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Equivalence of stochastic quantization to field theories from supersymmetry

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Abstract. Using the Ward-Takahashi identities from the hidden supersymmetry in Langevin equation we present a very simple proof of the equivalence of stochastic quantization to field theories.

Keywords. Stochastic quantization; supersymmetry; Ward-Takahashi identities.

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The stochastic quantization of Parisi and Wu (1981) scheme has been a subject of intensive study over the past few years. Several authors have proved the equivalence of this method to the conventional quantization method (Cardy 1983; Nakazato *et al* 1983; Grimus and Huffel 1983; Gozzi 1984; Krischner 1984; Gangopadhyay *et al* 1986). It is now well known that there is a superspace of formulation of the Langevin equation which brings out the hidden supersymmetry (SUSY) associated with the Langevin equation (Chaturvedi *et al* 1984a; Gozzi 1983; Egorian and Kalitzin 1983; Feigelman and Tsevlik 1982). This SUSY implies certain Ward-Takahashi identities that were derived by the authors and were used to give a very simple proof of the fluctuation dissipation theorem (Chaturvedi *et al* 1984b). In this paper we shall use the SUSY identities alone to give a very simple and direct proof of the equivalence.

Consider the Langevin equation

$$\frac{\partial\varphi(x,t)}{\partial t}=-\frac{\delta S}{\delta\varphi(x,t)}+\eta(x,t) \tag{1}$$

where $\varphi(x, t)$ is a scalar field and $\eta(x, t)$ a gaussian white noise source. The generating functional for the Green functions $Z(j)$, has been shown to be given by (Chaturvedi *et al* 1984a; Gozzi 1983; Egorian and Kalitzin 1983; Feigelman and Tsevlik 1982).

$$Z(j)=\int\mathscr{D}\varphi\mathscr{D}\pi\mathscr{D}\psi\mathscr{D}\bar{\psi}\exp[\int \mathrm{d}x\mathrm{d}t\mathrm{d}\alpha\mathrm{d}\bar{\alpha}\{-\mathscr{L}_{ss}+\mathscr{J}\Phi\}]. \tag{2}$$

Here, $\alpha,\bar{\alpha}$ are anticommuting Grassman variables and

$$\mathscr{L}_{ss}=\frac{1}{2}\Phi\frac{\partial}{\partial t}\Phi-\Phi\frac{\partial^2}{\partial\bar{\alpha}\partial\alpha}\Phi-\Phi\alpha\frac{\partial^2}{\partial\alpha\partial t}\Phi+\mathscr{S}(\Phi), \tag{3}$$

where Φ is the superfield defined by

$$\Phi=\varphi+\bar{\alpha}\psi+\bar{\psi}\alpha+\alpha\bar{\alpha}\pi \tag{4}$$

and the source $\mathscr{J}$ is

$$\mathscr{J}(x,t)=j(x)\delta(t). \tag{5}$$

The superspace Lagrangian is invariant under the following SUSY transformations.

$$\delta\varphi=\bar{\varepsilon}\psi+\bar{\psi}\varepsilon;\qquad \delta\psi=\varepsilon(\dot{\varphi}-\pi);$$
$$\delta\bar{\psi}=-\bar{\varepsilon}\pi;\qquad \delta\pi=-\varepsilon\dot{\bar{\psi}}. \tag{6}$$

Taking the source to be $\mathscr{J}=K+\bar{\alpha}L+\bar{L}\alpha+\alpha\bar{\alpha}J$, the Ward-Takahashi relations associated with these sources are obtained in an earlier paper (Chaturvedi *et al* 1984b), and are given by

$$\int \mathrm{d}x\,\mathrm{d}t\left[K(x,t)\frac{\partial}{\partial t}\frac{\delta Z}{\delta L(x,t)}+J(x,t)\frac{\delta Z}{\delta L(x,t)}\right.$$
$$\left.-\bar{L}(x,t)\left(\frac{\partial}{\partial t}\frac{\delta Z}{\delta J(x,t)}-\frac{\delta Z}{\delta K(x,t)}\right)\right]=0 \tag{7}$$

and

$$\int \mathrm{d}t\,\mathrm{d}x\left[J(x,t)\frac{\delta Z}{\delta\bar{L}(x,t)}-L(x,t)\frac{\delta Z}{\delta K(x,t)}\right]=0. \tag{8}$$

Differentiating (4) with respect to $\bar{L}(x,t)$ and setting $\bar{L}=L=0$ we arrive at

$$\int \mathrm{d}y\,\mathrm{d}\tau\left[-\frac{\partial}{\partial\tau}K(y,\tau)+J(y,\tau)\right]\frac{\delta^2 Z}{\delta\bar{L}(x,t)\,\delta L(y,\tau)}\bigg|_{L=\bar{L}=0}$$
$$+\left(\frac{\partial}{\partial t}\frac{\delta Z}{\delta J(x,t)}-\frac{\delta Z}{\delta K(x,t)}\right)\bigg|_{L=\bar{L}=0}=0. \tag{9}$$

The equation of motion for the π field is

$$2\frac{\delta Z}{\delta K(x,t)}-\frac{\partial}{\partial t}\frac{\delta Z}{\delta J(x,t)}+\left\langle\frac{\delta S(\varphi)}{\delta\varphi(x,t)}\right\rangle+K(x,t)=0, \tag{10}$$

where we have used the notation

$$\langle f(\varphi)\rangle=\int\mathscr{D}\varphi\mathscr{D}\pi\mathscr{D}\psi\mathscr{D}\bar{\psi}\exp\left[\int\mathrm{d}x\,\mathrm{d}t\,\mathrm{d}\alpha\,\mathrm{d}\bar{\alpha}\,(-\mathscr{L}_{ss}+\mathscr{J}\Phi)\right]f(\varphi). \tag{11}$$

Equations (9) and (10) with $K=0$ imply

$$\frac{\partial}{\partial t}\frac{\delta Z_0}{\delta J(x,t)}+2\frac{\delta S}{\delta\varphi}\bigg|_{\varphi=\delta/\delta J}Z_0=2\int\mathrm{d}y\,\mathrm{d}\tau\,J(y,\tau)\frac{\delta^2 Z_0}{\delta\bar{L}(x,t)\,\delta L(y,\tau)}, \tag{12}$$

where Z_0 is the generating functional Z for $K=L=\bar{L}=0$. If we take J as in (5) the right hand side of (12) involves the value of $\delta^2 Z_0/\delta\bar{L}\delta L$ at equal times $t=\tau=0$. It can be easily proved that (Nakano 1983)

$$\left.\frac{\delta^2 Z_0}{\delta\bar{L}(x,t)\,\delta L(y,\tau)}\right|_{t=\tau}=\frac{1}{2}\delta(x-y)Z_0. \tag{13}$$

Also in the steady state we must have

$$\frac{\partial}{\partial t}\frac{\delta Z_0}{\delta J(x,t)}=0. \tag{14}$$

Using (12), (13) and (14) we get

$$\left.\frac{\delta S}{\delta\varphi}\right|_{\varphi=\delta/\delta j}Z_0=j(x)Z_0. \tag{15}$$

The above equation implies that the generating functional $Z(j)$ has the form

$$Z_0(j)=\int\mathscr{D}\varphi\exp[-S(\varphi)+\int j(x)\varphi(x)\,\mathrm{d}x]. \tag{16}$$

This completes the desired proof.

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