The co-optimization of floral display and nectar reward

Prajakta V Belsare, Balasubramanian Sriram and Milind G Watve

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Appendix

Article 1

Here, we attempt to establish analytically the relationship of R with N as affected by p.

$R = f(p)$, *i.e.*

$$R = BR + \frac{(1-BR)\times N^p}{A^p + N^p} - C(N+D)$$

$$R' = \frac{N^p}{A^p + N^1}$$ and compute dR'/dp.

$1\text{n}\, R' = 1\text{n}\, N^p - 1\text{n}\,(A^p + N^p)$

now differentiating with respect to p

$$\frac{1}{R'}\frac{dR'}{dp} = \frac{1}{N^p}\frac{dN^p}{dp} - \frac{1}{(N^p + A^p)}\frac{d(N^p + A^p)}{dp}$$

applying

$$\frac{dx}{dp} = x\,1\text{n}\,N = N^P\,1\text{n}\,N$$

thus we get

$$\frac{1}{R'}\frac{dR'}{dp} = \frac{1}{N^p}N^p\ln N - \frac{1}{(N^p + A^p)}(A^p\ln A + N^p\ln N)$$

$$\frac{1}{R'}\frac{dR'}{dp} = \frac{A^p}{(A^p + N^p)}\ln\frac{N}{A}$$

$$\frac{dR'}{dp} = \frac{A^p N^p}{(A^p + N^p)^2}\ln\frac{N}{A}$$

$$\Rightarrow \frac{dR}{dp} = \frac{(1-BR)A^p N^p}{(A^p + N^p)^2}\ln\frac{N}{A}$$

Thus, for $N > A$ as per the assumption in the simulations, we find that the rate of change in R with respect to p is positive tending towards 0 as $N \rightarrow A$ and for $N < A$ the rate of change is negative. This results in the curves shown in figure 1.

Article 2:
Analytical treatment of the optimization problem

R is a function of N and D. Thus, we are interested in finding the optimum value of R in the surface obtained by plotting R versus N and D. We assume $C = 1$ for simplicity of solutions.

We optimize in two dimensions.

To do this, we compute

$$\frac{dR}{dD}, \frac{dR}{dN}, \frac{d^2R}{dD^2}, \frac{d^2R}{dN^2}, \frac{d^2R}{dDdN} = \frac{d^2R}{dNdD}.$$

Next, we put $\frac{dR}{dD} = 0\, and\, \frac{dR}{dN} = 0$ to obtain the critical point.

In order to obtain the nature of the critical point, we compute the Hessian determinant $(H = (\frac{d^2R}{dD^2}.\frac{d^2R}{dN^2} - (\frac{d^2R}{dDdN})^2)$ to check its sign and check for the sign of $\frac{d^2R}{dD^2}$.

$$\text{Thus, } \frac{dR}{dD} = \frac{aA^{p+1}}{(a + A\times D)^2(A^p + N^p)} - 1,$$

$$\frac{dR}{dN} = \frac{apA^pN^{p-1}}{(a + A\times D)(A^p + N^p)^2} - 1,$$

$\frac{dR}{dD} = 0\, and\, \frac{dR}{dN} = 0$ yields the critical point which is computed numerically for different values of p. Thus, the values of N and D are obtained.

$$\frac{d^2R}{dD^2} = \frac{-2aA^{p+2}}{(a + A\times D)^3(A^p + N^p)} < 0$$

$$\frac{d^2R}{dN^2} = \frac{apA^pN^{p-2}(A^p(p-1) - N^p(p+1))}{(a + A\times D)(A^p + N^p)^3},$$

$$\frac{d^2R}{dDdN} = \frac{-apA^{p+1}N^{p-1}}{(a + A\times D)^2(A^p + N^p)^2}$$

Therefore

$$H = (2A^{3p+2}N^{p-2}a^2p(1-p)A^{2p+2}N^{2p-2}a^2p(p+2))/((a+A\times D)^4(A^p+N^p)^4)$$

Thus for all $p = \in (0,1)\, H > 0$ and as $\frac{d^2R}{dD^2} < 0$ maxima is possible for every p in this range at the critical points.

At equilibrium $A = N$ and substituting this in H we get

$$H = 2A^{4p}a^2p(1-p)+A^{4p}a^2p(p+2)/(16a^4(1+A)^4(A^p)^4)$$

$$H = (2p(1-p)+p(p+2))/(16a^2(1+A)^4) = (4p-p^2)\,/\,(16a^2(1+A)^4)$$

Thus, for all $p > 4$ H <0 and $p < 4$ H >0.

This implies that at critical points less than 4, we get maxima when stability is achieved when $N = A$, and for all critical points greater than 4, we get a saddle point as the sign of the Hessian changes at 4. The precise position of the critical p will change if we change the assumed parameters. The behaviour that there will be stability below a critical p remains invariant.