BOUNDARY LAYER ON A FLAT PLATE WITH SUCTION

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Introduction

In this paper we discuss the problem of evaluating the thickness of boundary layer and skin friction on a thin plate with suction, placed parallel to an otherwise uniform flow. A number of workers have studied this problem. For example, Blasius' classical work discusses the boundary layer on a plate at some distance from the leading edge, while Schlichting has given the solution of the problem taking into account uniform suction in which the velocity distribution is independent of the distance from the leading edge. Recently Carrier and Lin[1] have obtained the solution in the neighbourhood of the leading edge, while Torda[2] has deduced the necessary equations giving the distribution of suction at the plate which will ensure constant thickness of boundary layer. He has represented this suction velocity distribution in a graphical form for an aerofoil for which experimental data are available. We have obtained the stream function by the method of successive approximations assuming a linear law of suction which holds strictly in the neighbourhood of the leading edge. In passing, we may mention that in the absence of suction the present solution reduces to a form similar to that given by Carrier and Lin. We have obtained the thickness of the boundary layer and the skin friction explicitly in terms of suction velocity and distance from the leading edge without putting any restriction on the distribution of suction speed on the plate. To ensure symmetry we have assumed the suction to take place with the same speed at the corresponding points on the two sides of the plate.

It is evident that our aim is in a sense different from that of Torda. While Torda has attempted to specify the distribution of suction speed over the plate to ensure a constant thickness of boundary layer, we have attempted to evaluate the boundary layer thickness in terms of a specified distribution of suction speed. The present point of view may be found more useful in practice besides being more general than that of Torda.

The explicit expressions for velocity boundary layer thickness $\delta(x)$ and the skin friction $\tau_0(x)$ for the downstream flow as obtained in this paper are:

$$\delta(x) = \frac{\sqrt{x}}{a} + \frac{v_0(x)}{2}\left(\frac{\sqrt{x}}{a}\right)^2 + \left\{\frac{1}{48} + \frac{v_0^2(x)}{3}\right\}\left(\frac{\sqrt{x}}{a}\right)^3 + \frac{v_0^3(x)}{4}\left(\frac{\sqrt{x}}{a}\right)^4 + \ldots,$$

and

$$\tau_0(x) = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=0} = \mu\left(\frac{a}{\sqrt{x}} - v_0'(x)\right),$$

where

$$a = 0{\cdot}332.$$

1. Description of the Flow Near the Leading Edge

The equations governing an incompressible, viscous, fluid flow in two-dimensional cartesian co-ordinates x_1, y_1 are:

Equation of continuity:

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} = 0, \tag{1.1}$$

Equations of momentum:

$$u_1\frac{\partial u_1}{\partial x_1} + v_1\frac{\partial u_1}{\partial y_1} + \frac{1}{\rho}\frac{\partial p_1}{\partial x_1} = \nu\nabla^2 u_1, \tag{1.2}$$

$$u_1\frac{\partial v_1}{\partial x_1} + v_1\frac{\partial v_1}{\partial y_1} + \frac{1}{\rho}\frac{\partial p_1}{\partial y_1} = \nu\nabla^2 v_1. \tag{1.3}$$

The introduction of the following dimensionless variables:

$$p = \frac{p_1}{\frac{1}{2}\rho u_0^2}, \quad x = \frac{x_1 u_0}{\nu}, \quad y = \frac{y_1 u_0}{\nu}, \quad u = \frac{u_1}{u_0} = \frac{\partial\psi}{\partial y},$$

$$v = \frac{v_1}{v_0} = -\frac{\partial\psi}{\partial x}, \tag{1.4}$$

where u_0 is the free stream velocity, reduces the above equations to:

$$\nabla^4\psi = \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\psi) - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\psi), \tag{1.5}$$

Supplement to the

"Proceedings of the Indian Academy of Sciences"

Proceedings of the Twenty-fifth Annual Meeting of the Indian Academy of Sciences, held from the 26th to the 28th December 1959 in the University Campus, Annamalai University, Annamalainagar.

The Session was attended by forty-one Fellows and numerous Delegates from different parts of India.

Saturday, the 26th December 1959

Between 10 A.M. and 12 Noon, the Fellows and the Delegates visited the Chidambaram Nataraja Temple.

All the Functions and the Meetings were held in various halls in the University Campus.

BUSINESS MEETING

(SENATE HALL)

(*3 to 4 p.m.*)

1. Thirty-six Fellows were present at the Meeting. Sir C. V. Raman presided.

2. The Report for the year 1959, which had already been circulated, was taken as read. Prof. C. Mahadevan moved and Dr. P. Kutumbiah seconded the adoption of the Report, which was then unanimously adopted.

3. Dr. K. S. G. Doss and Prof. C. Mahadevan were appointed scrutators for scrutinising the voting papers relating to the election of Fellows. The following were declared elected:—

(i) Dr. S. S. Dharmatti, M.SC., PH.D. (Bombay), F.INST.P., F.A.P.S., Reader in Physics and Head of Nuclear and Electron Magnetism Group, Tata Institute of Fundamental Research, Bombay 1.

(ii) Miss Anna Mani, B.SC. (HONS.), A.I.I.SC., Meteorologist, Instruments Division, Meteorological Office, Poona 5.

(iii) Dr. G. S. Puri, M.SC. (HONS.), PH.D. (Lucknow), PH.D. (London), F.G.S., F.B.S., F.L.S., F.G.M.S., F.H.S. (Lond.), Director of the Central Botanical Laboratory, Allahabad.

(iv) Dr. G. Rangaswami, B.SC. (AG.) (Madras), ASSOC.I.A.R.I., PH.D. (Rutgers), Professor and Head of the Department of Agriculture, Annamalai University, Annamalainagar P.O.

(v) Dr. T. H. Rindani, M.D., Professor of Physiology, National Medical College, Bombay 8.

(vi) Dr. K. S. Thind, B.Sc. (Hons.), M.Sc. (Hons.) (Punjab), Ph.D. (Wisconsin), Reader in Botany, University of Punjab, Khalsa College, Amritsar.

(vii) Dr. M. K. Vainu Bappu, M.Sc., A.M. (Harvard), Ph.D. (Harvard), Director, Uttar Pradesh State Observatory, Nami Tal.

4. Dr. B. V. Thosar and Shri V. Ramanathan were appointed scrutators for scrutinising the voting papers relating to the election of Honorary Fellows. The following were declared elected:-

(i) Professor Albert Frey-Wyssling, Professor of General Botany at the Federal Institute of Technology, Zurich, Switzerland.

(ii) Professor Nikolai Nikolaevich Semenov, Institute of Chemical Physics, Moscow, U.S.S.R.

(iii) Professor Arne Wilhelm Kaurin Tiselius, Institute of Biochemistry, Uppsala, Sweden.

5. Prof. T. S. Sadasivan announced that the Vice-Chancellor of the Madras University had asked him to convey to the Academy the informal invitation of the Madras University to hold the 26th Annual Meeting of the Academy in 1960 at Madras under the auspices of the University and that the formal invitation from the Syndicate would follow in due course. The invitation was accepted with acclamation.

6. The President moved the following resolutions and they were carried with acclamation:-

(*a*) The Indian Academy of Sciences desire to convey to Dr. Rajah Sir A. Muthiah Chettiar of Chettinad, Pro Chancellor of the Annamalai University, their grateful appreciation of his kind consent to inaugurate the Twenty-fifth Annual Meeting of the Academy.

(*b*) The Indian Academy of Sciences desire to convey to the Vice-Chancellor and the Syndicate of the Annamalai University, their grateful thanks for the kind invitation which enabled the Academy to hold its Session at Annamalainagar.

(*c*) The Indian Academy of Sciences desire to convey to the Chairman and the Members of the Reception Committee of the Twenty-fifth Annual Meeting of the Academy, their grateful appreciation of the many acts of kindness and hospitality which have made their stay at Annamalainagar most pleasant and agreeable.

(*d*) The Indian Academy of Sciences desire to convey their grateful thanks to the authorities of the Annamalai University for providing the necessary facilities for the conduct of the meetings of the Academy and for various other kinds of help which have all greatly contributed to the success of the Session.

(*e*) The Indian Academy of Sciences wish in particular to convey to Prof. G. Rangaswami and Dr. J. Jacob, their warmest appreciation of the labour and thought devoted by them to the work of organising the Session, making arrangements for the conduct of the meetings and for the comfortable stay of the Fellows and the Delegates, which have all contributed to the success of the Session.

(*f*) The Indian Academy of Sciences wish to place on record their grateful thanks to the Leader and other Members of the Volunteer Organisation for the invaluable help rendered by them which have greatly contributed to the success of the Session.

(*g*) The Indian Academy of Sciences desire to convey to the organisers, the Academy's great appreciation of the entertainments provided during the Session.

(*h*) The Indian Academy of Sciences desire to convey to the Chairman, the Vice-Chairman and the Members of the Chidambaram Municipal Council, their great appreciation of the excellent Reception given by them to the President and the Fellows of the Academy.

7. Dr. B. Sundar Raj suggested that increased emphasis may be given to biological sciences. The President assured him that it was being done to the maximum extent possible.

AT HOME

(4 *p.m.*)

The Reception Committee was "AT HOME" to the Fellows and the Delegates.

INAUGURAL FUNCTION

(SASTRI HALL)

(5–30 *to* 7–30 *p.m.*)

The function commenced with an invocation.

Shri T. M. Narayanaswamy Pillai, Vice-Chancellor of the Annamalai University and Chairman of the Reception Committee, in his Welcome Address stated that it was a great pleasure and privilege for him to extend a most cordial welcome to the distinguished Founder-President, Sir C. V. Raman, and the Members of the Indian Academy of Sciences for the 25th Annual Session.

Dr. Rajah Sir M. A. Muthiah Chettiar of Chettinad, Pro-Chancellor of the Annamalai University, inaugurating the Session said that it was his privilege and honour as Pro-Chancellor of the University to extend a warm welcome to the distinguished Chief Guest, Sir C. V. Raman, the President of the Academy, and other distinguished Fellows of the Academy and various delegates and members attending the Session.

Sir C. V. Raman, in accordance with the time-honoured practice at the Annual Meetings of the Academy, introduced to the audience the Fellows and the special invitees for the symposia, who had come from various parts of India to take part in the meetings.

Before delivering his Presidential Address on "The Origin of Colour and the Nature of the Visual Mechanism", Sir C. V. Raman expressed on his own behalf and on behalf of the Fellows of the Academy, grateful thanks to the authorities of the University for their kind invitation which enabled the Academy to meet at Annamalainagar.

Dr. G. Rangaswami, Local Secretary, on behalf of the Reception Committee, proposed a hearty Vote of Thanks to Sir C. V. Raman, the distinguished Fellows and the Delegates. He expressed his grateful thanks to the Pro-Chancellor for his ready acceptance of the invitation to inaugurate the Session. He thanked the Vice-Chancellor, the University Syndicate, the authorities of the Colleges and the Heads of the Departments, especially, Profs. R. V. Seshaiya, T. N. Muthuswami and K. Venkateswarlu and Dr. J. Jacob, the various Committees, the student-members and volunteers and several others who gave their willing help and hearty co-operation in making the Session a grand success.

With the singing of the National Anthem, the function came to a close.

ENTERTAINMENT

(SASTRI HALL)

(9 *to* 11–45 *p.m.*)

Shri A. K. C. Natarajan and party entertained the Fellows and the Delegates with a performance on the clarionet.

Sunday, the 27th December 1959

Symposium on "Tectonics in Relation to India"

(LIBRARY HALL)

(9–30 *a.m. to* 12–30 *p.m.*)

Dr. S. Bhagavantam presided over the symposium. After the Chairman's introductory remarks on "Storage of Strain Energy in an Elastic Medium", Dr. M. S. Krishnan opened the symposium with his address on "Tectonics with Special

reference to India". The following papers were then presented: Prof. C. Mahadevan on "Age Levels of Precambrian Orogenic Cycles of India", Dr. S. Balakrishna on "Granite Tectonics of Hyderabad", and Shri N. A. Vemban on "Structure and Tectonics of the Manganese Ore Belt of Madhya Pradesh and Adjoining Parts of Bombay". After the concluding remarks of the Chairman, there was a discussion for about 15 minutes and Dr. M. S. Krishnan replied to the various points raised during the discussion.

SCIENTIFIC MEETING IN SECTION "A"

(LIBRARY HALL)

(2–30 *to* 4–05 *p.m.*)

Dr. S. Bhagavantam presided over the meeting. The following papers were presented: Dr. S. Bhagavantam on "Elasticity of Metals and Alloys", Shri S. Pancharatnam on "Theory of the Dispersion of Infra-red Radiation in Crystals", Dr. H. G. Venkatesh on "Design and Operation of a Molecular Oscillator", Dr. S. K. Srinivasan on "Photo-production of Pion Pairs by Polarized Nucleons" and Dr. K. Venkateswarlu on "Intensity Problems in Raman Effect".

RECEPTION

(4–15 *p.m.*)

The Chidambaram Municipality gave a Reception to the President and the Fellows of the Academy.

PUBLIC LECTURE

(LIBRARY HALL)

(6–15 *p.m.*)

Dr. P. Nilakantan delivered an illustrated popular lecture on "Supersonic Flight". Dr. S. Bhagavantam occupied the Chair.

ENTERTAINMENT

(SASTRI HALL)

(9 *to* 11–30 *p.m.*)

Prof. M. M. Dandapani Desigar and party entertained the Fellows and the Delegates with music.

Monday, the 28th December 1959

Symposium on "Chemical and Biological Control of Insect Pests"

(LIBRARY HALL)

(9–30 *a.m. to* 12–30 *p.m.*)

Dr. K. Ramiah presided. After the Chairman's introductory remarks, the following papers were presented: Dr. M. Putturudriah on "Present Position with

Regard to the Control of Insect Pests Attacking Cereals, Oilseeds, Spices, Plantation Crops, Sugarcane, Cotton, Vegetables and Root Crops, Store and Household pests", "Plant Protection Services with Regard to Pest Control, Difficulties Experienced and Suggestions to Overcome Them", "General Effects of Intensifying Chemical Methods of Insect Control on Insect Population and Human Welfare, including Hazards in Transport and Use"; Dr. Sardar Singh on "Effective Pest Control through Spray Programmes", "Changed Outlook in Plant Protection", "Need for Regulation of the Pesticide Industry in India" and Dr. K. V. Joseph on "Present Position with Regard to the Control of Rice Pests in Kerala". After discussion, the Chairman made his concluding remarks summing up the discussion.

SCIENTIFIC MEETING IN SECTIONS "A" AND "B"

(LIBRARY HALL)

(2-30 *to* 4-30 *p.m.*)

From 2-30 to 3-15 P.M., the following papers in Section 'B' were presented: Dr. M. K. Subramaniam on "The Yeast Nucleus", Prof. G. Rangaswami on "Studies on the Microbial Spoilage of Canned Foods, Parts I and II", and Dr. J. V. Bhat on "The Characteristics and Questionable Taxonomic Position of the Oxalate-Decomposing Bacterium, *Vibrio Extorquens*". Dr. K. Ramiah occupied the Chair.

From 3-15 to 4-30 P.M., the following papers in Section 'A' were presented: Prof. G. N. Ramachandran on "Conservative Systems in Physics", Shri T. N. Seshadri on "Certain Fundamental Equations in the Study of Day Lighting in Buildings" and Dr. R. D. Desai on "Latent Heat of Vaporisation and Composition". Dr. S. Bhagavantam occupied the Chair.

PUBLIC LECTURE

(LIBRARY HALL)

(6-15 *p.m.*)

Dr. S. Bhagavantam delivered an illustrated popular lecture on "Radio Astronomy". Sir C. V. Raman occupied the Chair.

At the conclusion of the lecture, Dr. Bhagavantam on behalf of the visiting Fellows expressed his grateful thanks to the Vice-Chancellor, the Reception Committee and others for the excellent hospitality which the visitors had enjoyed at their hands. He earnestly wished that the Annamalai University might flourish under the benign guidance of the Vice-Chancellor.

Before winding up the Session, Sir C. V. Raman once again expressed, both on his own behalf and on behalf of the visiting Fellows, their most heart-felt appreciation of the extraordinary enthusiasm, friendliness, courtesy and hospitality they had at Annamalainagar. In moving terms, he expressed that he was deeply impressed with the Vice-Chancellor's kindness.

K. RAMIAH,
K. S. VISWANATHAN,
Secretaries.

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where

$$\nabla^2\psi \equiv \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}. \tag{1.6}$$

It is found convenient to work through polar co-ordinates defined by

$$x = r\cos\theta, \quad y = r\sin\theta.$$

With these, (1.5) and (1.6) reduce to:

$$\frac{\partial^4\psi}{\partial r^4} + \frac{2}{r}\frac{\partial^3\psi}{\partial r^3} - \frac{1}{r^2}\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r^3}\frac{\partial\psi}{\partial r} + \frac{2}{r^2}\left(\frac{\partial^4\psi}{\partial r^2\partial\theta^2} - \frac{1}{r}\frac{\partial^3\psi}{\partial r\partial\theta^2} + \frac{2}{r^2}\frac{\partial^2\psi}{\partial\theta^2}\right) + \frac{1}{r^4}\frac{\partial^4\psi}{\partial\theta^4}$$

$$= \frac{1}{r}\left\{\frac{\partial\psi}{\partial\theta}\frac{\partial}{\partial r}(\nabla^2\psi) - \frac{\partial\psi}{\partial r}\frac{\partial}{\partial\theta}(\nabla^2\psi)\right\}, \tag{1.7}$$

where

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial\theta} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2}. \tag{1.8}$$

Following Carrier and Lin we write this equation as

$$\mathrm{L}(\psi) = \mathrm{L}^*(\psi), \tag{1.9}$$

where L denotes the biharmonic operator and L^* denotes the non-linear operator on the right. For a linear law of suction,

$$v(x, 0) = -\mathrm{V}x, \tag{1.10}$$

the boundary conditions are

$$\left.\begin{aligned} \left(\frac{\partial\psi}{\partial r}\right)_{\theta=0} &= \mathrm{V}r, & \left(\frac{\partial\psi}{\partial r}\right)_{\theta=2\pi} &= -\mathrm{V}r, \\ \left(\frac{\partial\psi}{r\partial\theta}\right)_{\theta=0} &= 0, & \left(\frac{\partial\psi}{r\partial\theta}\right)_{\theta=2\pi} &= 0 \end{aligned}\right\}. \tag{1.11}$$

We assume a solution of (1.9) in the form:

$$\psi = \psi_0 + \psi_1 + \psi_2 + \ldots, \tag{1.12}$$

where

$$\psi_0, \psi_1, \psi_2, \ldots,$$

are defined by

$$L(\psi_0) = 0, \tag{1.13 a}$$

$$L(\psi_1) = L^*(\psi_0), \tag{1.13 b}$$

$$L(\psi_2) = L^*(\psi_0 + \psi_1) - L^*(\psi_0), \tag{1.13 c}$$

.....

$$L(\psi_n) = L^*(\psi_0 + \psi_1 + \ldots + \psi_{n-1}) - L^*(\psi_0 + \psi_1 + \ldots + \psi_{n-2}), \tag{1.13 n}$$

We shall choose ψ_0 such that it satisfies the boundary conditions (1.11) completely, *i.e.*,

$$\left.\begin{aligned} \left(\frac{\partial\psi_0}{\partial r}\right)_{\theta=0} &= Vr, & \left(\frac{\partial\psi_0}{\partial r}\right)_{\theta=2\pi} &= -Vr, \\ \left(\frac{\partial\psi_0}{\partial\theta}\right)_{\theta=0} &= 0, & \left(\frac{\partial\psi_0}{\partial\theta}\right)_{\theta=2\pi} &= 0 \end{aligned}\right\}. \tag{1.14}$$

Besides the velocity distribution (as determined by ψ_0) is to be symmetric about the plate. The boundary conditions to be satisfied by ψ_i for $i > 0$, are

$$\frac{\partial\psi_i}{\partial r} = \frac{\partial\psi_i}{\partial\theta} = 0, \quad \text{at} \quad \theta = 0, \quad 2\pi. \tag{1.15}$$

Also the velocity distribution (as determined from each one of the ψ_i's) is to be symmetrical about the plate.

The first two conditions in (1.14) suggest a solution for (1.13 *a*) in the form

$$\psi_0 = r^2 f(\theta). \tag{1.16}$$

Using this form we find that $f(\theta)$ has to satisfy

$$f^{IV} + 4f'' = 0, \tag{1.17}$$

with the conditions

$$\left.\begin{aligned} f(0) &= \frac{V}{2}, & f(2\pi) &= -\frac{V}{2}, \\ f'(0) &= 0, & f'(2\pi) &= 0. \end{aligned}\right\} \tag{1.18}$$

With these we easily see that the solution of (1.13 *a*) of the assumed type is

$$\psi_0 = \frac{Vr^2}{2} + \frac{Vr^2}{4\pi}(\sin 2\theta - 2\theta) - 2Cr^2 \sin^2\theta, \tag{1.19}$$

where the constant C is yet undetermined.

The symmetry of velocity distribution demands

$$\left.\begin{aligned} \left(\frac{\partial\psi_0}{r\partial\theta}\right)_{\substack{r=\rho \\ \theta=\alpha}} &= \left(\frac{\partial\psi_0}{r\partial\theta}\right)_{\substack{r=\rho \\ \theta=2\pi-\alpha}}, \\ \left(\frac{\partial\psi_0}{\partial r}\right)_{\substack{r=\rho \\ \theta=\alpha}} &= -\left(\frac{\partial\psi_0}{\partial r}\right)_{\substack{r=\rho \\ \theta=2\pi-\alpha}}, \end{aligned}\right\} \tag{1.20}$$

for all ρ and α.

These conditions yield

$$C = 0. \tag{1.21}$$

Thus

$$\psi_0 = \frac{Vr^2}{2} + \frac{Vr^2}{4\pi}(\sin 2\theta - 2\theta). \tag{1.22}$$

If we put V = 0 in (1.22), we get,

$$\psi_0 \equiv 0. \tag{1.23}$$

Hence we find that the above stream function is unable to account for the flow without suction.

The boundary conditions for the flow without suction are

$$\left.\begin{aligned} \left(\frac{\partial\psi}{\partial r}\right)_{\theta=0} &= 0, \quad \left(\frac{\partial\psi}{\partial r}\right)_{\theta=2\pi} = 0, \\ \left(\frac{\partial\psi}{\partial\theta}\right)_{\theta=0} &= 0, \quad \left(\frac{\partial\psi}{\partial\theta}\right)_{\theta=2\pi} = 0. \end{aligned}\right\} \tag{1.24}$$

Besides the velocity distribution is to be symmetric about the plate. It is found by trial that the least value of n for which a function of the form $r^n f(\theta)$ satisfies these conditions is 3/2 and the corresponding function $f(\theta)$ is given by

$$f(\theta) = \cos\frac{\theta}{2} - \cos\frac{3\theta}{2}. \tag{1.25}$$

Thus to get a complete expression for ψ_0 which will also account for the flow in the absence of suction without affecting the boundary conditions we must add to (1.22).

$$2Ar^{3/2}\left(\cos\frac{\theta}{2} - \cos\frac{3\theta}{2}\right), \tag{1.26}$$

where A is yet an undetermined constant which is independent of suction.

Hence

$$\psi_0 = \frac{Vr^2}{2} + \frac{Vr^2}{4\pi}(\sin 2\theta - 2\theta) + 2Ar^{3/2}\left(\cos\frac{\theta}{2} - \cos\frac{3\theta}{2}\right). \tag{1.27}$$

The presence of the constant A allows us a choice to fix up the flow without suction.

With this value of ψ_0, (1.13 *b*) becomes

$$\begin{aligned} L(\psi_1) = {} & \frac{2A^2}{r}(2\sin\theta - 3\sin 2\theta) \\ & + \frac{AV}{\pi r^{1/2}}\left(2\pi\sin\frac{\theta}{2} + 7\cos\frac{\theta}{2} - 2\theta\sin\frac{\theta}{2} - 6\cos\frac{3\theta}{2}\right. \\ & \left. - \cos\frac{5\theta}{2}\right) \\ & + \frac{2V^2}{\pi}\left(1 - \frac{\theta}{\pi} + \frac{1}{2\pi}\sin 2\theta\right) \end{aligned} \tag{1.28}$$

It appears that a solution of (1.28) is of the form

$$\psi_1 = r^3 f_1(\theta) + r^{7/2} f_2(\theta) + r^4 f_3(\theta); \tag{1.29}$$

but such a solution is found not to satisfy boundary conditions (1.15) and the symmetry conditions unless $V = 0$. Thus we cannot proceed with the above form of solution for ψ_1 if we are interested in a solution for the problem with suction. It is found that a function of the form

$$\begin{aligned} \psi_1 = {} & A^2 r^3 \{f_1(\theta) + \phi_1(\theta)\log r\} \\ & + \frac{AV}{\pi} r^{7/2}\{f_2(\theta) + \phi_2(\theta)\log r\} \end{aligned}$$

$$+\frac{2V^2}{\pi}r^4\{f_3(\theta)+\phi_3(\theta)\log r\}, \tag{1.30}$$

satisfies (1.28) and the conditions (1.15) along with the symmetry of the velocity distribution. Using this we find that f_1, f_2, f_3, ϕ_1, ϕ_2, ϕ_3 have to satisfy

$$\phi_1^{IV}+10\phi_1''+9\phi_1=0, \tag{1.31}$$

$$f_1^{IV}+10f_1''+9f_1=-24\phi_1-8\phi_1''+4\sin\theta-6\sin 2\theta, \tag{1.32}$$

$$\phi_2^{IV}+\frac{29}{2}\phi_2''+\frac{441}{16}\phi_2=0, \tag{1.33}$$

$$f_2^{IV}+\frac{29}{2}f_2''+\frac{441}{16}f_2=-\frac{105}{2}\phi_2-10\phi_2''+2\pi\sin\frac{\theta}{2}+7\cos\frac{\theta}{2}-2\theta\sin\frac{\theta}{2}-6\cos\frac{3\theta}{2}-\cos\frac{5\theta}{2}, \tag{1.34}$$

$$\phi_3^{IV}+20\phi_3''+64\phi_3=0, \tag{1.35}$$

$$f_3^{IV}+20f_3''+64f_3=-96\phi_3-12\phi_3''+1-\frac{\theta}{\pi}+\frac{1}{2\pi}\sin 2\theta, \tag{1.36}$$

with the conditions

$$\left.\begin{aligned} &f_i(0)=f_i(2\pi)=f_i'(0)=f_i'(2\pi)=0,\\ &\phi_i(0)=\phi_i(2\pi)=\phi_i'(0)=\phi_i'(2\pi)=0,\\ &\qquad\qquad i=1,\ 2,\ 3.\end{aligned}\right\} \tag{1.37}$$

These equations admit of direct solution and we get

$$\begin{aligned}\psi_1=Ar^3\Big[&\frac{\pi-\theta}{8}(\cos 3\theta-\cos\theta)+\frac{2}{5}(\sin 2\theta-2\sin\theta)\\&+\frac{1}{8}(3\sin\theta-\sin 3\theta)\log\kappa r\Big]\\+\frac{AV}{\pi}r^{7/2}\Big[&\frac{1}{144}\Big(49\cos\frac{\theta}{2}+6\cos\frac{5\theta}{2}-55\cos\frac{7\theta}{2}\Big)\\&+\frac{\pi-\theta}{240}\Big(20\sin\frac{\theta}{2}+89\sin\frac{3\theta}{2}-41\sin\frac{7\theta}{2}\Big)\\&+\frac{41}{240}\Big(\cos\frac{3\theta}{2}-\cos\frac{7\theta}{2}\Big)\log\lambda r\Big]\end{aligned}$$