A HYDRODYNAMICAL INVESTIGATION OF THE SUB-SOIL FLOW FROM CANAL BEDS BY MEANS OF MODELS.

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Some of the important applications of hydrodynamics to practical problems are the study of the effect of pumping and draining on the water-table in the sub-soil and the determination of the nature of percolation of water from beds, such as those of canals. These investigations are specially of importance in areas where irrigation is practised on a large scale, as for instance in the Punjab. In such areas where the rain and canal water enters the soil, there may be a gradual rise of water-table in the sub-soil and in the course of time, the water-table reaches a level where it begins to be injurious to the soil. There is no absolute depth which can be fixed for this stage, as it depends on a number of factors. Vast areas of canal-irrigated lands in the Punjab have gone out of cultivation as a result of the rise of the sub-soil water-table.

To counteract this rise of water-table, two of the methods employed are to pump out the sub-soil water and to adopt drainage schemes.

Pumping was tried as an experimental measure in the Punjab in the districts of Gujranwala and Ferozepur. But it was found that the radius of influence of the pump on the lowering of the water-table was small and pumping had no permanent effect on lowering the water-table. It was not possible, therefore, to adopt it as a scheme. Drainage schemes also exist on a small scale. Drains are sometimes cut parallel to the canals to carry away the seepage water that enters the soil from the bed of canals. They are also cut across the fields to drain away the water from the water-table and thus lower the same. The physical aspects of these problems have not so far been investigated. If these schemes are to be really effective, there are many points to be decided by investigations in the field and in models. As for example, the correct distances at which the drains are most effective, the increase in seepage caused by drains, the transmission coefficient of the soil where the drains are cut, the relation of the direction of seepage flow from the bed of canals to the sub-soil water gradient, and many other aspects have to be determined by investigations. Model and field experiments to decide these questions are among the schemes of research, undertaken in the Irrigation Research Institute at Lahore.

The present work is undertaken to study the effect of the gradient of the water-table in the sub-soil, on the direction of flow from the bed of canals and to test how far the conclusions of theoretical hydrodynamics can be applied to the same. The experiments were carried out in models in the Laboratory.

Experimental Arrangements.

The experimental arrangement consists of a glass tank $90 \times 60 \times 60$ cms. having three compartments. The central one is 60 cms. long and is to contain sand as a model of the sub-soil. The side compartments are each 15 cms. and are fitted with inlets and outlets for water. The inlet and outlet pipes could be so adjusted as to keep the required gradient of the water-table in the sand contained in the central compartment. The sand is packed under water and a small model of a canal section of base 3×4 cms., surface $7 \cdot 5 \times 4$ cms. and side 5 cms. is then embedded into the sand. This model section is made up of wire-gauze, to enable the water to percolate into the sand without disturbing the packing. The model section is seen in the centre of the photographs (Fig. 1).

After fixing the model in the sand, a slow steady stream of water is allowed to run into the model of the canal and a steady head is maintained in the same. The water percolates from the model through the sand and joins the water-table. This corresponds to the seepage taking place in the actual field. The water-table in the model is also maintained constant by suitable adjustments of the inlets and outlets. The direction of flow of the seepage water has now to be determined. This was done by the following method.

When the sand was being packed, a few crystals of silver nitrate were embedded in the sand, just below the level of the bed of the canal section in the model. Instead of water, a constant head of a dilute solution of potassium chromate was maintained in the model. When the solution enters the sand, it reacts with the silver nitrate, giving a precipitate along the line of flow of the seepage water. When the head of potassium chromate had been maintained for five hours, it was found that the lines of flow in the sub-soil model had been completely traced. The sand was then cut in vertical section in the centre, in a plane along the length of the tank and one-half of the sand carefully removed. The traces of seepage flow indicated by the precipitate of silver chromate under the bed of the model section could now be seen. These were then photographed. The sand was packed again as before and the experiment repeated with a different gradient in the sub-soil.

Results.

The results are shown in Figs. 1 to 5.

Fig. 1 shows the direction of flow when the gradient of the sub-soil water-table is zero. Figs. 2 and 3 are for gradients of 4/15 and 6/15 respectively of the sub-soil water-table in the model. They show the progressive distortion of the lines of flow when the gradients are increased. Experiments were also conducted with gradients of 1/15, 2/15, 3/15 and 5/15, but the results are not shown here. Those given in the plates are sufficient to illustrate the general nature of the distortions of the lines. The nature of the forces in relation to the direction of flow are discussed later.

Fig. 4 shows the effect of pumping on the direction of seepage flow from the bed of the model. The tube representing the model of the pump, marked T in the figure, was inserted at a distance of 10 cms. from the model. The water-table was 5 cms. below the bed of the model of the canal. It could be seen from the photographs that the lines of flow near the end of the tube T are distorted considerably. The other lines also reveal slight distortions and these are comparable to those in Fig. 2. When a similar experiment was carried out with the tube T at a distance of 22 cms. from the model of the canal, none of the lines was distorted and the lines compared with that in Fig. 1.

Fig. 5 shows the effect of a drain 10 cms. distant from the model. The water-table in this case was 5 cms. below the bed of the model of the canal and the bed of the drain 10 cms. below that of the model. In this case the distortion of the lines of flow are of a nature different from the previous case, *i.e.*, the case of pumping. The lines farther away from the drain are more affected than in the corresponding case of pumping. The effect on other lines are also more gradual. Only the line very near the drain seems to be less affected than the corresponding line in the case of pumping. These points are of technical application when the relative merits of pumping and drainage schemes are discussed.

Theoretical.

It is well known that the movement of water in the medium in which the above experiments were conducted follows the relationship deduced by Darcy in 1856, *e.g.*,

$$u = k \cdot \frac{h}{l}$$

where u is the average velocity of flow, h the loss of pressure head between two points in the same horizontal line at a distance l apart, and k a constant for the medium. If this relationship be extended to three dimensions and put in differential form we get,

$$u = k\frac{\partial p}{\partial x}$$

$$v = k\frac{\partial p}{\partial y}$$

$$w = k\frac{\partial p}{\partial z},$$

so that assuming that the law of continuity holds we get

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = 0.$$

The streamlines given by

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

are perpendicular to equi-pressure surfaces and will be known as soon as a solution of

$$\nabla^2 p = 0$$

is obtained that will satisfy the given boundary conditions.

In the above experiments the movement was essentially two dimensional so that a two dimensional solution of $\nabla^2 p = 0$ consistent with the boundary condition will be obtained in each case.

Case No. 1.

Canal over a Sub-soil of Great Depth.—Here, in order to get any spreading out of the water in horizontal directions we must assume that the pressure at the bottom of the canal is proportional to the height of the liquid above it and normal to the bottom, which gives

$$\text{for} \quad r = a, \quad H - h = 0$$

when we assume the section of the canal as circular of radius a. The curves $H - h$ of equi-pressure and stream function ψ are given by

$$H - h = \frac{r^2 - a^2}{r}\cos\theta,$$

$$\psi = -u_\infty \frac{r^2 + a^2}{r}\sin\theta.$$

$$= -u_\infty\left(y + \frac{a^2 y}{r^2}\right);$$

so that

$$u = -\frac{\partial\psi}{\partial y} = u_\infty\left(1 + \frac{a^2}{r^2} - \frac{2a^2y^2}{r^4}\right)$$

$$v = \frac{\partial\psi}{\partial x} = u_\infty\frac{2a^2xy}{r^4}$$

$$\frac{\partial\psi}{\partial r} = -u_\infty\left(1-\frac{a^2}{r^2}\right)\sin\theta$$

$\therefore$ when $r = a,\quad \frac{\partial\psi}{\partial r} = 0;$

therefore the flow is along the radius of the circular section of the channel.

When $r = \infty$, $u = u_\infty = k$, $v = 0$. Hence, the streamlines are given by

$$\psi = -k\left(y+\frac{a^2y}{r^2}\right);$$

if the total flow across the canal section be $2q$, the bounding streamlines will be given by

$$\psi = \pm q \quad = \mp k\left(y+\frac{a^2y}{r^2}\right),$$

or

$$y+\frac{a^2y}{r^2} = \pm q/k = b \text{ (say)};$$

and they will be between the asymptotes

$$y = \pm b = \pm q/k.$$

This is the case treated in Fig. 6.

Case No. 2.

Canal over a Sub-soil of Finite Depth.—In general the expression sub-soil of finite depth means that the depth of the natural water-table below the ground surface is not very great and in the absence of any definite knowledge about what happens when a sheet of water meets another at an angle, the lower boundary has been treated as impervious and plane, which in the case of sub-soil hydraulic is approximately true even though the lower layer might not be an impervious soil. In the present case the canal is treated as a thin sheet of water of length $2a$ from which percolation is taking place into the sub-soil. This approximation will not affect the streamlines at a distance from the canal, only in its vicinity will the theoretical lines diverge from the actuals. Hence the boundary conditions are,

$$x = 0,\quad y = 0,\quad \psi = 0,\quad \text{and } u = u_0;$$

also

$$x = b,\quad u = 0,\quad \psi = 0$$

So the solution of $\nabla^2\psi = 0$ with the above boundary conditions is

$$\psi = -u_0\frac{2b}{\pi}\sinh\frac{y\pi}{2b}\cos\frac{x\pi}{2b}.$$

with

$$u = u_0\cosh\frac{y\pi}{2b}\cos\frac{x\pi}{2b}$$

and

$$v = u_0\sinh\frac{y\pi}{2b}\sin\frac{x\pi}{2b},$$

so that the streamline $\psi = 0$ coincides with the impervious boundary. This refers to Fig. 7 and is shown in the photograph Fig. 1.

The above case is not of very great practical importance where the sub-soil water-table is very seldom horizontal. We now generalise this case by taking into consideration the slope of the water-table. If we denote the slope to the horizontal as α we get as a solution with axes perpendicular and parallel to the water-table passing through the centre of the canal

$$\psi = - v_0 \frac{2b}{\pi} \cos\alpha \sinh \frac{y\pi}{2b} \cos \frac{x\pi}{2b} + v_0 \sin\alpha\, x$$

which reduces to the previous case when $\alpha = 0$. Here the streamlines corresponding to the downward free surface is given by $\psi = -q$ and upward free surface $\psi = q$ whereas the middle one is given by $\psi = 0$. This is given in Fig. 8. Similarity between these streamlines and those given in the photos (Figs. 2 and 3) is striking.

Some very interesting deductions can be made from the above theoretical relationship.

1. There is a critical slope in the water-table given by $\sin\alpha = q/bu_0$. If the slope is flatter than this then the upward free surface $\psi = q$ will not turn down but begin to pile up and waterlogging will be the result.

2. If the canal be brought nearer the water-table then the absolute value of the seepage from the canal will fall off; but whether this will result in a diminution of waterlogging will depend on the rate of variation of the ratio $\frac{q}{b}$. If $\frac{q}{b}$ increases, *i.e.*, if the fall in the rate of seepage be less than the corresponding change in b, then the critical slope required will be more than the existing slope and waterlogging will result in spite of the fact that there is an absolute fall in seepage. This statement is not so paradoxical as it would appear. Though there might be a drop in the actual seepage, the slope required in the water-table to drain off being more than the existing slope there will be a heaping up of the seepage water on the upstream side of the canal and waterlogging will be the result.

The qualitative agreement found here between the experimental results and theory lends support to the applicability of theoretical Hydrodynamics to simple cases in models.

We wish to thank Dr. E. McKenzie Taylor, the Director of the Institute, for his keen and continued interest in the progress of the investigation.

Summary.

Experiments have been conducted in models to study the direction of flow of water from earthen beds such as those of canals. The photographs of streamlines in the sub-soil are shown. The theory of the flow is discussed

and qualitative agreements between theory and experiment have been observed.

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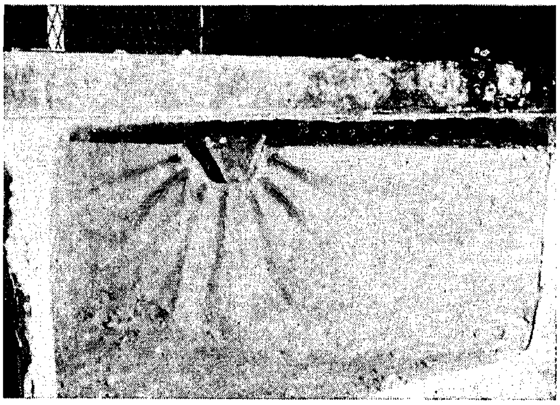

FIG. 1.

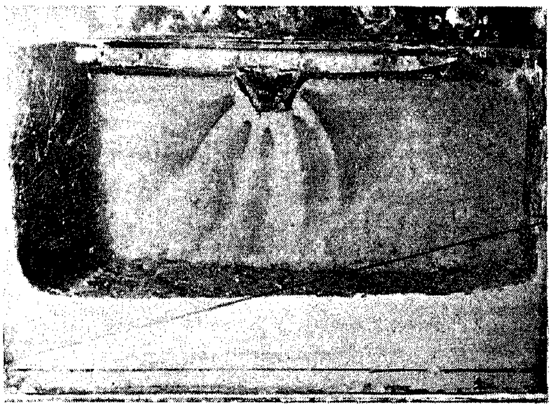

FIG. 2.

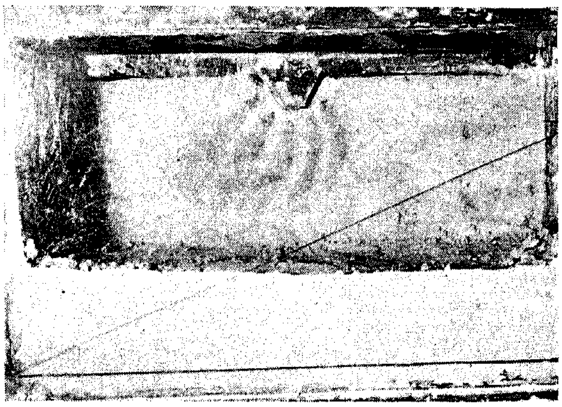

FIG. 3.

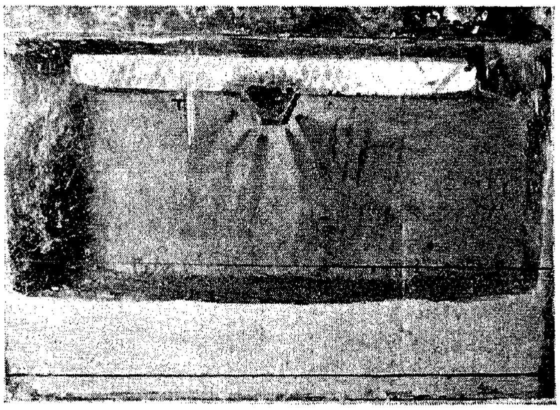

FIG. 4.

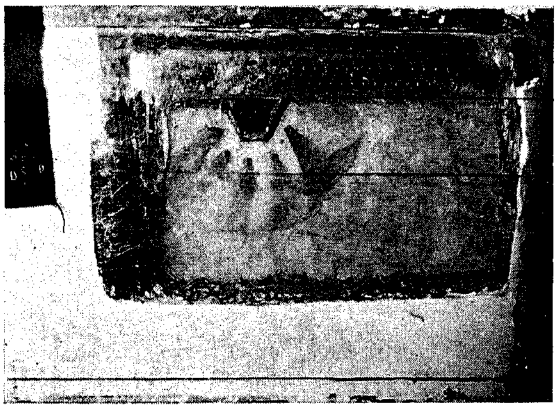

FIG. 5.

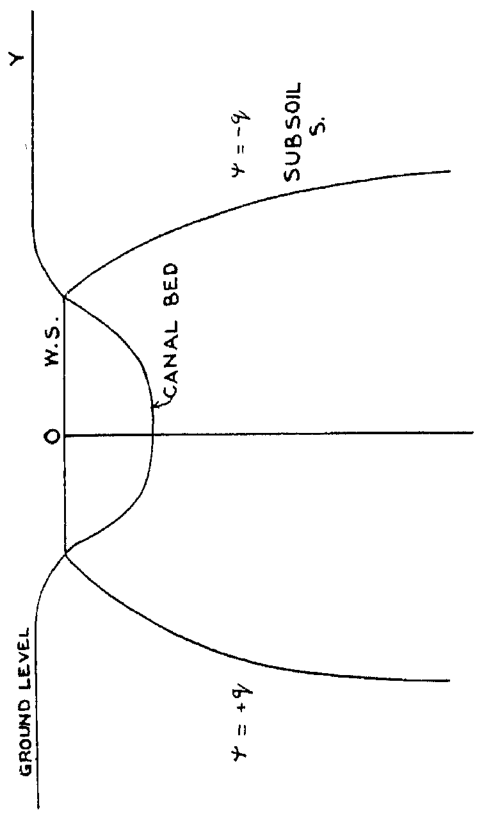

FIG. 6

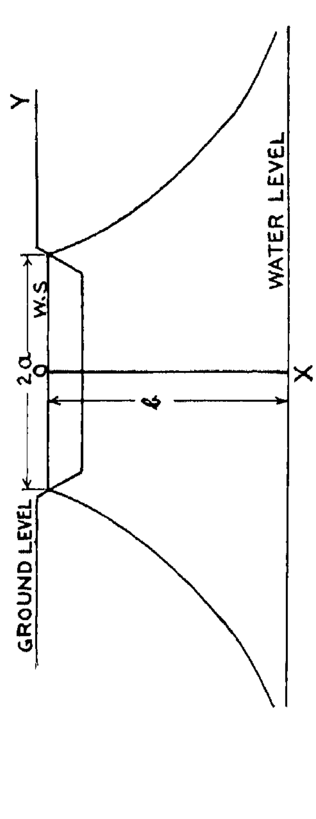

FIG. 7

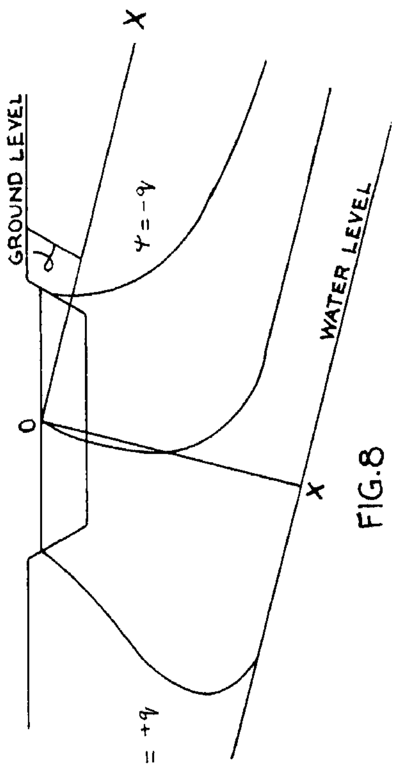

FIG. 8