the lightcurve. However the neutralinos under consideration here have a mass which is of the same order as the core temperature [12] so one must reconsider their production rate in order to quantify this potentially important constraint.

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[6] See for example the third paper in Ref. [30] for a derivation of this result.

V. OTHER IMPLICATIONS FOR R-PARITY VIOLATING PHENOMENOLOGY

A. The HERA high-Q^2 anomaly

In 1997, the HERA collaborations reported an anomaly in their high Q^2 data [38] whose most likely explanation was in terms of R-parity violation [39]. However, this required a significant $L_eQ_iD^c_j$ operator. While this is not excluded by our model it is a completely distinct possibility. Together the two operators might contribute to the decay $\mu \to e\gamma$. However this does not lead to a significant new bound since the bound on the coupling λ'_{211} is already so strict [18].

B. Neutrino masses

The trilinear R-parity violating couplings we have introduced also generate Majorana masses for neutrinos through one-loop self-energy diagrams [40]:

$$m_{ji} = \sum_{k,a,b} \frac{N_c}{16\pi^2} \frac{\lambda'_{ikb}\lambda_{jak}}{m^2_{\tilde{f}}} (m^2_{\rm LR})_{ab} m_{f_k}, \tag{26}$$

where N_c is a colour factor, and $(m^2_{LR})_{ab}$ is the left-right mixing term in the sfermion sector. The question then arises whether our model can account also for the SuperKamiokande data suggesting oscillation of atmospheric ν_μ into ν_τ [41]. The results indicate that the neutrinos mix almost maximally and that they are nearly mass-degenerate, $\delta\, m^2 \sim 10^{-3}-10^{-2}$ eV^2. If neutrino masses are hierarchical then the natural interpretation is that one of the neutrinos (presumably the ν_τ) has a mass of $\mathcal{O}(0.1)$ eV (although the possibility of a close mass-degeneracy for a heavier ν_μ–ν_τ pair is not excluded). Now the sfermion left-right mixing is not well determined, however within a given framework, e.g. supergravity-inspired models, approximate relations such as $(m^2_{LR})_{aa} \approx m_{f_a} m_{\tilde{f}}$ arise. Thus λ'_{211}, the coupling responsible in our model for the pion decaying to the neutralino, generates a mass

$$m_{\mu\mu} \approx 1.5 \times 10^{-7} \text{ eV} \left(\frac{\lambda'_{211}}{10^{-4}}\right)^2 \left(\frac{m_{\tilde{f}}}{150 \text{ GeV}}\right)^{-1}, \tag{27}$$

which is too small to be of phenomenological interest. The couplings $\lambda_{1\{2,3\}1}$ responsible for neutralino decay also generate rather small masses:

$$m_{\mu\mu,\tau\tau} \approx 10^{-6} \text{ eV} \left(\frac{\lambda_{121,131}}{10^{-2}}\right)^2 \left(\frac{m_{\tilde{f}}}{150 \text{ GeV}}\right)^{-1}. \tag{28}$$

Thus the absolute scale of the masses seems too low to explain the data on atmospheric neutrinos. However if other R-parity violating couplings are also present, it may well be possible to generate a neutrino mass pattern consistent with the observations, both of atmospheric and solar neutrinos [42].

VI. FUTURE TESTS

Experimentally, our model largely looks like the MSSM with non-universal gaugino masses and with a very light LSP. Thus most future tests of the MSSM also apply to our model, e.g. chargino pair production. A specific test would be to identify a very light LSP for example via neutralino pair production [43]. At an e^+e^- collider one can study the process

$$e^+ + e^- \to \tilde{\chi}^0_2 + \tilde{\chi}^0_1, \tag{29}$$

where $\tilde{\chi}^0_2$ subsequently decays visibly [44]. In Fig. 6 we show the cross section evaluated along our MSSM solution curves for both LEP2 ($\sqrt{s} = 200$ GeV) and the NLC ($\sqrt{s} = 500$ GeV). This should be directly observable, provided it is kinematically accessible, i.e. $m_{\tilde{\chi}^0_2} < \sqrt{s}$.

Experimentally the main difference between R-parity violation with a long-lived neutralino LSP and the MSSM is the possibility of resonant sparticle production. The value of $\lambda'_{211} \lesssim 10^{-4}$ is too small for the observation of resonant slepton production at hadron colliders [45]. However values of $\lambda_{1\{2,3\}1} > 10^{-3}$ should allow a test for resonant second or third generation sneutrino production at e^+e^--colliders for masses upto $\sim \sqrt{s}$ [46]. One can also test for the first generation via the mechanism described in Ref. [47].

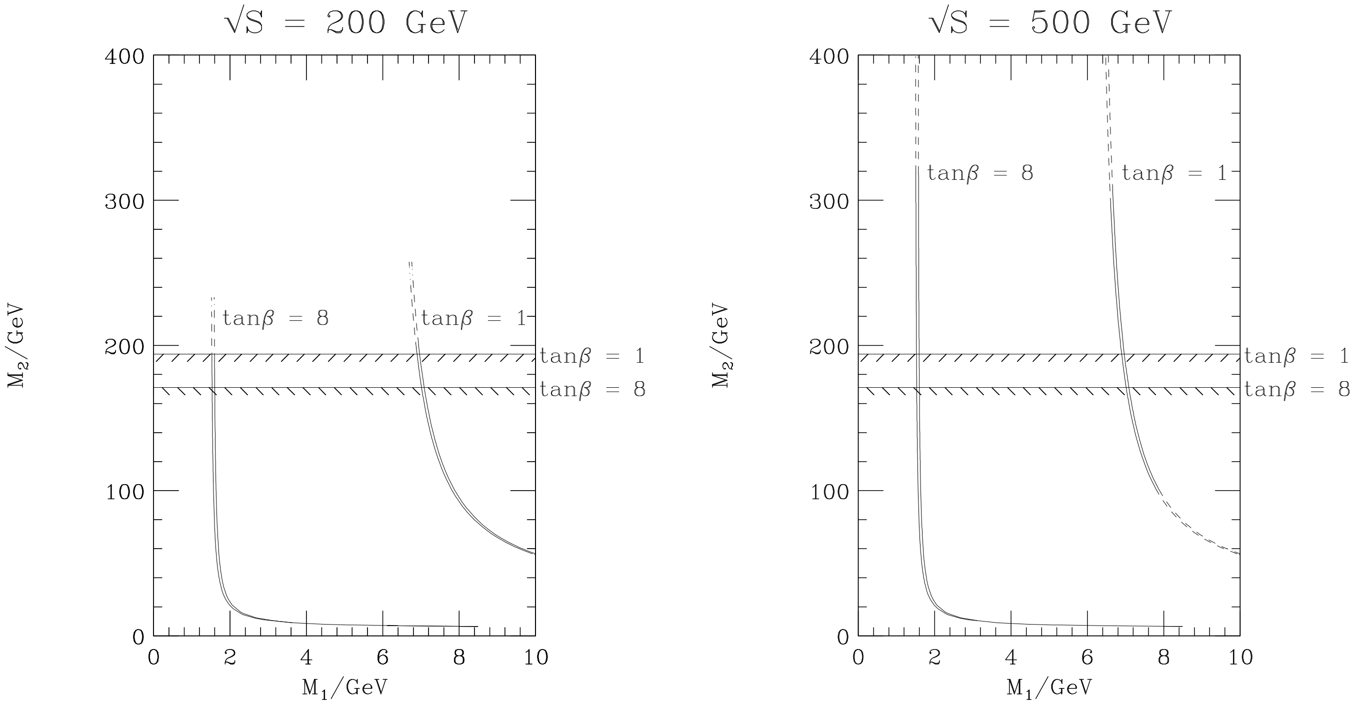

FIG. 6. Cross-Sections for $e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_1^0$ for the solutions in Fig. 5. The solid lines correspond to 0.1 pb< σ <1 pb, the dashed lines to 10 fb< σ <0.1 pb, the dotted lines to 1 fb< σ <10 fb, and the dot-dashed lines to σ <1 fb.

A further upgrade of the KARMEN detector may allow a better resolution of the decay of the x particle, in particular the angular distribution of the decay products. For reference we show in Fig. 7 the differential decay rate of the LSP in our model as a function of the angle between the two final state electrons.

VII. CONCLUSIONS

The KARMEN time anomaly is particularly intriguing because contrary to several other reported $3-4\sigma$ effects in the literature, its significance has not diminished with improved statistics, nor has it been explained away as a systematic effect. In fact the anomaly persists in the KARMEN-2 data, which has a much reduced background [2], with the same characteristics as in the KARMEN-1 data [1]. It would appear that there is no independent experiment which has the sensitivity to reproduce this result. In particular although the LSND experiment also studies pions and muons decaying at rest, it lacks the distinctive time-structure of the beam in the KARMEN experiment necessary to isolate the anomaly. Since KARMEN-2 acquires only of $\mathcal{O}(10)$ anomaly events per year of running, it is clear that a definitive resolution of the problem will have to await an upgraded detector with tracking capability.

Phenomenological models for the anomaly as due to the production and decay of a new particle are very tightly constrained. The only viable proposals at present are a singlet neutrino decaying through its large mixing with the ν_τ [3,7], or a neutralino decaying through violation of R-parity [4] which we have extended and investigated in detail. An important lesson from our investigation is that contrary to popular belief a neutralino lighter than even the pion is *not* excluded by present accelerator data unless a GUT relation between gaugino masses is assumed. Whether the KARMEN anomaly is indeed the first evidence for such a particle is a matter for future experiments to decide.

Note Added: While this manuscript was under review, the E815 (NuTeV) experiment at Fermilab reported a search for a 33.9 MeV neutral particle produced in pion decay decaying to a partially electomagnetic state such as $e^+e^-\nu$ or $\gamma\nu$ (J.A. Formaggio *et al.*, hep-ex/9912062). No evidence was found for such a particle but the lifetimes probed ($\sim 10^{-9} - 10^{-3}$ s) are much smaller than the lower limits (15) on the neutralino lifetime in our model so there are no implications. We note however that the exclusion of such short lifetimes is relevant in the context of the constraints from SN 1987a on the decaying particle hypothesis [3,4]. These constraints have been investigated further in two other recent reports (I. Goldman, R. Mohapatra and S. Nussinov, hep-ph/9912465, M. Kachelriess, hep-ph/0001160) which conclude that our model is excluded by the observations of SN 1987a. We reserve judgement on this issue for the reasons mentioned earlier.

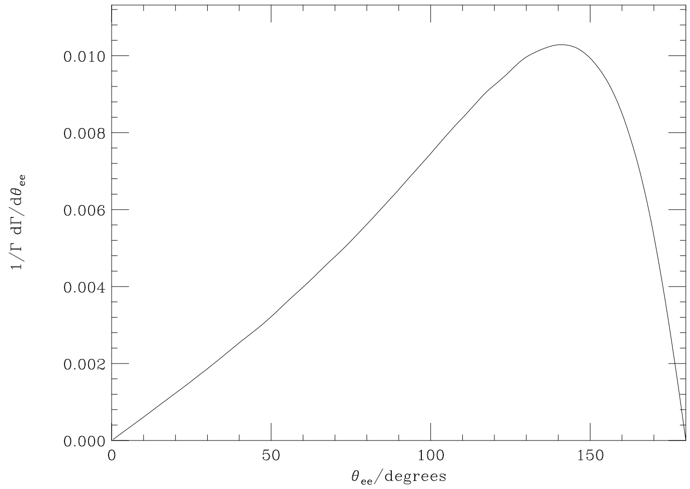

FIG. 7. Decay rate of the LSP versus the angle between the final state electrons.

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APPENDIX A: CALCULATION OF THE PION DECAY RATE

The rate of the pion decay $\pi \to \mu + \tilde{\chi}^0_1$ can be calculated using chiral perturbation theory. To do this we need an effective Lagrangian for the 4-fermion interaction u, $\bar{d}$, $\tilde{\chi}^0_1$ and μ^+ with the sfermion degrees of freedom integrated out (analogous to using the Fermi Lagrangian in the SM calculation). This gives

$$\mathcal{L} = \sqrt{2}\lambda'_{211}\left(\frac{A_e}{M^2_{\tilde{\mu}}} - \frac{A_u}{2M^2_{\tilde{u}}} - \frac{A_d}{2M^2_{\tilde{d}}}\right)(\bar{\mu}P_{\rm R}\tilde{\chi}_0)\,(\bar{u}P_{\rm R}d)\,, \tag{A1}$$

where we have also Fierz-reordered the Lagrangian (and neglected some tensor-tensor interaction terms which cannot contribute to the pion decay rate). The matrix element of the axial-vector current between the pion and the vacuum is

$$\langle 0|j^{\mu 5a}(x)|\pi^b(p)\rangle = -ip^\mu f_\pi \delta^{ab}{\rm e}^{-ipx}, \tag{A2}$$

where a and b are isospin indices. Using Eq. (A2) we obtain

$$\langle 0|\bar{u}\gamma^\mu\gamma_5 d|\pi^-\rangle = -i\sqrt{2}p^\mu f_\pi {\rm e}^{-ipx}. \tag{A3}$$

Contracting this with the pion 4-momentum and using the Dirac equation for the up and down quarks yields

$$\langle 0|\bar{u}\gamma_5 d|\pi^-\rangle = +\frac{i\sqrt{2}f_\pi m^2_\pi {\rm e}^{-ipx}}{(m_d+m_u)}, \tag{A4}$$

where m_d, m_u are thus the current quark masses. The amplitude for the decay (5) is then,

$$\mathcal{A} = -\frac{\lambda'_{211}f_\pi m^2_\pi}{(m_d+m_u)}\left(\frac{A_e}{M^2_{\tilde{\mu}}} - \frac{A_u}{2M^2_{\tilde{u}}} - \frac{A_d}{2M^2_{\tilde{d}}}\right)(\bar{u}_\mu(p_1)P_{\rm R}v_{\tilde{\chi}}(p_2))\,(2\pi)^4\delta\,(p_0-p_1-p_2)\,. \tag{A5}$$

This gives the decay rate quoted in Eq. (9). The additional contribution to the decay rate $\pi^+ \to e^+\nu_e$ given in Section III can be calculated in the same way.

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