The quark gluon plasma: Lattice computations put to experimental test

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Abstract.   I describe how lattice computations are being used to extract experimentally relevant features of the quark gluon plasma. I deal specifically with relaxation times, photon emissivity, strangeness yields, event-by-event fluctuations of conserved quantities and hydrodynamic flow. Finally I give evidence that the plasma is rather liquid-like in some ways.

Keywords. Heavy-ion collisions; lattice quantum chromodynamics; fluctuations; flow; transport coefficients.

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1. Introduction

> *The quagma engineers? That huge ugly brown thing we saw? That was one of them?*
> Gregory Benford, in "*Around the curve of a cosmos*"

QCD has been tested at zero temperature by its predictions for 'hard processes', i.e., processes in which all relevant scales are much larger than the intrinsic scale, Λ_{QCD}. This convenience is due to asymptotic freedom in QCD; at scales much larger than Λ_{QCD} the coupling α_S is small. At finite temperature, T, the scale relevant to most thermodynamic variables is of order T. Since $T_c/\Lambda_{\text{QCD}} = 0.5$ for QCD with two light flavours of quarks [1], at experimentally accessible temperatures $T/T_c \sim 1$–3, the scales are comparable to Λ_{QCD}, $g \equiv \sqrt{4\pi\alpha_S} = \mathscr{O}(1)$, and one deals with soft physics [2]. Perturbation theory may remain a rough guide to intuition. However, since it is sensitive to the infrared, i.e., non-perturbative length/mass scales, its domain of applicability really is $g \ll 1$, i.e., $T \geq 10^9 T_c$. As a result, lattice gauge theory is the only theoretical tool of direct relevance to experiments currently being performed at the relativistic heavy-ion collider (RHIC) at the Brookhaven Laboratory.

Until recently, the agreement of the energy density at freeze-out in relativistic heavy-ion collisions with that predicted at T_c by lattice computations, and the connection between Debye screening and J/ψ suppression, have been the main points of contact between fundamental QCD computations and experiments. In this talk I will concentrate on other comparisons, all potentially precise confrontations of lattice QCD predictions against experiments. Many of these have emerged in the last few years and are therefore less known.

Specifically, I will deal with predictions of strangeness yields, event-to-event fluctuations of conserved quantities, extraction of the speed of sound from the centrality dependence of elliptic flow and the first estimates of relaxation times and photon/dilepton production rates. A secondary motive for this talk is to identify the ways in which thermal perturbation theory may guide our thinking even in the domain where it is not expected to work.

For $T \ll T_c$ strongly interacting matter is in the confined phase. Chiral symmetry is spontaneously broken, with pions emerging as pseudo-Goldstone boson. Since the Dirac operator for quarks has nearly vanishing eigenvalues, accurate lattice computations are hard. In this range of temperatures it may be much easier to use effective theories such as chiral perturbation theory to extract quantities of interest to experiments. Interesting predictions exist for a lukewarm pion gas [4] and for the phases of cold and dense QCD [5]. At this time it seems that the role of lattice computations is to validate and determine some of the crucial inputs into such models. A discussion of this lies outside the scope of this talk.
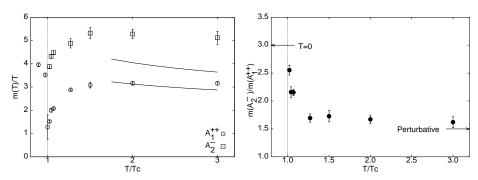
QCD matter undergoes a phase transition, or at least a rapid cross-over at $T = T_c$. This was the region in which the earliest lattice computations concentrated – successfully extracting T_c with high precision, and estimating the order of the phase transition [6]. The universality class of the phase transition in the chiral limit still remains to be reliably extracted – the main problem here is that extracting physics at small quark masses requires very large lattices, thus pushing up the time required to perform accurate numerical lattice computations. This region of temperature remains of great interest, since the transition from quarks to hadrons stamps the physics of this region onto many observables studied at RHIC. Since highly accurate lattice computations for this region are still underway, this talk will touch only briefly on this.

Most of the material in this talk is of relevance to the physics of the temperature range $1.5 \leq T/T_c \leq 3$, where $g = \mathcal{O}(1)$, and the perturbative and non-perturbative scales cannot be separated. As a result, perturbation theory cannot be numerically accurate and lattice computations are essential to extract the physics of the plasma. This talk is divided into four main sections. We begin by an examination of the quasiparticle modes of the plasma, which allows us to test perturbative expansions in a theoretically clean setting. The next two sections concentrate on two thermodynamic quantities of direct relevance to experiments – the equation of state and quark number susceptibilities. The following section is devoted to off-equilibrium phenomena such as relaxation times, electrical conductivity and photon (and dilepton) production rates in the plasma.

2. Perturbation theory: Is the QCD plasma a quark gluon plasma?

> *... quagga, extinct African wild ass like the zebra. quagma, hypothetical matter made up of quarks and gluons. quahog, type of edible clam. ...*
>
> http://phrontistery.50megs.com, "*List of unusual words*"

Perturbation theory is an expansion of the free energy of QCD in a series in g, and is effectively an expansion in terms of gluon and quark fields. One of the most basic quantities in Euclidean high temperature perturbation theory is the Debye screening mass. At leading orders in the perturbative series this has contribution only from the electric polarisation of the gluon [7]. However, at higher orders, magnetic polarisations also contribute [8] and, as a result, the perturbation expansion breaks down at finite order [9]. Perturbative predictions

Figure 1. The screening masses in quenched QCD from correlations of the two operators in eq. (1) on the left and their ratio on the right. The band is the one-sigma error band on the perturbative prediction along with an estimate of the non-perturbative terms from a numerical computation in a DR theory [21].

for the Debye screening mass do not exist close to T_c, and lattice studies of Debye screening can give no meaningful test of perturbation theory [10]. A couple of more limited tests are possible.
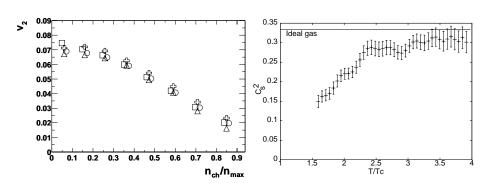
The first is to check whether a 'constituent' gluon picture works [11]. Correlations of the operators

$$A_1^{++} = \mathscr{R}e\,\mathrm{Tr}\,L \quad \text{and} \quad A_2^{--} = \mathscr{I}m\,\mathrm{Tr}\,L \tag{1}$$

(L is the Polyakov line operator, i.e., the flux due to a static quark) are obtained by two and three electric gluon exchanges to leading order. If this continues to be true in some sense non-perturbatively, then the screening masses obtained in these two channels should be in the ratio 3/2. A recent lattice computation (see figure 1) shows that this is actually true in the range $1.25 \leq T/T_c \leq 3$ [12]. However, detailed studies of other screening masses on the lattice show that no 'constituent' picture can be built-up in the sector of magnetic gluons [13]. In fact, magnetic Wilson loops have been shown to confine [14]. This is consistent with the picture of an effective theory for finite temperature QCD in which electric gluons and magnetic glueballs are the degrees of freedom [15]. A detailed model consistent with the lattice data is under investigation [16].

The second is to test a systematic reduction of the theory which goes by the name of dimensional reduction (DR) [17]. This attempts to integrate out the high frequency ($\omega \geq 2\pi T$) components of the theory and produces a long distance effective theory. The couplings in this effective theory are computed at the scale $2\pi T$ and hence perturbation theory should be fine as long as α_S is small enough. However, the effective theory is fairly complicated (probably confining) [18] and its long distance properties have to be extracted by lattice computation. For quenched QCD, the spectrum of screening masses obtained from DR [19] agrees with that from the full theory for $T \geq 2T_c$ [20]. One such test is shown in figure 1.

For physics in thermal equilibrium, it seems fruitful to think of the quenched QCD plasma above $1.25T_c$ as containing electric gluons. The magnetic sector seems confined, thus solving the infrared (Linde) problems of hot perturbative QCD through the non-perturbative mechanism of generating 'thermal glueballs' [22]. Closer to T_c there is not even any evidence for electric gluons. QCD with dynamical quarks may have a quantitative description in terms of gluons only for $T > 6T_c$ [1,23].

Figure 2. On the left are STAR collaboration results on v_2 as a function of centrality [37], from the correlation between particle pairs consisting of randomly chosen particles ($\circ$), particles with opposite sign of charge ($\bullet$), particles with the same sign of charge ($\triangle$) and particles with opposite sign of pseudorapidity ($\square$). On the right is a lattice estimate of the speed of sound in quenched QCD.

3. Flow and the equation of state

> *When quagma is allowed to cool and expand its binding superforce decomposes into four sub-forces. To my surprise, I understood some of this.*
> Stephen Baxter, in '*On the Orion line*'

A clear signal of collective effects in the final state of a relativistic heavy-ion collision would be the hydrodynamical flow. If flow can be unambiguously identified in experiments, then the equation of state (EOS) of QCD matter becomes accessible to measurement, since it is an input to the hydrodynamical equations. The EOS, i.e., the temperature dependence of pressure (P), energy (E) and entropy (S) densities, have been extracted on the lattice in quenched QCD [24] as well as in QCD with two [25] or four [26] flavours of dynamical quarks. It is a remarkable lacuna that this EOSs has not yet been put through the machinery of hydrodynamical codes to confront experiments [27].

P, S and E deviate from the Stefan–Boltzmann limit strongly near T_c and by about 20% even at the highest temperatures at which lattice computations exist (about $4T_c$). This seems to have no explanation within perturbation theory, since the perturbative series for P fluctuates wildly as more terms are added; a Borel [28] or Padé [29] summation of the series does not help. Screened perturbation theory [30] applied to the hard thermal loop resummation does not produce agreement with the lattice results [31]. On the other hand, there have been reasonably successful attempts to fit the pressure by a gas of quasiparticles whose masses are the fit parameters [32]. A partially self-consistent resummation also gives good agreement with the lattice data [33]. More recently the pressure has been obtained in the DR theory [34].

Signatures of hydrodynamic flow have been sought in particle spectra and in HBT radii in the past. At present, one of the most promising signals is the elliptic flow [35]. If hydrodynamics can be trusted, then, in off-center collisions of the two nuclei, the spatial anisotropy leads to pressure gradients. These drive momentum anisotropies, whose second Fourier coefficient, v_2, is called elliptic flow [36]. This has been observed in experiments over a wide range of collider energies [37].

At RHIC energies, the variation of v_2 with the impact parameter b (which determines the charged multiplicity n_{ch}) is claimed to have a good explanation in terms of hydrodynamic flow [38]. So does the variation of v_2 with the transverse momenta, p_t, of the particles used to measure it [39]. If the initial temperature is determined independently, then the slope of v_2 against b depends on the speed of sound, c_s, since the pressure drives the evolution of v_2. In principle, then, c_s can be measured directly from RHIC experiments and compared to predictions from the lattice.

Lattice predictions for c_s can be obtained as a byproduct for the extraction of the EOS. In figure 2 we show our extraction of c_s from the data in [24]. This computation is preliminary (a more detailed computation is underway), and the main uncertainty is connected with the fact that the lattice data used have finite lattice spacing artifacts which need to be compensated for. However, a dip in c_s near T_c has been seen with two-flavour dynamical quarks [40], and argued to follow from thermodynamic considerations [41]. The most interesting observation is that at the highest temperatures c_s is close to its ideal gas value, although both P and E are far from ideal. This has also been seen with two flavours of dynamical quarks [40].

4. Fluctuations, strangeness yields, and quark number susceptibilities

We were inducted here by some curious property of the quagma, so I suppose.
Gregory Benford, in "*Around the curve of a cosmos*"

Event-by-event fluctuations in conserved quantities such as the charge or baryon number [42] are proportional to quark number susceptibilities

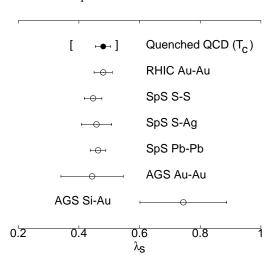$$\chi_{fg} = -\frac{T}{V}\frac{\partial^2 \log Z}{\partial \mu_f \partial \mu_g}\bigg|_{\mu_f=\mu_g=0}, \tag{2}$$

where Z is the partition function of QCD and μ_f is the chemical potential for flavour f [43]. Further details, including those of the evaluation of these susceptibilities on the lattice can be found in several recent reviews [44]. It is interesting to note that recent lattice computations [45] for the diagonal susceptibilities (χ_{ff}) can be reproduced in a skeleton graph resummation [46], dimensional reduction [47] and also in a quasiparticle picture [48]. The off-diagonal susceptibilities are found to be zero in lattice computations; there seems to be no explanation for this in models.

Measured fluctuations [49] are thought to be proportional to the ratio χ/S. Lattice computations for these are under good control for $T > T_c$, but the region $T < T_c$ requires more work. Present day lattice data [45] indicate a hierarchy of fluctuations for baryon number (χ_B), electric charge (χ_Q) and strangeness (χ_s):

$$\begin{aligned} \chi_B < \chi_Q < \chi_s \quad (T > T_c), \\ \chi_B > \chi_Q > \chi_s \quad (T < T_c). \end{aligned} \tag{3}$$

The inversion of the hierarchy as one crosses T_c may be a possible experimental signal of the phase transition.

Figure 3. The Wroblewski parameter extracted from the data [52] compared to a quenched lattice computation [53]. The bars are statistical errors. The brackets denote the envelope of uncertainties due to extrapolations from $T \geq 1.5T_c$ down to T_c.

One of the most interesting pieces of information that the lattice can supply is the strangeness yield, which is measured very accurately in experiments, and hence has attracted much attention [50]. This yield is parametrised as the Wroblewski parameter, λ_s, which is the relative number of primary-produced strange to light quarks [51,52]. Clearly, λ_s is the ratio of imaginary parts of the complex susceptibilities in these flavour channels. Under reasonable (and testable) assumptions [53]

$$\lambda_s = \frac{2\chi_{ss}}{\chi_{uu} + \chi_{dd}}, \tag{4}$$

thus allowing us to compute this quantity on the lattice. Results obtained in quenched QCD [53] are exhibited in figure 3. We expect this ratio to be fairly insensitive to quenching artifacts. A computation in dynamical QCD with two flavours at T_c is now underway.

5. Relaxation times, photon emissivity and the electrical conductivity of a plasma

Quagma ... was both the Red Dragon and the Green Dragon. It was light and the light was good.
Jonathan S McDermott, in *http://caraig.home.mindspring.com/rant020206.html*

We next turn to non-equilibrium phenomena in the QCD plasma. These are of very direct relevance to heavy-ion experiments, since the matter formed in the fireball is fully out of equilibrium initially. Of interest are limits on how fast it equilibrates with respect to the strong interactions, how fast local thermal fluctuations diffuse away, how quickly a hard probe (such as a jet) loses energy, whether the system remains forever out of equilibrium in electroweak interactions, and if so, the rate at which it radiates leptons and photons. Over the last two years, perturbation theory and lattice computations have reached a stage where we can begin to constrain the answers seriously.

The most crucial piece of information that is required is of the equilibration time. Hydrodynamic explanations for particle spectra, HBT radii and, especially, elliptic flow, all require relatively small equilibration times in the plasma (0.6–1 fm) [54], implying that transport related cross-sections are huge. Experimental evidence for jet quenching [55], particularly the damping of away-side jets [56], are also indicative of small relaxation times or rapid energy flows. These time scales, or the corresponding transport coefficients are intimately related to large angle or multiple small angle (Landau–Pomeranchuk–Migdal, LPM) scattering and are of the order of $1/g^4 \log(1/g)T$ when g is small enough [57]. The Kubo formulae relate these transport coefficients to the zero energy ($\omega = 0$) limits of the imaginary parts of certain retarded correlators. When these correlators are evaluated in perturbation theory, the multiparticle states which contribute to it have momenta $(k_0^i, \mathbf{k}^i)$ which sum up to zero (i labels particles). However, when these intermediate states are massless, each of the $k_0^i \simeq \omega$ can be zero. Then while integrating over k_0^i, the contour is pinched between these poles. Interactions, specifically the transport cross-sections, throw these poles slightly off-axis, but the pinch still gives a bump in the imaginary part of the correlators. The effect of such bumps, which are seen to persist beyond the pertubative regime, is to give rise to transport coefficients [58].

The simplest of this class of problems deals with electromagnetic interactions. The transport coefficient one deals with is the ohmic conductivity, σ, i.e., the response of the QCD plasma to an external static and spatially uniform electric field, E. The result of applying such a field is to set up a current $j = \sigma E$ in the direction of the field. A Kubo formula relates σ to the imaginary part, ρ, of the retarded current–current correlator in equilibrium
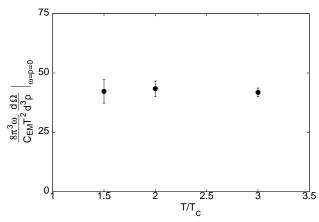
$$\sigma(T) = \frac{1}{6} \frac{\partial}{\partial \omega} \rho_i^i(\omega, \mathbf{p} = \mathbf{0}, T) \Big|_{\omega=0}, \tag{5}$$

where all spatial components i are summed over. There is a finite and non-vanishing ohmic conductivity as long as ρ_i^i is linear near zero energy. The photon emissivity is given by

$$\omega \frac{\mathrm{d}\Omega}{\mathrm{d}^3 \mathbf{p}} = \frac{1}{8\pi^3} n_{\mathrm{B}}(\omega, T) \rho_\mu^\mu(\omega, \mathbf{p}, T), \tag{6}$$

where Ω is the number of photons produced per unit volume per unit time. This is equal to the observed photon rate if the reabsorption rate is very small – in which case the medium is out of equilibrium with respect to the EM coupling α. In this work we shall take $\omega = \mathbf{p} = 0$, and hence obtain the soft photon production rate. Since $\rho_{00} = 0$ for $\mathbf{p} = 0$, the soft photon rate can be obtained once σ is computed. Extracting ρ_i^i from lattice computations needs the maximum entropy method [59] or other Bayesian techniques [60].

The soft photon production rate from the plasma phase of hadronic matter has long been of importance to searches for the QCD phase transition, especially due to persistent observations of enhancements in heavy-ion collisions over proton–proton rates [61]. Consequently, there has been a number of attempts at perturbative computations of this rate [62]. The first lattice computation in quenched QCD of dilepton (off-shell photon) rates [63] showed good agreement with perturbative results for $\omega > 3T$. Recently the leading order computation of the photon production rate was completed [64]. For the transport coefficient one has $\sigma \propto \alpha T/g^4 \log g^{-1}$, to leading-log accuracy, with a known proportionality constant [65]. The first computation of σ and hence of the soft photon emissivity from a

Figure 4. The scaled soft photon emissivity obtained from a lattice computation [60]. The quantity on the abscissa is equal to $6\sigma/T$.

quenched lattice computation has now been performed for $1.5 \le T/T_c \le 3$ [60]. It turns out that

$$\frac{\sigma}{T} \approx 7C_{EM}, \quad (\text{for } 1.5 \le T/T_c \le 3), \quad \text{where} \quad C_{EM} = 4\pi\alpha \sum_f e_f^2, \tag{7}$$

and e_f is the charge of a quark of flavour f. The corresponding soft photon emissivity is shown in figure 4. Clearly, for fireball dimensions less than $1/\sigma = 1/7C_{EM}T \approx 3$ fm, the plasma is transparent to photons and this emissivity is also the detection rate of photons.

The diffusion coefficient of quarks can also be obtained in the same computation using the Einstein relation $\sigma = 4\pi\alpha \sum_f e_f^2 \chi_{ff} D_f$.

$$TD_f = \left(\frac{T^2}{\chi_{ff}}\right)\left(\frac{\sigma}{C_{EM}T}\right), \tag{8}$$

where χ_{ff} is the quark number susceptibility defined in eq. (2) [65]. A characteristic relaxation time, τ_R, is the time for quarks to diffuse a distance equal to the screening length $1/T$. Then, we have

$$\tau_R \approx \frac{1}{DT^2} \approx \frac{1}{7T}. \tag{9}$$

For $1.5 \le T/T_c \le 3$ this is much smaller than a fermi. However, the relaxation time for charge carries an extra power of α in the denominator and hence is two orders of magnitude larger. This is the reason why charge fluctuations may be detectable.

The relaxation time required in jet quenching has to do with the gluon-dominated transport coefficient $\hat{q}$, which measures momentum transport transverse to the external force [66]. This transport coefficient remains to be measured on the lattice, but there is no reason to suspect that it leads to a significantly longer relaxation time. A complete theory of equilibration does not exist at this time [67], but given such small relaxation times near equilibrium, it does not seem implausible that equilibration times are also small.

On purely phenomenological grounds it is clear that extremely fast thermalization and jet quenching is not compatible with a fireball that is very transparent to photons. The ratio of the relevant scales is just $C_{\mathrm{EM}} \approx 1/20$. If the former scale is about 0.1–0.15 fm, then the latter scale must be in the range 2–3 fm. Thus, the fireball produced at RHIC is marginally transparent to soft photons, whereas the larger expected size of a fireball at LHC would only allow photon detectors to look 2–3 fm inside the surface of the fireball.

6. (not the) Conclusion

> *... the Earth is a type-13 civilization. Type 13 civilizations destroy themselves by turning their*
> *planet into degenerate matter looking for the Higgs boson.*
> murphy@panix.com posted on *Slashdot*

Let me introduce a dimensionless parameter which classifies several aspects of the physics that I have been talking about – the liquidity, defined by

$$\ell = \tau S^{1/3} \approx \tau E^{1/4}, \tag{10}$$

where τ is the transport mean free time. The non-relativistic analogue of S is the number density, so that ℓ is the mean free path in units of the interparticle spacing. For gases we expect this number to be large. A liquid would be characterised by values of ℓ close to unity.

In the perturbative expansion, when $g \ll 1$, we have $S \simeq T^3$, $\tau \simeq 1/Tg^4 \log(1/g)$, and hence $\ell \simeq 1/g^4 \log(1/g) \gg 1$. As a result, perturbation theory describes only the dilute, gaseous, phase of the QCD plasma. In experiments one finds $E \simeq 1$ GeV/fm^3 and $\tau < 1$ fm, giving $\ell < 1.5$, and matter that is definitely liquid. We shall continue to call this phase a plasma, in view of the screening phenomena that occur (but remain to be rigorously demonstrated in experiments). However it is important to remember that transport coefficients are dominated by interactions, as in liquids, and not by long mean free paths, as in gases. The lattice studies now seem to indicate liquid-like behaviour for $T \leq 3T_{\mathrm{c}}$, thus bringing us closer to an interpretation of heavy-ion collisions as quark matter.

The departure of c_s^2 from its gas value for $T < 2T_{\mathrm{c}}$ and the rapid fall in S, also indicate that the plasma changes character in the temperature region 2–3T_{c}. However, there is no evidence of a phase transition between the gaseous and liquid-like extremes of the QCD plasma. This is likely to be the reason that perturbative expansions around some quasiparticle pictures give a qualitative description of static quantities such as S, E or χ, not far from T_{c}. However, the experimental numbers indicate that this is unlikely to be the case for dynamics.

Liquid-like behaviour means that dissipative effects are important to the fluid dynamics – in the relation between the HBT, single-particle spectra and elliptic flow. In addition, the supersonic motion of jets through the liquid should give rise to many interesting colour-MHD effects apart from jet quenching. One near-term target for the lattice theory is to estimate the various transport coefficients and thereby determine the relative efficiency of various physical mechanisms for entropy production.

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