

# Unparticles and Muon Decay

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## Abstract

Recently Georgi has discussed the possible existence of ‘Unparticles’ describable by operators having non-integral scaling dimensions. With the interaction of these with the Standard Model (SM) particles being constrained only by gauge and Lorentz symmetries, it affords a new source for lepton flavour violation. Current and future muon decay experiments are shown to be very sensitive to such scenarios.

The notion of scale invariance in the description of a physical system is a very powerful one and has found wide applications in many different subdisciplines. A very well-known manifestation is afforded by phase transitions wherein the existence of a critical temperature is but a reflection of fluctuations at all length scales being equally important. In field theoretic models, scale invariance has traditionally been a powerful tool in the analysis of the asymptotic behaviour of correlation functions. And as is well known, conformal invariance plays an even more fundamental role in string theories.

In the regime of particle physics though, the existence of many different particles (elementary or composite) with a very wide range of masses, manifestly breaks scale invariance. Indeed, an interacting scale invariant theory in four space-time dimensions is, by definition, bereft of particles thus running counter to our understanding of nature. Nonetheless, it is quite possible that there could exist a different sector of the theory that is so weakly coupled to the Standard Model (SM) particles that we have been unable to probe it experimentally. Clearly, this new physics is allowed to be described by a nontrivial scale invariant theory sector with an infrared fixed point. A concrete example is afforded by a vector-like non-abelian gauge theory with a large number of massless fermions as studied by Banks and Zaks (BZ) [1]. Supersymmetric nonlinear sigma models with similar features have also been considered in the literature [2].

In a recent paper, Georgi [3] investigated the consequences of such a nontrivial scale invariant (BZ) sector interacting with the SM fields through the exchange of (unspecified) very heavy particles. Below the messenger scale, then, such interactions between the BZ and the SM fields may be parametrized in terms of nonrenormalizable interactions. As scale-invariance in the BZ sector emerges at an energy scale  $\Lambda_{\mathcal{U}}$ , this sector should no longer be described in terms of conventional particles, but rather in terms of massless “unparticles”. The renormalizable couplings of the BZ fields cause a dimensional transmutation [5], and in the effective theory operative below the scale  $\Lambda_{\mathcal{U}}$ , the BZ operators match onto corresponding unparticle operators. The aforementioned nonrenormalizable interactions, written in terms of unparticles, would, in general, have non-integral scale dimensions, with very unexpected phenomenological consequences [3, 4, 6–16].

Since we have no direct information on either the unparticle or the messenger sector, the only recourse for us in the exploration of the interaction with the SM sector is to consider all possible operators in an effective theory consistent with the symmetries of the SM. In particular, this includes flavour-changing operators [3, 7], and, more precisely, those that violate lepton flavour conservation.

While lepton flavour violation (LFV) is absent in the minimal version of the SM, it can be easily accommodated by extending the SM to include neutrino masses. Indeed, the very observation of neutrino oscillations [17] implies LFV. Including finite mass differences from neutrino mixing imply  $\ell_i - \ell_j$  mixing is generated at the one-loop level and is thus suppressed by a factor of  $(m_\nu^2/m_W^2)^2$ . However, various extensions of the SM naturally incorporate large LFV effects. The simplest examples are afforded by the inclusion of heavy singlet Dirac neutrinos [18], heavy right-handed Majorana neutrinos or left-handed and right-handed neutral isosinglets [19]; or even dimension-six effective fermionic operators [20]. More ambitious models consider see-saw mechanism with or without grand unification [21], supersymmetry [22], technicolor [23], models of compositeness [24]. Higgs [25] or a  $Z'$ -mediated [26] LFV has also been considered in the literature.

The great theoretical interest in LFV has been reflected in various experimental efforts as well. For example, each of the four LEP collaborations have investigated such scenarios at length [27]. This has also constituted an important component of the two collaborations at HERA [28] and, perhaps

more expectedly, at the two  $B$ -factories [29]. And finally, several dedicated experiments have been designed to explore LFV. Of particular interest to us is the MEG experiment at the PSI [30], designed to detect forbidden decays of the muon down to the  $10^{-14}$  level.

Quite understandably, the study of muon decays have been a bedrock of the investigations into lepton flavour violation. Apart from the experimental ease, the very smallness of the muon decay width in the SM makes it particularly amenable to look for small new physics effects. This is a feature that we wish to exploit.

For unparticles, a discussion of LFV must necessarily be attempted in the effective Lagrangian framework and hence part of it bears resemblance to some of the above mentioned analyses, although with very significant differences. Given our ignorance of the unparticle sector, all we can aver is that the unparticle operators in the effective Lagrangian must be SM gauge singlets and must have a mass dimension larger than one. They might have any Lorentz structure themselves as long as the overall operator is a Lorentz scalar.

Given that the only decay mode allowed to the muon within the SM is that into an electron and missing energy-momentum ( $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ ), the unparticle mode that could possibly fake it is  $\mu^- \rightarrow e^- + \mathcal{U}$  and we shall start our analysis with this. Since the effective Lagrangian is perforce restricted to terms of the form

$$\mathcal{L} \supset \mathcal{O}_{SM}^i \mathcal{O}_{\mathcal{U}}^i$$

where  $i$  runs over Lorentz as well as flavour indices, the simplest term that one can write for the process under consideration involves a scalar unparticle operator  $\mathcal{O}_{\mathcal{U}}$  and can be expressed as

$$\mathcal{L}_1 = \Lambda^{-d_u} \bar{e} \gamma_\eta (c_1 + c_2 \gamma_5) \mu \partial^\eta \mathcal{O}_{\mathcal{U}} \quad (1)$$

where  $c_i$  are constants,  $\Lambda$  ( $\equiv \Lambda_{\mathcal{U}}$ ) is the scale of new physics, and  $d_u > 1$  is the mass dimension of the operator  $\mathcal{O}_{\mathcal{U}}$ . Note that this Lagrangian had been considered in Ref. [3] in the context of the  $t \rightarrow c + \mathcal{U}$  decay wherein a choice  $c_1 = -c_2 = 1$  was made for the analogous coefficients. For reasons mentioned above, muon decay is expected to be a far more sensitive probe of such couplings.

Using scale invariance to fix the two point correlators of the unparticle operators [3], viz.

$$\langle 0 | \mathcal{O}_{\mathcal{U}}(x) \mathcal{O}_{\mathcal{U}}^\dagger(0) | 0 \rangle = \int \frac{d^4 p}{(2\pi)^4} e^{-i P \cdot x} |\langle 0 | \mathcal{O}_{\mathcal{U}}(0) | P \rangle|^2 \rho(P^2) \quad (2)$$

where  $|P\rangle$  is the unparticle state of momentum  $P^\mu$  created from the vacuum by the operator  $\mathcal{O}_{\mathcal{U}}$ , and  $\rho(P^2)$  is the density of states, we have [3]

$$\begin{aligned} |\langle 0 | \mathcal{O}_{\mathcal{U}}(0) | P \rangle|^2 \rho(P^2) &= A_{d_u} \theta(P^0) \theta(P^2) (P^2)^{d_u-2}, \\ \text{with } A_{d_u} &\equiv \frac{16 \pi^{5/2}}{(2\pi)^2 d_u} \frac{\Gamma(d_u + \frac{1}{2})}{\Gamma(d_u - 1) \Gamma(2 d_u)} \end{aligned} \quad (3)$$

normalised to give the phase space for  $d_u$  massless particles. The decay profile can then be computed in a straightforward manner to yield [3]

$$\begin{aligned} \frac{d\Gamma_S}{dE_e}(\mu \rightarrow e + \mathcal{U}) &= \frac{A_{d_u}}{4\pi^2} (c_1^2 + c_2^2) m_\mu^2 E_e^2 (m_\mu^2 - 2 m_\mu E_e)^{d_u-2} \Lambda^{-2d_u} \Theta(m_\mu - 2 E_e) \\ \Gamma_S(\mu \rightarrow e + \mathcal{U}) &= \frac{A_{d_u}}{16\pi^2} \frac{c_1^2 + c_2^2}{d_u^3 - d_u} m_\mu \left(\frac{m_\mu}{\Lambda}\right)^{2d_u} \end{aligned} \quad (4)$$

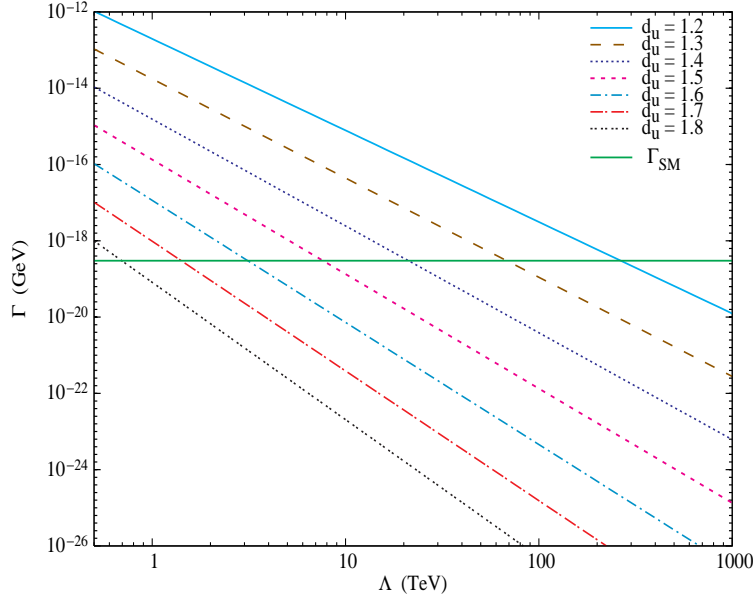


Figure 1: The muon decay width into  $(e^- + \mathcal{U})$  as a function of the unparticle physics scale  $\Lambda$  for various values of the mass dimension  $d_u$  of the scalar operator  $\mathcal{O}_{\mathcal{U}}$ . We have adopted the convention  $c_1^2 + c_2^2 = 1$ . Also shown is the SM width for the muon.

where the mass of the electron has been neglected and the second equality follows only for  $d_u > 1$ .

In Fig.1, we display the total width as a function of  $\Lambda$  for different choices of  $d_u$ . Since the dependence on the coefficients  $c_i$  is trivial, we have made the simplifying assumption of

$$c_1^2 + c_2^2 = 1$$

(a convention often adopted in effective field theories). To draw conclusions about the sensitivity of this measurement to unparticle physics, it needs be remembered that muon decay is one of the best measured observables and, in fact, essentially constitutes the measurement of the Fermi coupling constant [31, 32]. Furthermore,  $G_F$  is an input for various other precision measurements, a notable one being the coupling of the  $W$ -boson to the first generation quarks [31, 33], viz,

$$V_{ud} = 0.97377 \pm 0.00027 .$$

Since the latter is determined from superallowed nuclear beta decays, the couplings  $c_{1,2}$  have no rôle to play here. Thus, barring magical conspiracies between different terms in the effective Lagrangian, we may safely demand

$$\Gamma(\mu^- \rightarrow e^- + \mathcal{U}) \leq 10^{-3} \Gamma(\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu) , \quad (5)$$

and the consequent bounds are presented in Fig.2. These, expectedly, are quite strong, especially for small  $d_u$ . And, while these have been derived for  $c_1^2 + c_2^2 = 1$ , the dependence is quite mild, with the bounds obtained on  $\Lambda$  scaling as  $(c_1^2 + c_2^2)^{1/2 d_u}$ .

We now consider a different possible coupling of the unparticles to the muon-electron current, namely a vector one:

$$\mathcal{L}_2 = \Lambda^{1-d_u} \bar{e} \gamma_\eta (c_3 + c_4 \gamma_5) \mu \mathcal{O}_{\mathcal{U}}^\eta \quad (6)$$

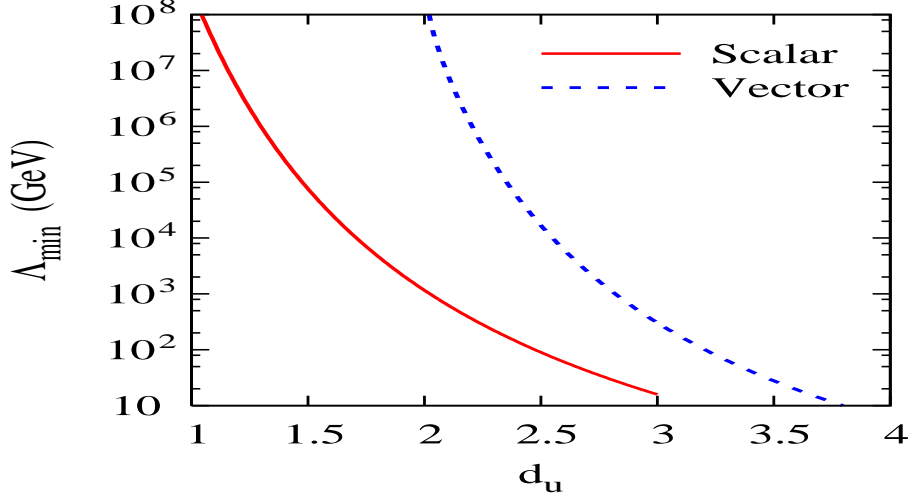


Figure 2: The lower limit on the scale of the effective theory for scalar ( $\mathcal{O}_{\mathcal{U}}$ ) and vector ( $\mathcal{O}_{\mathcal{U}}^{\eta}$ ) operators as a function of their mass dimension and assuming that  $Br(\mu^- \rightarrow e^- + \mathcal{U}) \leq 10^{-3}$ . In either case, we have assumed that  $c_1^2 + c_2^2 = 1$  and  $c_3^2 + c_4^2 = 1$  respectively.

where  $\mathcal{O}_{\mathcal{U}}^{\eta}$  is a transverse and Hermitian operator. The transversality condition, alongwith scale invariance, now stipulates that

$$\langle 0 | \mathcal{O}_{\mathcal{U}}^{\eta}(0) | P \rangle \langle P | \mathcal{O}_{\mathcal{U}}^{\omega}(0) | 0 \rangle \rho(P^2) = A_{d_u} \theta(P^0) \theta(P^2) (-g^{\eta\omega} + P^{\eta} P^{\omega} / P^2) (P^2)^{d_u-2}. \quad (7)$$

This leads to

$$\begin{aligned} \frac{d\Gamma_V}{dE_e}(\mu \rightarrow e + \mathcal{U}) &= \frac{A_{d_u}}{4\pi^2} (c_3^2 + c_4^2) m_{\mu} E_e^2 (m_{\mu}^2 - 2m_{\mu} E_e)^{d_u-3} \Lambda^{2-2d_u} (3m_{\mu} - 4E_e) \Theta(m_{\mu} - 2E_e) \\ \Gamma_V(\mu \rightarrow e + \mathcal{U}) &= \frac{3A_{d_u}}{16\pi^2} \frac{c_3^2 + c_4^2}{d_u^3 - d_u^2 - 2d_u} m_{\mu} \left(\frac{m_{\mu}}{\Lambda}\right)^{2d_u-2} \end{aligned} \quad (8)$$

once again neglecting the electron mass. The second equality holds only for  $d_u > 2$ . The resultant total width is displayed in Fig.3.

Certain differences with the scalar case (Eq.4) are easy to appreciate. The coupling of the fermion current to the scalar operator  $\mathcal{O}_{\mathcal{U}}$ —Eq.1—is a helicity suppressed one, leading to the amplitude being proportional to  $m_{\mu}$ . With the coupling to the vector operator being free of this suppression, one would naively expect an enhancement, relative to the scalar case, by roughly a factor of  $(\Lambda/m_{\mu})^2$ . In other words, the constraints on  $\Lambda$ , for identical values of  $d_u$ , are expected to be much stronger for the vector case than that for the scalar one. That this is indeed true can be easily divined from a comparison of Figs.1&3.

On the other hand, note that the differential width is now proportional to  $(m_{\mu}^2 - 2m_{\mu} E_e)^{d_u-3}$ , or, in other words, has an extra factor of  $1/P^2$ . This, of course, can be traced to Eq.7. While this term would not contribute when  $\mathcal{O}_{\mathcal{U}}^{\eta}$  couples to a conserved current, in the present context it leads to an enhanced density of states in the small  $P^2$  regime. Consequently, the total width is divergent unless  $d_u > 2$ . This constitutes a key result of our study and we shall return to it later.

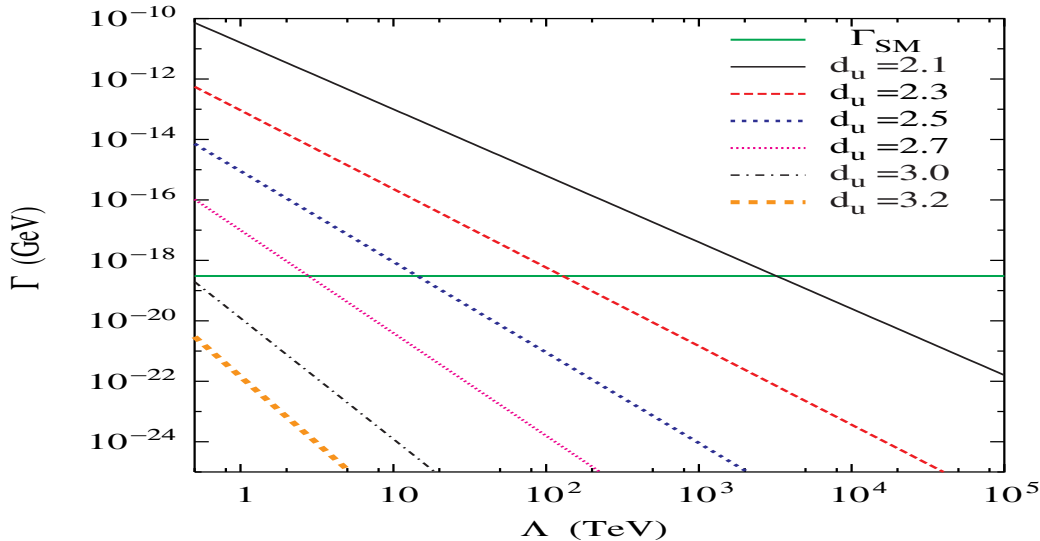


Figure 3: The muon decay width into  $(e^- + \mathcal{U})$  as a function of the unparticle physics scale  $\Lambda$  for various values of the mass dimension  $d_u$  of the vector operator  $\mathcal{O}_U^\eta$ . We have adopted the convention  $c_3^2 + c_4^2 = 1$ . Also shown is the SM width for the muon.

The limits on the effective scale for vector-like couplings are displayed in Fig.2. Once again, we have assumed that  $c_3^2 + c_4^2 = 1$ . While it may seem that the limits are much stronger for the vector case, note that, given the structure of  $\mathcal{L}_1$  and  $\mathcal{L}_2$ , it is only fair to compare  $\Gamma_V(d_u, \Lambda)$  with  $\Gamma_S(d_u - 1, \Lambda)$ . Shifting the curve for the vector coupling in Fig.2 to the left by one unit shows that the new curve would, for the most part, fall below that for the scalar. This can be easily understood by considering the ratio

$$\frac{\Gamma_S(d_u - 1, \Lambda)}{\Gamma_V(d_u, \Lambda)} = \frac{16\pi^2}{3} \frac{c_1^2 + c_2^2}{c_3^2 + c_4^2} (d_u - 2)(d_u + 1) \quad (d_u > 2)$$

which is larger than unity unless  $d_u$  is very close to 2.

It is amusing to consider the hypothetical case of an observed discrepancy in the decay  $\mu^- \rightarrow e^- + \text{nothing}$  in the forthcoming experiments. For example, can MEG [30] distinguish between the possible unparticle operators if such deviation were to be seen? A possible means is provided by the shape of the energy distribution. In Fig.4, we display the same for both cases considered above. For small values of  $d_u$ , the distributions are naturally peaked at  $E_e = m_\mu/2$  as is expected for a decay into two massless particles. While it might seem that, for the vector case, the peaking persists to much larger values of  $d_u$ , it is but a reflection of the differing powers of  $P^2$  in the two cases ( $d_u - 3$  for vector vs.  $d_u - 2$  for scalar).

It should be noted here that, while the limit  $d_u \rightarrow 1^+$  for the scalar case corresponded to the two-body decay, in the case of the vector, it is instead the limit  $d_u \rightarrow 2^+$  that corresponds to the same (namely, a muon decaying to an electron and a vector particle). Similarly,  $d_u \rightarrow 2$  in the scalar case corresponds to a three-body decay (and hence the close identification with the SM curve in Fig.4a). For the vector case, this feature is exhibited in the  $d_u \rightarrow 3$  limit instead. Both these correspondences in the vector case owe themselves to the form of Eq.7 and are reflective of the fact that, in this case,

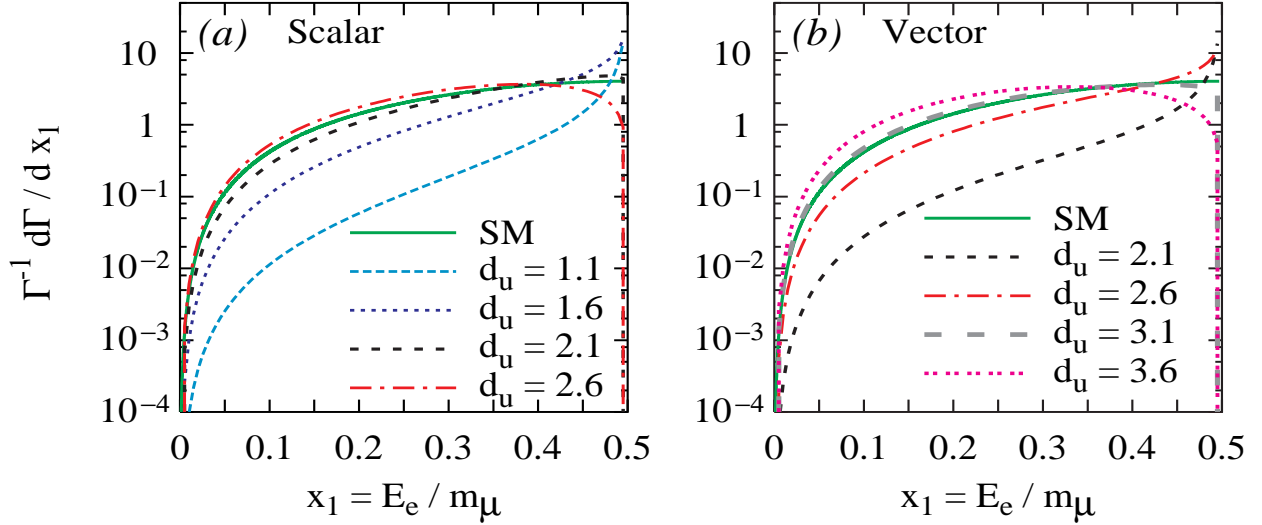


Figure 4: The energy distribution for the electron in  $\mu^- \rightarrow e^- + \mathcal{U}$  decay for various values of  $d_u$ . The left (right) panels refer to scalar (vector) unparticle operators respectively. Also shown, in each case, is the energy distribution for the SM process  $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ .

it is  $d_u \rightarrow 2^+$  that goes over to the one-particle description and hence the theory makes sense only for  $d_u > 2$ .

Until now, we have been considering only LFV couplings of the unparticle sector with SM matter. Of course, this sector could couple to lepton flavour conserving currents as well. As far as muon decays are concerned, the only such coupling that is of relevance is the one with the electrons. Restricting ourselves to the vector operator, we may now write an additional term of the form

$$\mathcal{L}_3 = \Lambda^{1-d_u} \bar{e} \gamma_\eta (c_5 + c_6 \gamma_5) e \mathcal{O}_U^\eta \quad (9)$$

Such terms would immediately manifest themselves in observables pertaining to the electron, in particular low-energy ones. In Ref. [10], effects on both the anomalous magnetic moment of the electron and the decays of ortho-positronium was examined. The bounds were found to be quite stringent, in particular those emanating from the latter set of observables. It should be noted that unparticle contributions to such observables are not proportional to the combination  $(c_5^2 + c_6^2)$ . For example, a value  $d_u = 1.5$  would imply  $\Lambda/c_5 \geq 4.3 \times 10^5$  TeV, or  $\Lambda/c_6 \geq 510$  TeV, as long as only one of the two coefficients were to be non-zero<sup>1</sup>. Of course, large cancellations between several such contributions are possible, but would represent a fine-tuned situation.

Simultaneous presence of both sets of operators  $\mathcal{L}_2$  and  $\mathcal{L}_3$  would immediately engender unparticle-mediated  $\mu \rightarrow 3e$  decays. The calculation is straightforward and mirrors that in the presence of a LFV  $Z'$ . Since the vector propagator in this case is given by [4, 6]

$$\int e^{iPx} \langle 0 | T(\mathcal{O}_U^\mu(x) \mathcal{O}_U^\nu(0)) | 0 \rangle d^4x = \frac{i}{2} A_{d_u} \frac{-g^{\mu\nu} + P^\mu P^\nu / P^2}{\sin(d_u \pi)} (-P^2 - i\epsilon)^{d_u-2},$$

<sup>1</sup>Note that the use of  $d_u < 2$  may also be a cause for concern in this context.

the spin-summed and averaged matrix-element-squared for the decay  $\mu(p_1) \rightarrow e^-(p_2) + e^-(p_3) + e^+(p_4)$  can be computed to be

$$\begin{aligned}
\left[ \frac{4\Lambda^{4-4d_u} A_{d_u}^2}{\sin^2(d_u\pi)} \right]^{-1} \overline{|\mathcal{M}|^2} &= |\mathcal{P}_1|^2 [K_1(p_{23}p_{14} + p_{24}p_{13}) + K_2(p_{23}p_{14} - p_{24}p_{13})] \\
&+ |\mathcal{P}_1|^2 [K_1(p_{23}p_{14} + p_{34}p_{12}) + K_2(p_{23}p_{14} - p_{34}p_{12})] \\
&- 2 \operatorname{Re}(\mathcal{P}_1 \mathcal{P}_2^*) [K_1 + K_2] (p_{14}p_{23}) \\
\mathcal{P}_1 &\equiv [-(p_1 - p_2)^2 - i\epsilon]^{d_u-2} \\
\mathcal{P}_2 &\equiv [-(p_1 - p_3)^2 - i\epsilon]^{d_u-2} \\
K_1 &\equiv (c_3^2 + c_4^2) (c_5^2 + c_6^2) \\
K_2 &\equiv 4c_3 c_4 c_5 c_6
\end{aligned} \tag{10}$$

where  $p_{ij} \equiv p_i \cdot p_j$ , and we have suppressed terms of  $\mathcal{O}(m_e)$  in the first equation for reasons of brevity<sup>2</sup>. Integrating Eq.(10) over the phase space would give us the total partial width in this channel. It should be noted though that considering strictly massless electrons would lead to a divergent value of the matrix-element whenever the positron were to be collinear with either of the electrons. Although it is numerically sufficient to consider the phase space to be that for three massive particles, while continuing to neglect  $m_e$  in the first of Eqs.(10), in our calculations, we retain the full dependence on  $m_e$ .

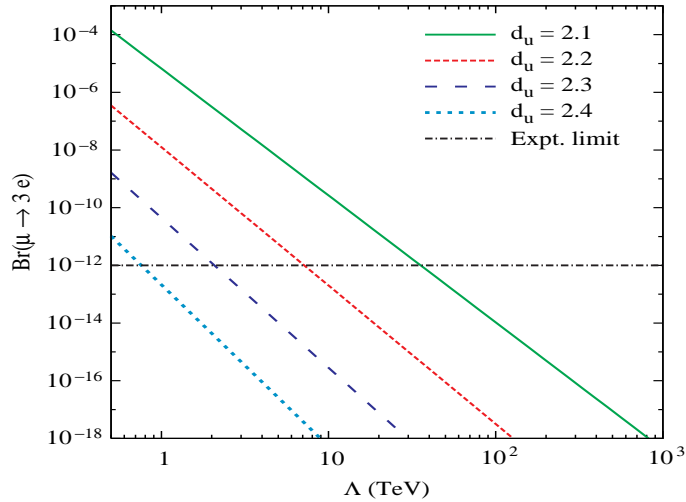


Figure 5:  $Br(\mu \rightarrow 3e)$  as a function of the unparticle physics scale  $\Lambda$  for various values of the mass dimension  $d_u$  of the vector operator  $\mathcal{O}_U^n$ . Both  $\mathcal{L}_2$  and  $\mathcal{L}_3$  terms appear in the effective Lagrangian with  $c_3 = c_4 = c_5 = c_6 = 1/\sqrt{2}$ . Also shown is the experimental upper limit for this channel.

<sup>2</sup> This process is also discussed in Ref. [11]. However, they concentrate on  $d_u < 2$ , a regime that is unphysical.



In Fig. 5, we present the branching ratio  $Br(\mu \rightarrow 3e)$  as a function of the scale  $\Lambda$  for different values of the scaling dimension  $d_u$ . To facilitate easy comparison with the limits obtained earlier, we maintain  $c_3 = c_4 = c_5 = c_6 = 1/\sqrt{2}$ . We concentrate on  $d_u > 2$  here for the aforementioned reasons. Although the experimental limit is [31]

$$Br(\mu \rightarrow 3e) < 10^{-12} ,$$

note that the constraints on  $\Lambda$  from this process are typically much weaker than those already obtained from  $\mu \rightarrow e + \mathcal{U}$ . The reason is not far to seek. Compared to the 2-body decay, the rate for  $\mu \rightarrow 3e$  process involves an extra factor of  $(m_\mu/\Lambda)^{2d_u-2}$ , apart from phase space factors and, consequently, the rate is suppressed even for the smallest of  $d_u$  allowed. It might seem then that it is pointless to consider  $\mu \rightarrow 3e$ , given the already existent constraints. But before we conclude so, it is of importance to ask whether  $\mu^- \rightarrow e^- + \mathcal{U}$  could fake  $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$  even in the presence of sizable  $c_{5,6}$ . Although it has been argued [6] that the imaginary part of the unparticle propagator does not correspond to a finite decay width, and that the unparticle, once produced, never decays. Remember though that the entire formalism corresponds to an effective theory and the details lie in the ultraviolet completion. In a very recent deconstruction of this theory, Stephanov [15] points out that the unparticle can be viewed as the limiting case of an infinite tower of particles of different masses with a regular mass spacing. If the spacing is small, but finite, then the unparticles are allowed to decay. In view of such subtleties and the lack of knowledge on our part as to the exact nature of unparticles (were they to be discovered), it seems contingent upon us to explore each constraint on its own.

It is both amusing and instructive to consider the phase space distributions for the  $\mu^- \rightarrow e^+ e^- e^-$  decay. Concentrating, for simplicity, on unpolarized muons, we present, in Fig.6, some of these distributions in the muon rest frame. The dependence on  $d_u$  is quite striking. For  $d_u = 3$ , each of these (and any other) matches the corresponding distributions for say a  $Z'$ -mediated  $\mu \rightarrow 3e$  decay. This, of course, is expected since  $d_u = 3$  corresponds to a single vector exchange. For  $d_u > 3$  ( $< 3$ ), the positron spectrum becomes harder (softer), while the reverse is true of the softer of the two electrons. Similarly,  $d_u > 3$  ( $< 3$ ) pushes the softer of the two electrons farther (closer) to the positrons.

The simultaneous presence of both  $\mathcal{L}_2$  and  $\mathcal{L}_1$  would also lead to processes like  $e^+ + e^- \rightarrow \mu^+ + e^-$  at high energy colliders and presumably used to look for unparticle effects at linear colliders. The amplitude for this can be obtained trivially by the use of crossing symmetry. A simple estimate shows though that, given the strong constraints already obtained, a first generation linear collider would not add to the sensitivity.

To summarize, we have studied a particularly intriguing aspect of low energy phenomena associated with Unparticle physics, namely nonconservation of lepton flavour. As unparticles are associated with a hidden scale invariant sector that communicates with the SM fields through a heavy messenger sector, at low energies such interactions are parametrized by generic operators in an effective field theory consistent with the symmetries of the SM. Of particular relevance here is the non-integral value of the scale dimensions of these operators, which could lead to very interesting phenomenology.

With lepton flavour violation being absent in the SM, it proffers an ideal theatre to look for signatures of physics beyond the SM. It is well known that, in the SM, the only decay mode allowed to the muon is that into an electron and missing energy-momentum ( $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ ); this channel could possibly be mimicked by  $\mu^- \rightarrow e^- + \mathcal{U}$ , where, the missing energy-momentum is carried by the Unparticle  $\mathcal{U}$ . While the scale dimension  $d_u > 1$  for the scalar operator, we demonstrate that

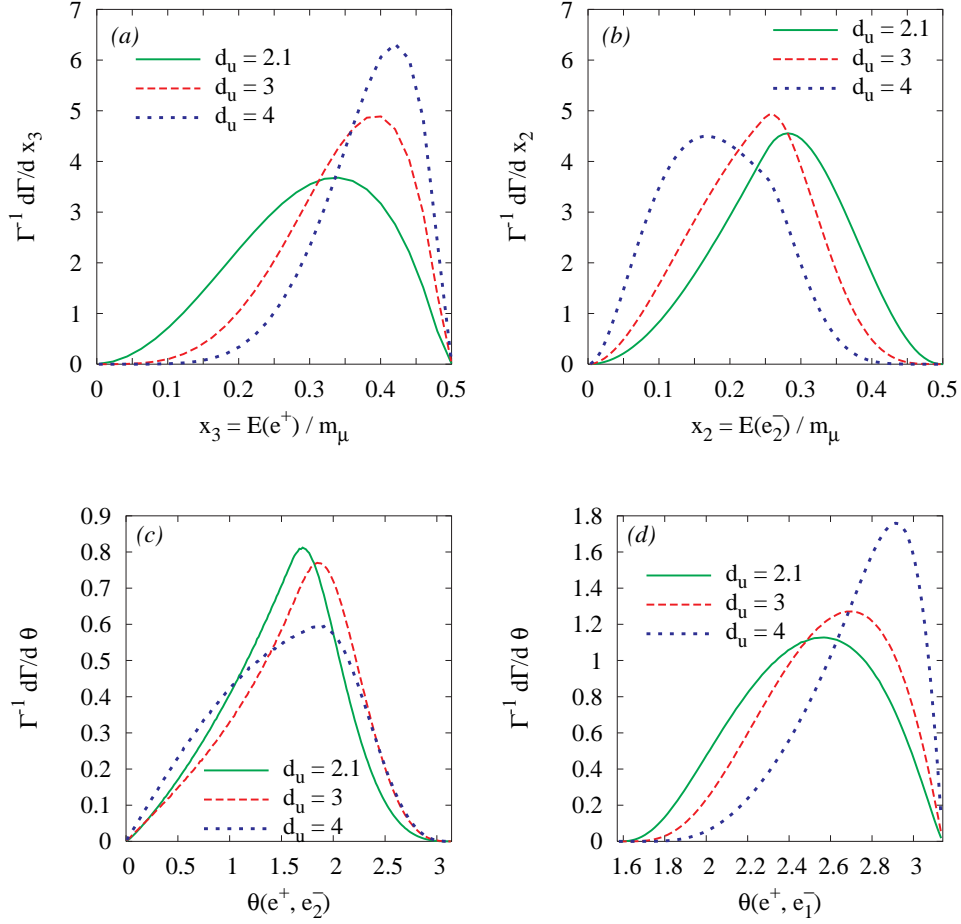


Figure 6: *Various normalized phase space distributions (in the muon rest frame) for the decay  $\mu^- \rightarrow e^+ e^- e^-$  mediated by vector unparticles. (a) the energy of the positron; (b) the energy of the softer electron; (c) the angle the softer electron subtends with the positron; (d) the angle the harder electron subtends with the positron.*

consistency of the vector operator requires the corresponding scaling dimension to be greater than 2. The present experimental accuracies on  $G_\mu$  and the nuclear beta decay measurements lead to very strong bounds on the unparticle scale  $\Lambda$ . In case a deviation is observed in future muon decay experiments, we demonstrate how the shape of the electron energy distribution could disentangle unparticle effects from other possible electroweak physics.

In addition to the real emission of unparticles, we reexamine  $\mu \rightarrow 3e$  decay mediated by a vector unparticle operator and find some disagreements with Ref. [11]. And although the constraints obtainable from present upper bounds on this decay mode are weaker than those derivable from  $\mu \rightarrow e + \mathcal{U}$ , this mode does offer an opportunity to probe some interesting issues, both theoretical and experimental.

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