

Charged particle motion in an electromagnetic field on Kerr background geometry

A R PRASANNA and C V VISHVESHWARA*

Physical Research Laboratory, Ahmedabad 380 009

*Raman Research Institute, Bangalore 560 006

MS received 5 April 1978

Abstract. In this paper we study the trajectories of charged particles in an electromagnetic field superimposed on the Kerr background. The electromagnetic fields considered are of two types: (i) a dipole magnetic field with an associated quadrupole electric field, (ii) a uniform magnetic field. The contribution of the background geometry to the electromagnetic field is taken through the solutions of Petterson and Wald respectively. The effective potential is studied in detail for the r -motion of the particles in the equatorial plane and the orbits are obtained. The most interesting aspect of the study is the illustration of the effect of inertial frame dragging due to the rotation of the central star. This appears through the existence of nongyrating bound orbits at and inside the ergo surface. The presence of the magnetic field seems to increase the range of stable orbits, as was found in a previous study involving the Schwarzschild background.

Keywords. Charged particle orbits; Kerr geometry; pulsars, general relativity.

1. Introduction

There has been considerable interest in the investigation of electromagnetic fields surrounding compact objects like neutron stars and black holes. Such studies shed light on the interplay of electromagnetism and the curvature of spacetime due to strong gravitational fields. Furthermore, they provide the basis for astrophysical applications if relevant situations occur. In this context, the investigation of the trajectories of charged particles can reveal the influence of the geometry on the electromagnetic fields, just as the study of geodesics yields important information regarding the structure of spacetime (Barden 1972, Bicak and Stuchilk 1976). At the same time, it would constitute the orbit theory of the test charges which is the first step towards the examination of possible plasma processes near collapsed objects.

The charged particle trajectories in a dipole magnetic field superposed on the Schwarzschild background geometry have already been considered in a recent paper (Prasanna and Varma 1977). There it was found that the presence of magnetic field increases the range of stable orbits. It is known that almost all of the celestial bodies do have a non-zero angular momentum and thus the Schwarzschild geometry does not tell the whole story. Thus it is necessary to extend the studies of the charged particle orbits to the case of Kerr background with an electromagnetic field. The electromagnetic fields that we consider are of two types:

(i) a dipole magnetic field with an associated (induced) quadrupole electric field due to the rotation of the central star; (higher multipoles) $\gtrsim O(a^2)$ are assumed to be negligible.

(ii) a uniform magnetic field surrounding the central star, which is constant at infinity.

2. Field structure

The space-time metric for the external field of a rotating star of mass M and angular velocity a is given by the Kerr solution which in Boyer-Lindquist coordinates may be written as,

$$ds^2 = - \left(1 - \frac{2mr}{\Sigma} \right) c^2 dt^2 - \frac{4mra}{\Sigma} \sin^2 \theta dt d\varphi + (\Sigma/\Delta) dr^2 + \Sigma d\theta^2 + (B/\Sigma) \sin^2 \theta d\varphi^2, \quad (1)$$

where

$$\Sigma = (r^2 + a^2 \cos^2 \theta), \quad \Delta = (r^2 + a^2 - 2mr), \quad m = \frac{MG}{c^2}$$

$$B = (r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta.$$

Case 1.

Several authors have obtained the solution for a stationary axisymmetric electromagnetic field around a rotating black hole by using Teukolsky's perturbation equations in the Newman-Penrose formalism (Chitre and Vishveshwara 1975; Petterson 1975; King *et al* 1975). We shall use the expression for the vector potential as given by Petterson restricting ourselves to the case of a dipole magnetic field with no electrostatic charge ($Q = 0$) which is given by

$$A_i = (A_r, 0, 0, A_\varphi) \quad (2)$$

$$A_r = \left(\frac{a\beta_1^t}{\Sigma} \right) \left\{ [r(r-m) + (a^2 - mr)\cos^2 \theta] \frac{1}{2\gamma} \ln \left(\frac{r-m+\gamma}{r-m-\gamma} \right) - (r-m\cos^2 \theta) \right\} - \left(\frac{\beta_1^r \cos \theta}{\Sigma} \right) \left\{ (a^2 \sin^2 \theta - r^2 + mr) + [(r^3 - 2mr^2 + ma^2) + (r-m)a^2 \cos^2 \theta] \frac{1}{2\gamma} \ln \left(\frac{r-m+\gamma}{r-m-\gamma} \right) \right\}. \quad (3)$$

$$A_\varphi = \left(\frac{\beta_1^t \sin^2 \theta}{2\Sigma} \right) \left\{ r(r^2 + mr + 2a^2) + (r-m)a^2 \cos^2 \theta - [r(r^3 - 2ma^2 + a^2r) + \Delta a^2 \cos^2 \theta] \frac{1}{2\gamma} \ln \left(\frac{r-m+\gamma}{r-m-\gamma} \right) \right\} + \left(\frac{a\beta_1^r \sin^2 \theta \cos \theta}{\Sigma} \right) \left\{ (a^2 + mr) + \frac{m(a^2 - r^2)}{2\gamma} \ln \left(\frac{r-m+\gamma}{r-m-\gamma} \right) \right\}. \quad (4)$$

with $\gamma = (m^2 - a^2)^{1/2}$, β_1^r , and β_1^t being constants.

From previous work (Prasanna and Varma 1977) concerning dipole magnetic field on the Schwarzschild background, we know that in the absence of rotation ($a = 0$) there is no electric field and $A_t = 0$. This requires immediately the constant $\beta_1^r = 0$. In order to determine β_1^t , we consider the boundary condition, that the Kerr geometry is asymptotically flat and thus as $r \rightarrow \infty$ the magnetic field and the electric field should have the structure of dipole and quadrupole fields on flat spacetime respectively. Accordingly we get,

$$\beta_1^t = \mp (3\mu/2\gamma^2) \tag{5}$$

depending on whether the dipole moment μ is parallel or antiparallel to the rotation axis. However in our study we shall consider only the case with $\beta_1^t = -3\mu/2\gamma^2$. Hence we get the components of the vector potential to be

$$A_t = \left(-\frac{3a\mu}{2\gamma^2\Sigma} \left\{ [r(r-m) + (a^2 - mr) \cos^2 \theta] \cdot \frac{1}{2\gamma} \ln \left(\frac{r-m+\gamma}{r-m-\gamma} \right) - (r-m \cos^2 \theta) \right\} \right) \tag{6}$$

$$A_\phi = \left(-\frac{3\mu \sin^2 \theta}{4\gamma^2\Sigma} \right) \left\{ (r-m)a^2 \cos^2 \theta + r(r^2 + mr + 2a^2) - [r(r^3 - 2ma^2 + a^2r) + \Delta a^2 \cos^2 \theta] \frac{1}{2\gamma} \ln \left(\frac{r-m+\gamma}{r-m-\gamma} \right) \right\} \tag{7}$$

Case 2. Uniform magnetic field

Wald (1974) has studied the case of a black hole in a uniform magnetic field, by using the fact that a Killing vector in an empty space time serves as a vector potential for a Maxwell test field. He has derived the solution for the electromagnetic field when a stationary, axisymmetric black hole is placed in an originally uniform magnetic field aligned along the symmetry axis of the black hole. We shall now use this solution for studying the charged particle orbits. We shall again assume that there is no electrostatic charge ($Q = 0$). This would give us the vector potential to be

$$A_i = (B_0/2) \left(\psi_i + \frac{2J}{M} \eta_i \right), \tag{8}$$

where ψ_i is the space-like and η_i the time-like Killing vectors. As the background geometry is that of Kerr we get the explicit solution as

$$A_t = -aB_0 \left[1 - \frac{mr}{\Sigma} (2 - \sin^2 \theta) \right]$$

$$A_\phi = \frac{B_0 \sin^2 \theta}{2\Sigma} \{ (r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta - 4ma^2r \} \tag{9}$$

B_0 being the magnetic field strength.

3. Equations of motion

As both the gravitational and the electromagnetic fields are axisymmetric and stationary, there exist two Killing vectors such that for the motion of a charged

particle of charge e and rest mass M_0 , there are two constants of motion, the canonical angular momentum l and energy E as given by

$$(U_\varphi + eA_\varphi) = l, \quad (10)$$

$$(U_t + eA_t) = -E, \quad (11)$$

wherein all quantities are normalised with respect to the particle rest mass M_0 . U_t and U_φ are connected with the components of particles proper velocity U^i as follows:

$$\begin{aligned} U^r &= -[BU_t + 2amrU_\varphi]/\Sigma\Delta \\ U^\varphi &= [(\Sigma - 2mr)U_\varphi/\sin^2\theta - 2amrU_t]/\Sigma\Delta. \end{aligned} \quad (12)$$

Using these in the general equations of motion of a charged particle in the presence of an external force field

$$U^b \nabla_b U_a = eF_{ab} U^b, \quad (13)$$

with $F_{ab} = (A_{b,a} - A_{a,b})$.

We get the explicit equations:

$$\begin{aligned} &\frac{d^2r}{ds^2} - \frac{[m(r^2 - a^2 \cos^2 \theta) - ra^2 \sin^2 \theta]}{\Sigma \Delta} \left(\frac{dr}{ds}\right)^2 \\ &- \frac{2a^2 \sin \theta \cos \theta}{\Sigma} \frac{dr}{ds} \frac{d\theta}{ds} - \frac{r\Delta}{\Sigma} \left(\frac{d\theta}{ds}\right)^2 + \frac{m\Delta}{\Sigma^3} (r^2 - a^2 \cos^2 \theta) \left(\frac{dt}{ds}\right)^2 \\ &- \frac{\Delta \sin^2 \theta}{\Sigma^3} \{r^5 + 2r^3 a^2 \cos^2 \theta - mr^2 a^2 \sin^2 \theta + (m-r)a^4 \sin^2 \theta \cos^2 \theta \\ &+ ra^4 \cos^2 \theta\} \left(\frac{d\varphi}{ds}\right)^2 \\ &- \frac{2\Delta am \sin^2 \theta (r^2 - a^2 \cos^2 \theta)}{\Sigma^3} \frac{d\varphi}{ds} \frac{dt}{ds} \\ &= \frac{e}{M_0} \frac{\Delta}{\Sigma} \left\{ A_{\varphi,r} \frac{d\varphi}{ds} + A_{t,r} \frac{dt}{ds} \right\}, \end{aligned} \quad (14)$$

$$\begin{aligned} &\frac{d^2\theta}{ds^2} + \frac{a^2 \sin \theta \cos \theta}{\Sigma \Delta} \left(\frac{dr}{ds}\right)^2 + \frac{2r}{\Sigma} \frac{dr}{ds} \frac{d\theta}{ds} - \frac{a^2 \sin \theta \cos \theta}{\Sigma} \left(\frac{d\theta}{ds}\right)^2 \\ &+ \frac{4mra(r^2 + a^2)}{\Sigma^3} \sin \theta \cos \theta \frac{d\varphi}{ds} \frac{dt}{ds} - \frac{2mra^2}{\Sigma^3} \sin \theta \cos \theta \left(\frac{dt}{ds}\right)^2 \\ &- \frac{\sin \theta \cos \theta}{\Sigma^3} [(r^2 + a^2)^3 - (r^2 + a^2 + \Sigma) \Delta a^2 \sin^2 \theta] \left(\frac{d\varphi}{ds}\right)^2 \\ &= \frac{e}{M_0} \frac{1}{\Sigma} \left\{ A_{\varphi,\theta} \frac{d\varphi}{ds} + A_{t,\theta} \frac{dt}{ds} \right\}, \end{aligned} \quad (15)$$

$$\frac{d\varphi}{ds} = \frac{1}{\Delta \sin^2 \theta} \left\{ \left(1 - \frac{2mr}{\Sigma}\right) (l - A_\varphi) + 2mra \sin^2 \theta (E + A_t)/\Sigma \right\} \quad (16)$$

$$\frac{dt}{ds} = \frac{1}{\Delta} \left\{ -\frac{2mra}{\Sigma} (l - A_\varphi) + [(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta] \sin^2 \theta (E + A_t)/\Sigma \right\}. \quad (17)$$

Introducing dimensionless quantities

$$\begin{aligned} \rho &= r/m, & \sigma &= s/m, & \alpha &= a/m, \\ L &= l/m, & \tau &= ct/m, & \bar{A}_\varphi &= A_\varphi/m, \end{aligned} \quad (18)$$

and rewriting the above equations we get:

$$\begin{aligned} \frac{d^2\rho}{d\sigma^2} &= \frac{[\rho^2 - \alpha^2 \cos^2\theta - \rho\alpha^2 \sin^2\theta]}{\Sigma \Delta} \left(\frac{d\rho}{d\sigma}\right)^2 - \frac{\rho \Delta}{\Sigma} \left(\frac{d\theta}{d\sigma}\right)^2 \\ &- \frac{2\alpha^2 \sin\theta \cos\theta}{\Sigma} \left(\frac{d\rho}{d\sigma}\right) \left(\frac{d\theta}{d\sigma}\right) - \frac{\Delta}{\Sigma^3} (\rho^2 - \alpha^2 \cos^2\theta) \left(\frac{d\tau}{d\sigma}\right)^2 \\ &- 2 \frac{\Delta \alpha \sin^2\theta}{\Sigma^3} (\rho^2 - \alpha^2 \cos^2\theta) \left(\frac{d\varphi}{d\sigma}\right) \left(\frac{d\tau}{d\sigma}\right) \end{aligned} \quad (19)$$

$$\begin{aligned} &- \frac{\Delta \sin^2\theta}{\Sigma^3} \{ \rho^5 + 2\rho^3 \alpha^2 \cos^2\theta - \rho^2 \alpha^2 \sin^2\theta + (1-\rho)\alpha^4 \sin^2\theta \cos^2\theta \\ &+ \rho \alpha^4 \cos^2\theta \} \left(\frac{d\varphi}{d\sigma}\right)^2 \\ &= \left(\frac{\Delta}{\Sigma}\right) \left\{ \frac{d\bar{A}_\varphi}{d\rho} \frac{d\varphi}{d\sigma} + \frac{dA_\tau}{d\rho} \frac{d\tau}{d\sigma} \right\} \cdot \frac{d^2\theta}{d\sigma^2} \\ &+ \frac{\alpha^2 \sin\theta \cos\theta}{\Sigma \Delta} \left(\frac{d\rho}{d\sigma}\right)^2 + \frac{2\rho}{\Sigma} \left(\frac{d\rho}{d\sigma}\right) \left(\frac{d\theta}{d\sigma}\right) \\ &- \frac{\alpha^2 \sin\theta \cos\theta}{\Sigma} \left(\frac{d\theta}{d\sigma}\right)^2 - \frac{2\rho \alpha^2 \sin\theta \cos\theta}{\Sigma^3} \left(\frac{d\tau}{d\sigma}\right)^2 \\ &+ \frac{4\rho \alpha (\rho^2 + \alpha^2)}{\Sigma^3} \sin\theta \cos\theta \left(\frac{d\varphi}{d\sigma}\right) \left(\frac{d\tau}{d\sigma}\right) - \frac{\sin\theta \cos\theta}{\Sigma^3} [(\rho^2 + \alpha^2)^3 \\ &- (\rho^2 + \alpha^2 + \Sigma) \Delta \alpha^2 \sin^2\theta] \left(\frac{d\varphi}{d\sigma}\right)^2 \\ &= \frac{1}{\Sigma} \left(\frac{d\bar{A}_\varphi}{d\theta} \frac{d\varphi}{d\sigma} + \frac{dA_\tau}{d\theta} \frac{d\tau}{d\sigma} \right) \end{aligned} \quad (20)$$

$$\frac{d\varphi}{d\sigma} = \left(\frac{1}{\Delta \Sigma} \right) \{ (\Sigma - 2\rho) (L - \bar{A}_\varphi) / \sin^2\theta + 2\rho \alpha (E + A_\tau) \}. \quad (21)$$

$$\frac{d\tau}{d\sigma} = \left(\frac{1}{\Delta \Sigma} \right) \{ [(\rho^2 + \alpha^2)^2 - \Delta \alpha^2 \sin^2\theta] (E + A_\tau) - 2\rho \alpha (L - \bar{A}_\varphi) \}. \quad (22)$$

wherein A_τ and \bar{A}_φ are taken appropriate to the case under consideration.

4. Motion in the equatorial plane

The motion in the equatorial plane may be characterised by the equations of motion obtained earlier for the particular θ value, $\theta = \pi/2$, $(d\theta/d\sigma) = 0$. The motion is completely determined by L , E and the space-time metric (1). Using (10) and (11) in (8) we get:

$$(U^\rho)^2 \equiv (d\rho/d\sigma)^2 = 1/\rho^3 \{ [\rho(\rho^2 + \alpha^2) + 2\alpha^2] (E + A_\tau)^2 - 4\alpha(E + A_\tau) (L - \bar{A}_\varphi) - (\rho - 2) (L - \bar{A}_\varphi)^2 - \rho \Delta \}. \tag{23}$$

Following Bardeen (1972) we define the effective potential for ρ -motion by solving for the turning points of the orbits given by $U^\rho = 0$. Thus we get

$$V_{\text{eff}} = E_{\text{min}} = -A_\tau + K/R, \tag{24}$$

with

$$K = [2\alpha (L - \bar{A}_\varphi) + \Delta^{1/2} \{ \rho^2 (L - \bar{A}_\varphi)^2 + \rho R \}^{1/2}]$$

$$R = (\rho^3 + \alpha^2 \rho + 2\alpha^2), \quad \Delta = (\rho^2 - 2\rho + \alpha^2) \tag{25}$$

Using the expressions for A_τ and \bar{A}_φ for the two different cases we can write $V_{\text{eff}} = F(\rho)$ as a function of ρ and study the effective potential curves.

Case 1

As given in section 2, we have for this case the vector potential A_i from (6) and (7) expressed in terms of ρ, σ, \dots by

$$A_\tau = \left(\frac{-3\lambda\alpha}{4(1-\alpha^2)^{3/2}} \right) \left\{ \left(1 - \frac{1}{\rho} \right) \ln \left(\frac{\rho - 1 + \sqrt{1-\alpha^2}}{\rho - 1 - \sqrt{1-\alpha^2}} \right) - (2\sqrt{1-\alpha^2})/\rho \right\} \tag{26}$$

$$\bar{A}_\varphi = \left(\frac{-3\lambda}{8(1-\alpha^2)^{3/2}} \right) \left\{ 2(\sqrt{1-\alpha^2}) (1 + \rho + (2\alpha^2/\rho)) - \left(\rho^2 + \alpha^2 - \frac{2\alpha^2}{\rho} \right) \ln \left(\frac{\rho - 1 + \sqrt{1-\alpha^2}}{\rho - 1 - \sqrt{1-\alpha^2}} \right) \right\}, \tag{27}$$

where $\lambda = e\mu/m^2$.

Figures 1 to 7 show the plots of F as a function of ρ for different values of α, λ and L . From the expressions for A_τ and \bar{A}_φ , it is apparent that both of them have logarithmic singularities at $\rho = 1 + \sqrt{1-\alpha^2}$, the event horizon. This reflects in the

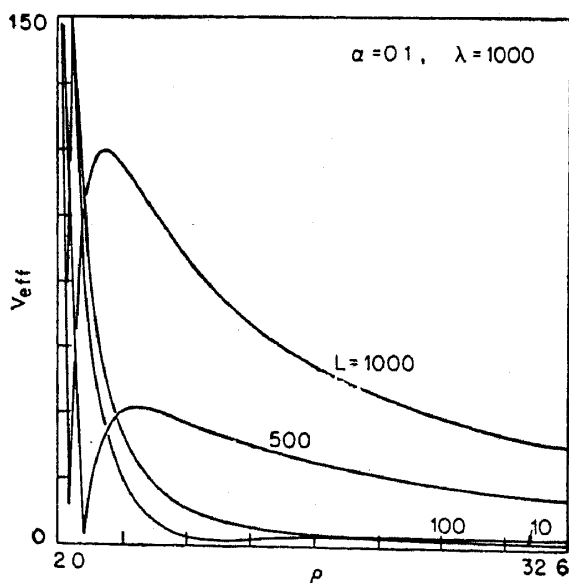


Figure 1.

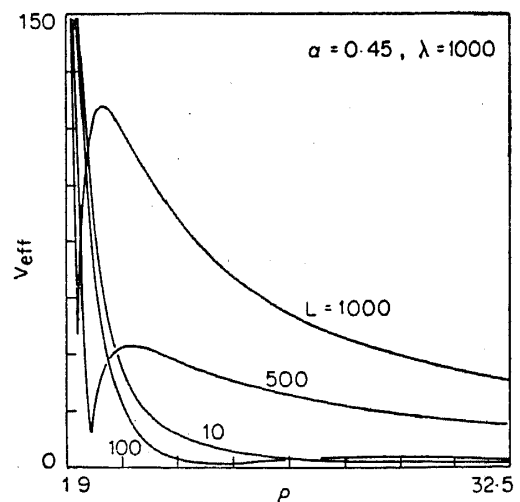


Figure 2.

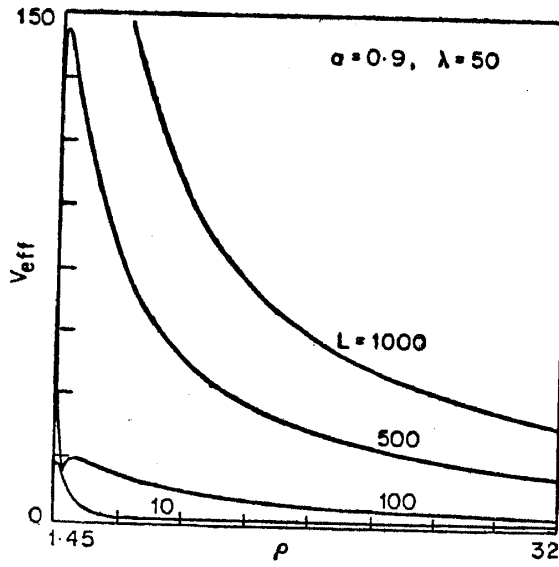


Figure 3.

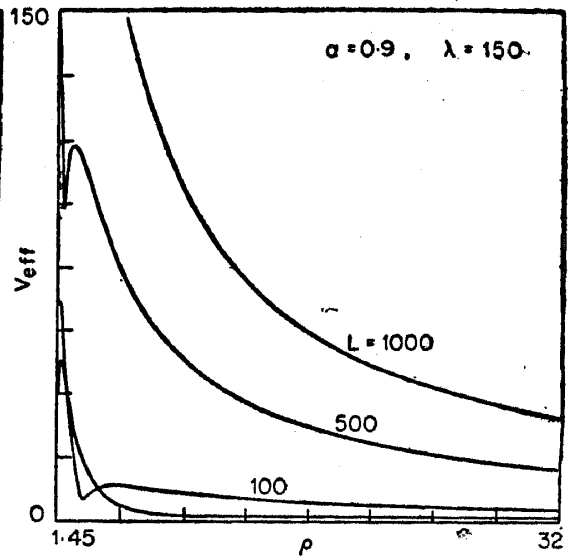


Figure 4.

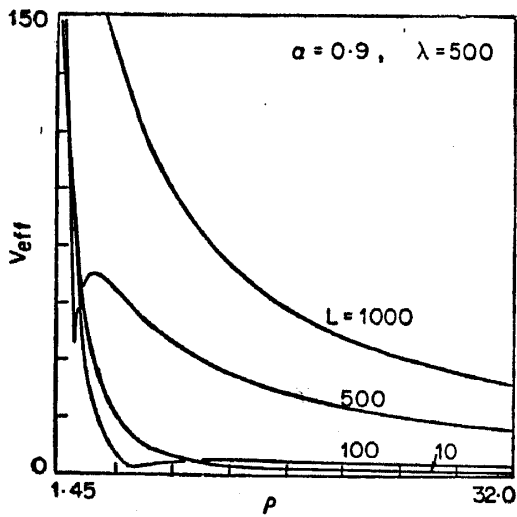


Figure 5.

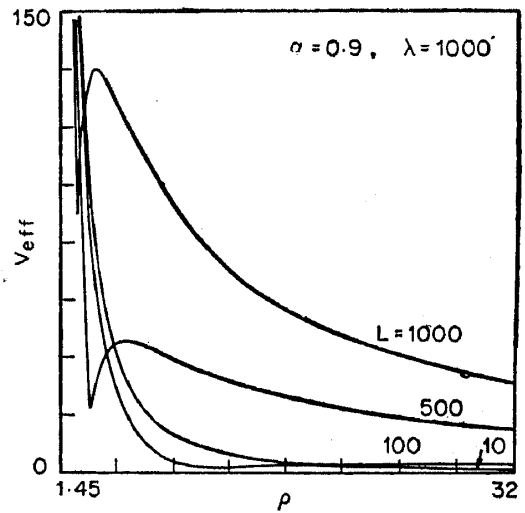


Figure 6.

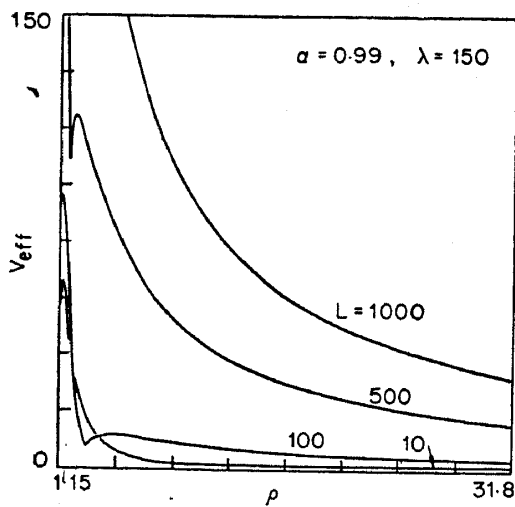


Figure 7.

Figures 1-7. Plots of effective potential as a function of ρ for different values of α , λ and L for the case of dipole magnetic field.

effective potential having a maximum very near the event horizon. The V_{eff} , then decreases sharply to a minimum and then rises to a second maximum which is indeed the centrifugal barrier (increases with increasing L) after which it smoothens out towards a flat minimum at large distances.

Table 1. $\alpha = 0.1$ $\bar{\rho} = 1.99508$

| λ | L | M_1 | ρ_{M_1} | m | ρ_m | M_2 | ρ_{M_2} |
|-----------|------|---------|----------------|----------|----------------|---------|--------------|
| 50 | 10 | 12.8192 | 2.08423 | 0.842611 | 6.28404 | 1.01198 | 16.8720 |
| | 100 | 8.59854 | 2.01456 | 1.84804 | 2.18804 | 14.4595 | 3.76862 |
| | 500 | | $< \bar{\rho}$ | 11.6013 | 1.99559 | 93.1164 | 3.04509 |
| | 1000 | | $< \bar{\rho}$ | | $< \bar{\rho}$ | 193.034 | 2.96226 |
| 150 | 10 | 40.0452 | 2.09604 | 0.936031 | 14.9605 | — | > 30 |
| | 100 | 33.7370 | 2.05342 | 1.24632 | 3.08303 | 9.21090 | 5.71800 |
| | 500 | 23.0221 | 2.00229 | 9.73257 | 2.05606 | 81.4313 | 3.39620 |
| | 1000 | 26.5905 | 1.99573 | 22.2353 | 2.00061 | 179.838 | 3.13020 |
| 300 | 10 | 80.9224 | 2.09926 | 0.965100 | 26.7526 | — | > 30 |
| | 100 | 73.9496 | 2.07384 | 1.02406 | 4.54706 | 5.98416 | 8.70652 |
| | 500 | 54.3433 | 2.02037 | 7.35614 | 2.26613 | 68.3250 | 3.95643 |
| | 1000 | 46.0439 | 2.00229 | 19.2934 | 2.05606 | 162.859 | 3.39615 |
| 500 | 10 | 135.433 | 2.10058 | — | > 30 | — | > 30 |
| | 100 | 128.176 | 2.08417 | 0.96063 | 6.53117 | 4.12090 | 12.7139 |
| | 500 | 103.591 | 2.03879 | 5.32077 | 2.61399 | 56.1497 | 4.72990 |
| | 1000 | 85.9785 | 2.01455 | 15.8164 | 2.18795 | 144.429 | 3.76680 |
| 1000 | 10 | 271.714 | 2.10159 | — | > 30 | — | > 30 |
| | 100 | 264.238 | 2.09291 | 0.948504 | 11.5165 | 2.42389 | 22.7725 |
| | 500 | 235.184 | 2.06276 | 3.01195 | 3.56470 | 38.7794 | 6.70403 |
| | 1000 | 207.182 | 2.03879 | 10.1563 | 2.61391 | 112.292 | 4.72980 |

Table 2. $\alpha = 0.45$ $\bar{\rho} = 1.893128$

| λ | L | M_1 | ρ_{M_1} | m | ρ_m | M_2 | ρ_{M_2} |
|-----------|------|---------|--------------|----------|----------------|---------|----------------|
| 50 | 10 | 12.8915 | 2.00838 | 0.890279 | 6.47308 | 1.01526 | 15.8422 |
| | 100 | 16.0621 | 1.91819 | 7.32076 | 2.13430 | 15.9340 | 3.34908 |
| | 500 | 55.4341 | 1.89328 | 54.4258 | 1.89491 | 108.095 | 2.58653 |
| | 1000 | 226.193 | 2.50157 | | $< \bar{\rho}$ | | $< \bar{\rho}$ |
| 150 | 10 | 38.8490 | 2.02607 | 0.944468 | 15.4044 | — | 30 |
| | 100 | 39.2957 | 1.96595 | 3.50699 | 3.07200 | 9.67194 | 5.36061 |
| | 500 | 62.9607 | 1.90344 | 43.9813 | 1.98042 | 91.6956 | 2.95633 |
| | 1000 | 111.207 | 1.89454 | 103.542 | 1.90469 | 206.972 | 2.67530 |
| 300 | 10 | 77.8445 | 2.03104 | 0.967891 | 27.4247 | — | > 30 |
| | 100 | 77.3244 | 1.99352 | 1.97846 | 4.55540 | 6.14818 | 8.38249 |
| | 500 | 90.4896 | 1.92509 | 31.8993 | 2.21900 | 74.6853 | 3.54696 |
| | 1000 | 125.921 | 1.90344 | 87.7699 | 1.98036 | 183.388 | 2.95630 |
| 500 | 10 | 129.850 | 2.03310 | — | > 30 | — | > 30 |
| | 100 | 128.898 | 2.00831 | 1.40363 | 6.55359 | 4.18971 | 12.4048 |
| | 500 | 135.665 | 1.94740 | 22.0918 | 2.58623 | 59.9668 | 4.35004 |
| | 1000 | 160.615 | 1.91818 | 70.4128 | 2.13311 | 159.191 | 3.34792 |
| 1000 | 10 | 259.873 | 2.03466 | — | > 30 | — | > 30 |
| | 100 | 258.577 | 2.02131 | 1.40363 | 6.55359 | 2.44318 | 22.4631 |
| | 500 | 259.138 | 1.97830 | 22.0918 | 2.58623 | 59.9668 | 6.36417 |
| | 1000 | 271.330 | 1.94740 | 43.6967 | 2.58581 | 119.926 | 4.34997 |

Table 3.

$\alpha = 0.9$

$\bar{\rho} = 1.435989$

| λ | L | M_1 | ρ_{M_1} | m | ρ_m | M_2 | ρ_{M_2} |
|-----------|------|---------|--------------|----------|----------|---------|--------------|
| 50 | 10 | 17.1867 | 1.59150 | 0.945115 | 6.77408 | 1.02038 | 14.5054 |
| | 100 | 36.2314 | 1.49113 | 14.3917 | 1.97054 | 18.4438 | 2.75584 |
| | 500 | 145.172 | 1.44383 | 128.928 | 1.49905 | 145.060 | 1.79727 |
| | 1000 | 293.991 | 1.43758 | 285.360 | 1.44918 | 316.975 | 1.66985 |
| 150 | 10 | 48.3268 | 1.61315 | 0.954271 | 16.0461 | — | >30 |
| | 100 | 64.4536 | 1.54413 | 6.25395 | 3.03131 | 10.3195 | 4.89495 |
| | 500 | 158.361 | 1.47070 | 91.9033 | 1.73788 | 111.455 | 2.29320 |
| | 1000 | 293.124 | 1.45100 | 234.375 | 1.55799 | 268.644 | 1.92509 |
| 300 | 10 | 95.1208 | 1.61951 | 0.971178 | 28.35334 | — | >30 |
| | 100 | 110.039 | 1.57438 | 3.15455 | 4.55925 | 6.36627 | 7.96648 |
| | 500 | 193.913 | 1.49940 | 62.9132 | 2.07667 | 85.0729 | 2.97920 |
| | 1000 | 316.722 | 1.47070 | 183.554 | 1.73757 | 222.907 | 2.29320 |
| 500 | 10 | 157.530 | 1.62218 | — | >30 | — | >30 |
| | 100 | 171.859 | 1.59147 | 1.95629 | 6.58202 | 4.27962 | 12.0086 |
| | 500 | 249.071 | 1.52421 | 42.5962 | 2.50276 | 65.6272 | 3.84514 |
| | 1000 | 362.312 | 1.49112 | 140.839 | 1.96584 | 184.298 | 2.75563 |
| 1000 | 10 | 313.567 | 1.62423 | — | >30 | — | >30 |
| | 100 | 327.401 | 1.60720 | 1.25833 | 11.6342 | 2.46823 | 22.0653 |
| | 500 | 397.422 | 1.55751 | 21.1167 | 3.52571 | 42.4210 | 5.92552 |
| | 1000 | 498.141 | 1.52421 | 84.7074 | 2.50169 | 131.247 | 3.84513 |

Table 4.

$\alpha = 0.99$

$\bar{\rho} = 1.141167$

| λ | L | M_1 | ρ_{M_1} | m | ρ_m | M_2 | ρ_{M_2} |
|-----------|------|---------|--------------|----------|----------|---------|--------------|
| 50 | 10 | 22.7096 | 1.28234 | 0.955089 | 6.84573 | 1.02155 | 14.2328 |
| | 100 | 53.6887 | 1.21282 | 15.7641 | 1.92745 | 19.0441 | 2.63176 |
| | 500 | 207.965 | 1.16871 | 149.605 | 1.36570 | 158.687 | 1.58328 |
| | 1000 | 410.321 | 1.15670 | 345.581 | 1.26304 | 358.907 | 1.40367 |
| 150 | 10 | 61.9672 | 1.29757 | 0.956090 | 16.1857 | — | >30 |
| | 100 | 90.8909 | 1.25012 | 6.76826 | 3.02038 | 10.4554 | 4.80183 |
| | 500 | 234.881 | 1.19658 | 101.916 | 1.67119 | 116.721 | 2.14703 |
| | 1000 | 426.653 | 1.17774 | 266.489 | 1.45206 | 288.604 | 1.73914 |
| 300 | 10 | 120.921 | 1.30215 | 0.971808 | 28.5487 | — | >30 |
| | 100 | 148.937 | 1.27062 | 3.37956 | 4.55934 | 6.41064 | 7.88363 |
| | 500 | 286.134 | 1.21896 | 68.8432 | 2.03911 | 87.4797 | 2.86203 |
| | 1000 | 469.762 | 1.19658 | 203.568 | 1.67075 | 233.439 | 2.14704 |
| 500 | 10 | 199.542 | 1.30409 | — | >30 | — | >30 |
| | 100 | 227.095 | 1.28234 | 2.06368 | 6.58777 | 4.29775 | 11.9296 |
| | 500 | 359.661 | 1.23651 | 46.4349 | 2.48079 | 66.8543 | 3.74335 |
| | 1000 | 536.886 | 1.21282 | 154.526 | 1.92139 | 190.298 | 2.63182 |
| 1000 | 10 | 396.103 | 1.30559 | — | >30 | — | >30 |
| | 100 | 423.254 | 1.29334 | 1.29227 | 11.6481 | 2.47327 | 21.9857 |
| | 500 | 550.610 | 1.25918 | 23.0041 | 3.61763 | 42.8539 | 5.83814 |
| | 1000 | 719.322 | 1.23651 | 92.3871 | 2.47952 | 133.701 | 3.74335 |

Asymptotically as $\rho \rightarrow \infty$ it may be seen that V_{eff} goes as $1 - (2/\rho) + a^2/\rho^2$, showing that it tends to 1 from below. Tables 1 to 4 give the actual values of the effective maxima (M_1 , M_2) and the minimum (m) and their locations ρ_{M_1} , ρ_m and ρ_{M_2} for different values of α , λ , and L , between $\rho = \bar{\rho} \equiv 1 + \sqrt{1 - \alpha^2} + 0.0001$, and $\rho = 30$.

For a given α and λ as L increases the M_2 increases continuously, whereas M_1 decreases for lower values of α (~ 0.1) but increases for other values of α (≥ 0.45). On the other hand, the effective minimum (m_1) increases as L increases for all α and λ and moves inwards in ρ .

When L is fixed, as λ increases, for all αM_1 increases whereas m_1 and M_2 keep decreasing monotonically. Thus as λ increases the potential well gets flatter and moves outward in ρ . If L and λ are fixed as α increases the entire potential well moves upwards in energy and inwards in ρ . In the equatorial plane we know that the ergosurface corresponds to $\rho=2$. From the figures and tables it is clear that for some set of parameters the entire potential well stays outside the ergosurface (e.g. $\alpha=0.1$, $\lambda > 300$) while in some other cases it stays completely within the ergosurface (e.g. $\alpha=0.9$, $L \geq 500$, $\lambda \leq 50$). For other set of values part of the potential well stays inside and part outside the ergosurface.

Case 2

The vector potential for the case of a black hole immersed in a uniform magnetic field as given in (11) expressed in dimensionless quantities take the form:

$$A_\tau = -\lambda\alpha(1-1/\rho), \quad \lambda = eB_0m \quad (28)$$

$$\bar{A}_\varphi = \frac{\lambda}{2} \left[\rho^2 + \alpha^2 \left(1 - \frac{2}{\rho} \right) \right]. \quad (29)$$

Substituting these in V_{eff} we get

$$V_{\text{eff}} = \lambda\alpha(1-1/\rho) + K/R \quad (30)$$

with K and R as defined earlier.

As we are taking α strictly less than 1, there is no singularity in the vector potential.

Asymptotically we find that

$$V_{\text{eff}} \approx 1 + \lambda\alpha + \left(\frac{\bar{A}_\varphi}{\rho} \right)^2 - \frac{\bar{A}_\varphi L}{\rho} \quad (31)$$

$$\approx 1 + \lambda\alpha + O(\rho^2) + \text{constant}.$$

Hence the effective potential increases as the square of the distance for large values of ρ . There are only two extrema (one maximum and one minimum) for the effective potential, which we have tabulated in tables 5 to 8 for different values of α , λ and L . As may be seen from the tables there always exist a potential well, but it is either completely away from the ergosurface ($\alpha=0.1$) or only a part of it lies within the ergosurface with the rest outside.

For a given α and λ as L increases the minimum moves outwards in ρ , whereas for fixed α and L as λ increases the minimum moves towards the event horizon. The same thing happens for fixed L and λ as α increases.

5. Orbits

In both the cases we have found existence of well defined potential wells. Naturally only those particles which are confined inside the potential well will have bound orbits. In order to get the actual orbits we integrate the set of equations :

Table 5. $\alpha = 0.1$ $\bar{\rho} = 1.99508$

| λ | L | M | ρ_M | m | ρ_m |
|-----------|------|---------|----------------|---------|----------------|
| 50 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | 69.0159 | 2.46470 | 4.62532 | 4.47162 |
| | 1000 | 166.155 | 2.61906 | 5.03626 | 6.32406 |
| 150 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | 30.1357 | 2.15137 | 9.66494 | 2.58173 |
| | 1000 | 114.066 | 2.35635 | 11.5640 | 3.65093 |
| 300 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 1000 | 60.2686 | 2.15136 | 18.8545 | 2.58165 |
| 500 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 1000 | 25.1003 | 1.99610 | 25.0477 | 2.00059 |
| 1000 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 1000 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |

Table 6. $\alpha = 0.45$ $\bar{\rho} = 1.893128$

| λ | L | M | ρ_M | m | ρ_m |
|-----------|------|---------|----------------|---------|----------------|
| 50 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | 12.1669 | 1.91115 | 11.4500 | 2.00754 |
| | 500 | 99.9600 | 2.20626 | 18.1985 | 4.46019 |
| | 1000 | 217.089 | 2.29252 | 19.7631 | 6.31386 |
| 150 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | 74.6676 | 2.01292 | 46.7548 | 2.57409 |
| | 1000 | 184.852 | 2.14186 | 49.6245 | 3.63929 |
| 300 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 1000 | 149.333 | 2.01292 | 83.0231 | 2.57367 |
| 500 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 1000 | 121.572 | 1.91044 | 112.691 | 2.00103 |
| 1000 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 1000 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |

$$\begin{aligned}
 & \frac{1}{\Delta} \frac{d^2 \rho}{d\sigma^2} - \frac{1}{\Delta^2} \left(1 - \frac{\alpha^2}{\rho}\right) \left(\frac{d\rho}{d\sigma}\right)^2 - \left(\frac{1}{\rho} - \frac{\alpha^2}{\rho^4}\right) \left(\frac{d\varphi}{d\sigma}\right)^2 \\
 & - \frac{2\alpha}{\rho^4} \left(\frac{d\varphi}{d\sigma}\right) \left(\frac{d\tau}{d\sigma}\right) + \frac{1}{\rho^4} \left(\frac{d\tau}{d\sigma}\right)^2 = \frac{1}{\rho^2} \left\{ \frac{d\bar{A}_\varphi}{d\rho} \frac{d\varphi}{d\sigma} + \frac{d\bar{A}_\tau}{d\rho} \frac{d\tau}{d\sigma} \right\}, \quad (32)
 \end{aligned}$$

$$\frac{d\phi}{d\sigma} = \frac{1}{\Delta} \left\{ \left(1 - \frac{2}{\rho}\right) (L - \bar{A}\phi) + \frac{2\alpha}{\rho} (E + A_\tau) \right\} \tag{33}$$

$$\frac{d\tau}{d\sigma} = \frac{1}{\Delta} \left\{ \left(\rho^2 + \alpha^2 + \frac{2\alpha^2}{\rho}\right) (E + A_\tau) - \frac{2\alpha}{\rho} (L - \bar{A}\phi) \right\}, \tag{34}$$

Table 7. $\alpha = 0.9$ $\bar{\rho} = 1.435989$

| λ | L | M | ρ_M | m | ρ_m |
|-----------|------|---------|----------------|---------|----------------|
| 50 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | 32.7177 | 1.47693 | 22.8575 | 2.00754 |
| | 500 | 172.774 | 1.53396 | 35.5685 | 4.42338 |
| | 1000 | 348.467 | 1.54506 | 38.6618 | 6.28128 |
| 150 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | 167.681 | 1.50034 | 82.5349 | 2.54942 |
| | 1000 | 342.790 | 1.52416 | 98.1962 | 3.60243 |
| 300 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | 161.858 | 1.46744 | 123.895 | 1.84617 |
| | 1000 | 335.362 | 1.50034 | 164.553 | 2.54873 |
| 500 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | 157.224 | 1.44032 | 151.016 | 1.51735 |
| | 1000 | 327.158 | 1.47686 | 225.354 | 2.00083 |
| 1000 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 1000 | 314.446 | 1.44032 | 302.013 | 1.51672 |

Table 8. $\alpha = 0.99$ $\bar{\rho} = 1.141167$

| λ | L | M | ρ_M | m | ρ_m |
|-----------|------|---------|----------------|---------|----------------|
| 50 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | 44.1595 | 1.16043 | 25.1348 | 2.00711 |
| | 500 | 221.784 | 1.16599 | 39.0247 | 4.41306 |
| | 1000 | 443.833 | 1.16680 | 42.4337 | 6.27213 |
| 150 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | 43.7661 | 1.15077 | 38.2329 | 1.35232 |
| | 500 | 221.273 | 1.16303 | 90.5889 | 2.54269 |
| | 1000 | 443.305 | 1.16521 | 107.831 | 3.59218 |
| 300 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | 43.4183 | 1.14232 | 42.7122 | 1.18158 |
| | 500 | 220.568 | 1.15922 | 136.592 | 1.84927 |
| | 1000 | 442.546 | 1.16303 | 180.653 | 2.54197 |
| 500 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | 219.729 | 1.15501 | 169.483 | 1.52132 |
| | 1000 | 441.590 | 1.16041 | 247.882 | 2.00077 |
| 1000 | 10 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 100 | — | $< \bar{\rho}$ | — | $< \bar{\rho}$ |
| | 500 | 218.084 | 1.14730 | 202.333 | 1.25899 |
| | 1000 | 439.458 | 1.15501 | 338.776 | 1.52055 |

with initial conditions

$$\begin{aligned}
 \rho &= \rho_0, \quad \varphi = \varphi_0 = 0 \\
 \left(\frac{d\rho}{d\sigma}\right)_0 &= \pm \left\{ \left(1 + \frac{\alpha^2}{\rho_0^2} + \frac{2\alpha^2}{\rho_0^3}\right) [E + (A_\tau)_0]^2 \right. \\
 &\quad - \frac{4\alpha}{\rho_0^3} [E + (A_\tau)_0] [L - (\bar{A}\varphi)_0] - \left(\frac{1}{\rho_0^2} - \frac{2}{\rho_0^3}\right) [L - (\bar{A}\varphi)_0]^2 \\
 &\quad \left. - \left(1 - \frac{2}{\rho_0} + \frac{\alpha^2}{\rho_0^2}\right) \right\} \\
 (A_\tau)_0 &= (A_\tau)_{\rho=\rho_0}, \quad (\bar{A}\varphi)_0 = (\bar{A}\varphi)_{\rho=\rho_0}.
 \end{aligned} \tag{35}$$

Figures 8 to 17 give the orbits for the case 1 whereas figures 18 to 26 give the orbits for the case 2. The most important point borne out in either case is that the particle executes Larmor motion (gyration) only outside the ergosurface. This is in fact the reflection of the effect of dragging of inertial frames by the rotating star. For we know that if the particle has to gyrate, then during every Larmor circle the particle angular velocity ($d\varphi/d\sigma$) will be prograde for one half and retrograde for the other half with respect to the angular velocity of the star. It is well known that in the Kerr geometry the ergosurface is the static limit surface on and behind (towards the event horizon) which no retrograde motion is possible. Thus the particle can gyrate only outside the ergosurface. This fact can also be seen analytically as follows. When a particle gyrates its ($d\rho/d\sigma$) has to go through zero for some $\rho = \rho_g$, for which $(d\rho/d\sigma)_{\rho=\rho_g}$ should be real and positive.

$$\begin{aligned}
 (d\varphi/d\sigma)_{\rho=\rho_g} &= 0 \text{ gives} \\
 (1 - 2/\rho_g) [L - (\bar{A}\varphi)_{\rho=\rho_g}] + \frac{2\alpha}{\rho_g} [E + (A_\tau)_{\rho=\rho_g}] &= 0.
 \end{aligned} \tag{36}$$

Using this in $(d\rho/d\sigma)_{\rho=\rho_g}^2$ we get

$$(d\rho/d\sigma)_{\rho=\rho_g}^2 = \frac{\Delta}{\rho_g^2} \left\{ (1 - 2/\rho_g)^{-1} [E + (A_\tau)_{\rho=\rho_g}]^2 - 1 \right\}.$$

Since $\Delta > 0$, in the region under consideration, $(d\rho/d\sigma)_{\rho=\rho_g}$ will be real if and only if

$$(1 - 2/\rho_g)^{-1} [E + (A_\tau)_{\rho=\rho_g}]^2 > 1 \tag{37}$$

This can be true only for $(1 - 2/\rho_g) > 0$, or $\rho_g > 2$, i.e. outside the ergosurface.

Another aspect of the dragging appears through the precession of the entire orbit for every revolution. Looking at the actual plots we see that as the magnetic field changes, for a given α and L , the nature of the orbit changes. For example, consider the dipole field (figures 8-12). When $\lambda = 1000, 500$ and 300 , both the turning points of the orbit lie outside $\rho = 2$ and thus there is gyration whereas when $\lambda = 150$, and 50 , both the turning points are inside $\rho = 2$ and thus there is no gyration. On the other hand, if we consider the cases of figures 13-17, here α, λ and L are kept constant and E alone varies. When E is large, (figures 13 and 14), the potential well is such that the particle moves in and out of the ergosurface without gyrating. As the energy is decreased it will be in the region of the potential well, which is completely

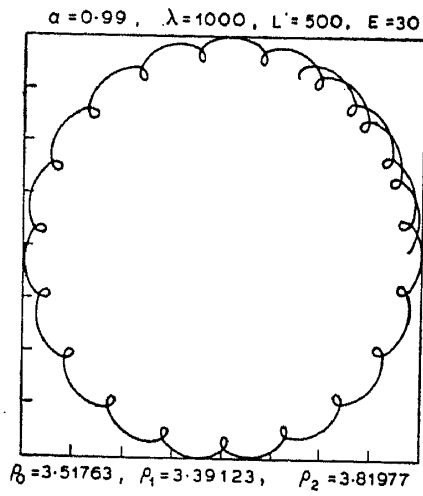


Figure 8.

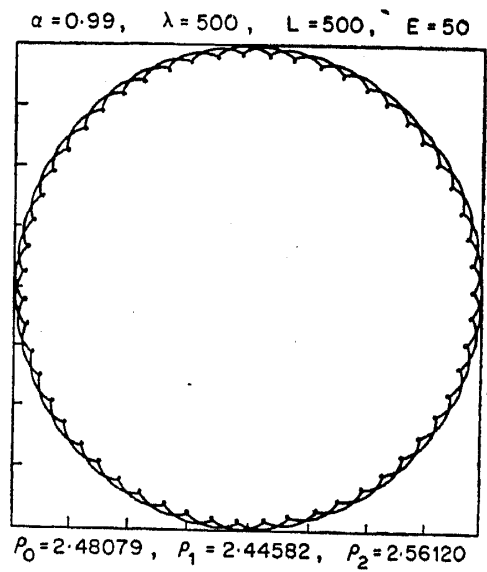


Figure 9.

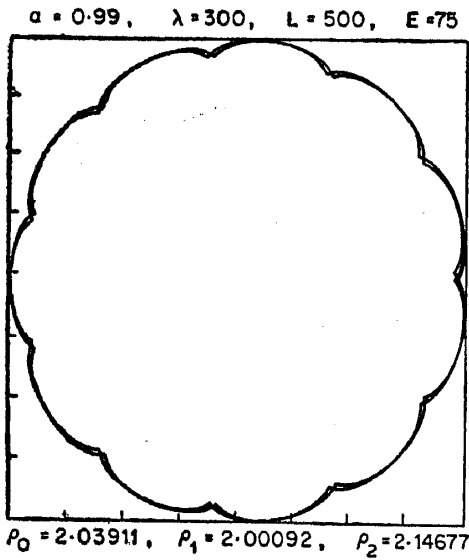


Figure 10.

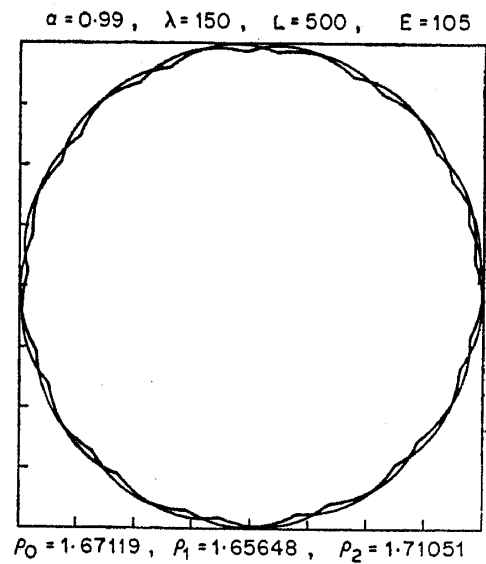


Figure 11.

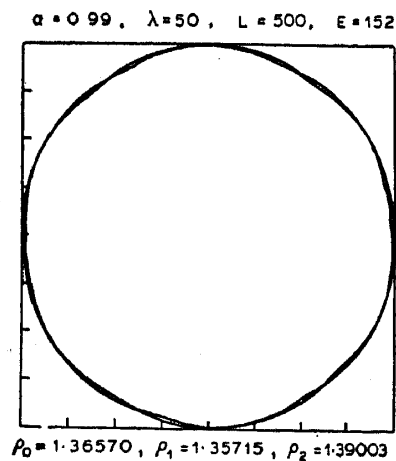


Figure 12.

Figures 8-12. Orbits in the equatorial plane for different values of λ but same α and L (dipole field).

outside $\rho = 2$ and thus the particle gyrates. As we go down in energy towards the potential minimum the Larmor circles get smaller and smaller. Figure 17 depicts the case when the energy is almost near the minimum and thus the orbit looks almost circular. At the potential minimum it is well known that the particle will have a stable circular orbit.

We have similar orbits for the case of the uniform magnetic field too. Here in the cases (figures 18 to 22) for the same α , L and E as λ changes from 50 to 1000 (increasing magnetic field) the motion is such that till $\lambda = 500$ the particle moves in and out of the ergosurface executing gyration while outside, and follows a purely prograde motion inside. For $\lambda = 1000$, the particle is completely within the ergosurface and thus it does not gyrate. Figures 23 to 25 depict the other aspect that for fixed α , λ and L as the energy changes the orbits change. At $E = 320$, the particle is moving in and out of the ergosurface while for $E = 200$ and 100 it stays completely outside the ergosurface executing fully Larmor motion.

6. Concluding remarks

In the foregoing we have considered two types of magnetic fields. The case of the uniform field could correspond to the galactic magnetic field surrounding the compact object. The dipole field considered could correspond to the intrinsic magnetic field of the compact object, if it is not a black hole. In such a case the vector potential is well behaved down to the surface of the star. In the case of a blackhole the dipole will have to be generated by external sources such as current rings exterior to the event horizon. The possible existence of these current rings has been pointed out by Petterson (1975). Our formalism applies to the region outside the current rings which could lie very close to the event horizon.

As had been found earlier in the case of dipole magnetic field on the Schwarzschild background, the essential role that the magnetic field plays is to stabilise the orbits.

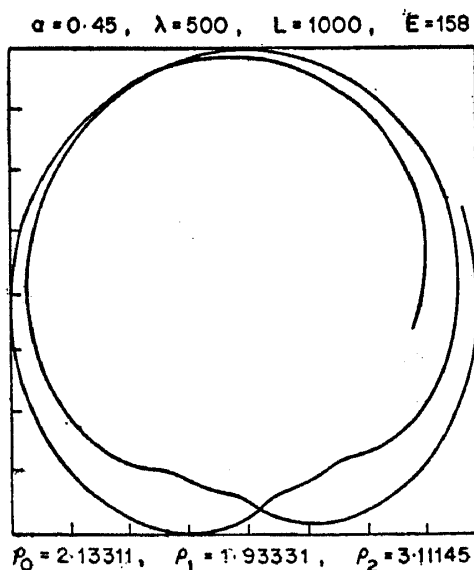


Figure 13.

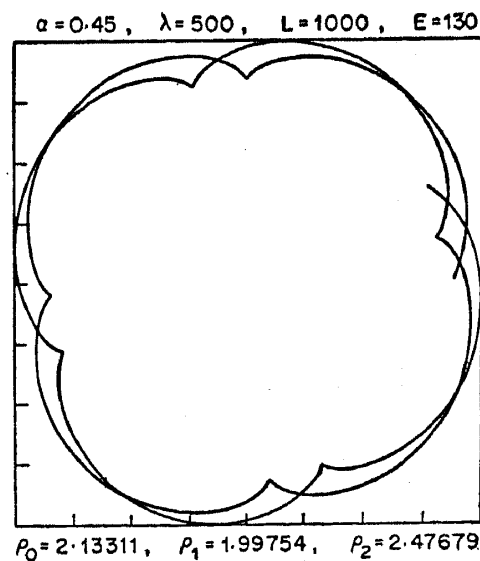


Figure 14.

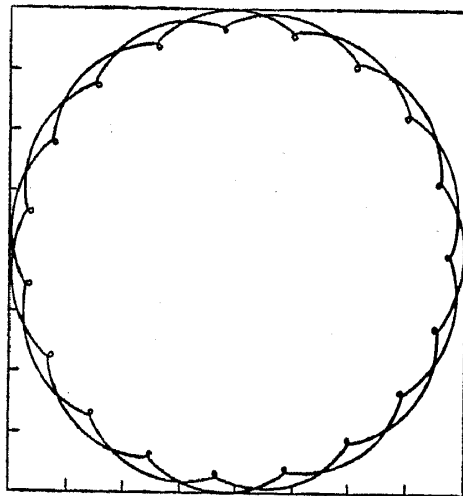
$\alpha = 0.45, \lambda = 500, L = 1000, E = 100$

 $\rho_0 = 2.13311, \rho_1 = 2.06173, \rho_2 = 2.26142$

Figure 15.

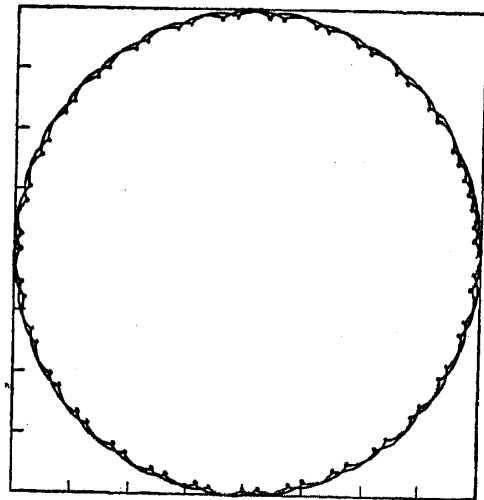
 $\alpha = 0.45, \lambda = 500, L = 1000, E = 80$

 $\rho_0 = 2.13311, \rho_1 = 2.10832, \rho_2 = 2.16992$

Figure 16.

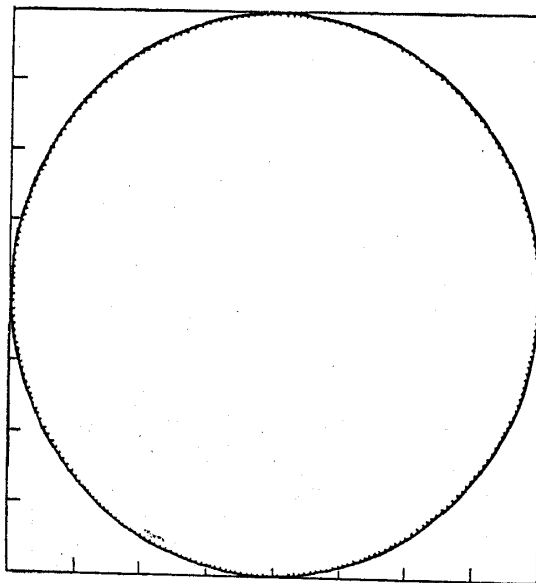
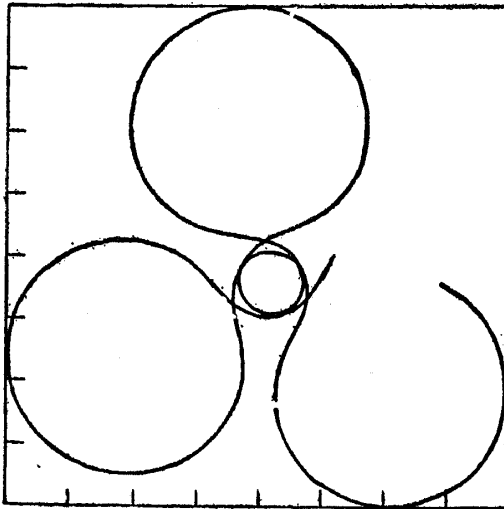
 $\alpha = 0.45, \lambda = 500, L = 1000, E = 72$

 $\rho_0 = 2.13311, \rho_1 = 2.12822, \rho_2 = 2.13963$

Figure 17.

Figures 13-17. Orbits for different energies but α, λ and L being the same (dipole field).

Existence of bound orbits depending on the structure of the potential well is similar to the earlier case. However, the important difference is the existence of non gyrating bound orbits within the ergosphere as a result of the dragging of inertial frames due to the rotation of the central star. Charged particles in the two different types of bound orbits—gyrating non-gyrating—could possibly generate significantly different radiation patterns. Further, the above study of orbits could also play an important role while considering plasma discs and particularly the question of transfer of angular momentum. The dynamics of the Penrose energy extraction process

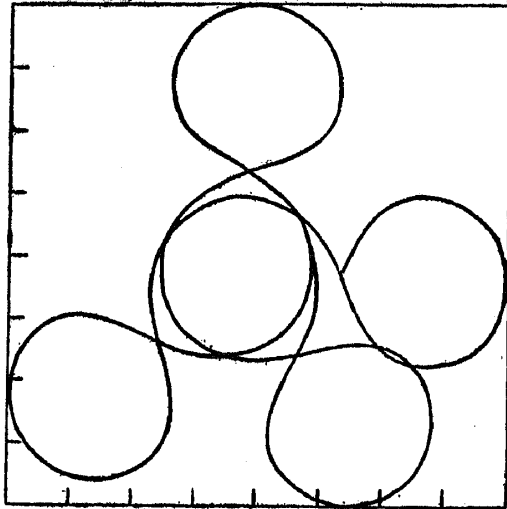
$\alpha = 0.99, \lambda = 50, L = 1000, E = 350$



$\rho_0 = 9.38640, \rho_1 = 1.79771, \rho_2 = 15.6497$

Figure 18.

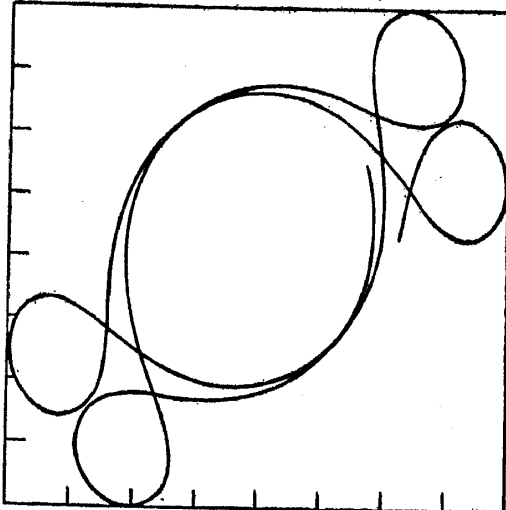
$\alpha = 0.99, \lambda = 150, L = 1000, E = 350$



$\rho_0 = 2.37979, \rho_1 = 1.72675, \rho_2 = 6.07672$

Figure 19.

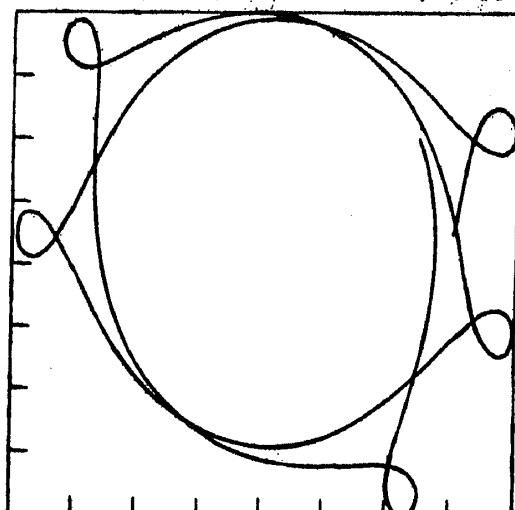
$\alpha = 0.99, \lambda = 300, L = 1000, E = 350$



$\rho_0 = 1.98156, \rho_1 = 1.65307, \rho_2 = 3.56260$

Figure 20.

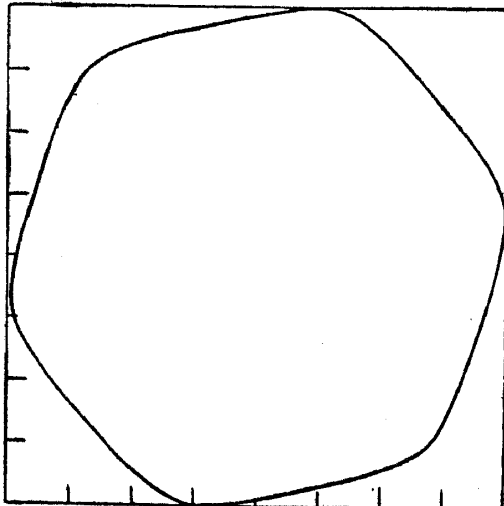
$\alpha = 0.99, \lambda = 500, L = 1000, E = 350$



$\rho_0 = 1.78721, \rho_1 = 1.58603, \rho_2 = 2.49805$

Figure 21.

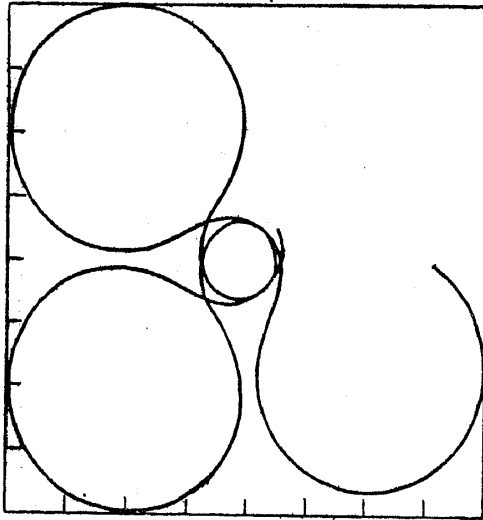
$\alpha = 0.99, \lambda = 1000, L = 1000, E = 350$



$\rho_0 = 1.54210, \rho_1 = 1.48561, \rho_2 = 1.60200$

Figures 18-22. Orbits for different values of λ but same values of L and E (uniform magnetic field).

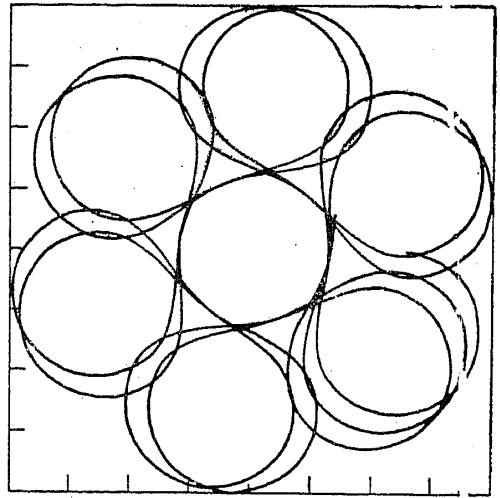
$$\alpha = 0.9, \lambda = 50, L = 1000, E = 320$$



$$\rho_0 = 10, \rho_1 = 1.90970, \rho_2 = 14.7571$$

Figure 23.

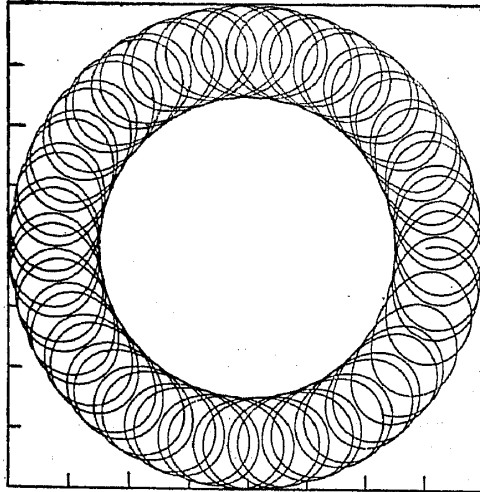
$$\alpha = 0.9, \lambda = 50, L = 1000, E = 200$$



$$\rho_0 = 6.5480, \rho_1 = 3.25721, \rho_2 = 10.8316$$

Figure 24.

$$\alpha = 0.9, \lambda = 50, L = 1000, E = 100$$

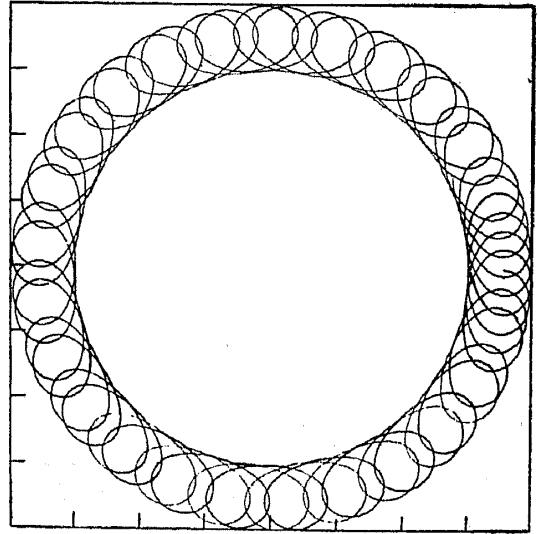


$$\rho_0 = 5.89400, \rho_1 = 4.92446, \rho_2 = 7.90786$$

Figure 25.

Figures 23-25. Orbits for different values of energy but same values of α , λ and L (uniform magnetic field).

$$\alpha = 0.9, \lambda = 150, L = 1000, E = 150$$



$$\rho_0 = 3.65400, \rho_1 = 3.13043, \rho_2 = 4.15418$$

Figure 26.

Figure 26. A typical plot of a gyrating orbit well outside the ergosurface (uniform magnetic field).

through injecting a particle into ergosphere should now be discussed with the help of the details of the possible orbits for different values of energy, angular momentum, magnetic field strength and the Kerr-angular velocity parameter a obtained in the above study.

Acknowledgements

It is a pleasure to thank Dr K S Rao for his valuable help in the numerical computations and Prof R K Varma for useful discussions. This work got started when

one of us (ARP) was visiting the Raman Research Institute and the Centre for Theoretical Studies, Indian Institute of Science, Bangalore, during the summer of 1977. He would like to thank Profs V Radhakrishnan and N Mukunda for the hospitality shown to him at their respective institutions.

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