LIQUID METAL POWER GENERATOR—2

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THE Magnetohydrodynamic (hereafter called MHD) power generator is an unconventional power generator in the sense that it is a direct conversion system where the thermal (or kinetic) energy is directly converted into electrical energy, whereas in the usual conventional power generator the thermal (or kinetic) energy is converted into mechanical energy and then to electrical energy. The advantages of the MHD power generator are many. Since the MHD power generator does not involve any moving mechanical part, it operates efficiently at higher altitudes compared to the usual conventional generators. Therefore, MHD power generator is very useful for defence and space research purposes. In recent years, MHD power generator has attracted many scientists (Rosa,[1] 1968). Recently, we (Chandrasekhara[2] and others, 1968 hereafter called Part, 1) discussed the liquid metal generator where we have derived expressions for the current and efficiency of the generator for arbitrary variation of density. However, in this article we consider the variation of density with height and its effect on the current and efficiency. We have also included the equivalent circuit of the generator and expressions for open circuit voltage, terminal voltage and short circuit current.

The results of Part 1 are true for any density distribution. But in most of the physical situations the variation of density decreases exponentially with height. Hence, we can write

$$\rho = \rho_0 e^{-\beta y} \tag{1}$$

where β is an arbitrary constant called the mixing parameter. In Part 1, we have derived an expression for the current in the form

$$J_L = \sigma\left[\frac{2\phi_w}{h} - \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} UB\right] L + \frac{4}{\pi}\sigma\,\phi_w \log 2 \tag{2}$$

where ϕ_w is the applied voltage, B is the applied magnetic field, σ is the electrical conductivity, ρ is the density of the fluid, L is the length and h is the height of the channel.

Equation (2), using (1) and integrating with respect to y from $-\pi/_2$ to $\pi/_2$ becomes

$$J_L = 2\sigma\left[\frac{\phi_w}{h} - \frac{2UB \sinh \frac{\beta\pi}{4}}{\pi\beta}\right] L + \frac{4\sigma\phi_w \log 2}{\pi}. \tag{3}$$

The change in total average pressure is

$$\Delta p = \pi^{-1}\int_{-\pi/2}^{\pi/2}\int_{0}^{L} (\vec{J}\times\vec{B})_x\, dx dy$$

$$\Delta p = 2\sigma BL\left[\frac{\phi_w}{h} - \frac{2UB \sinh \frac{\beta\pi}{4}}{\pi\beta}\right]. \tag{4}$$

The generator efficiency ϵ_g now becomes

$$\epsilon_g = \frac{P}{\Delta PUh} \tag{5}$$

where

$$P = -2\phi_w J_L$$

$$= \frac{2\phi_w\left[2\sigma\left\{\frac{\phi_w}{h} - \frac{2UB \sinh\frac{\beta\pi}{4}}{\pi\beta}\right\}L + \frac{4\sigma\phi_w \log 2}{\pi}\right]}{2UBhL\sigma\left[\frac{\phi_w}{h} - \frac{LUB \sinh\frac{\beta\pi}{4}}{\pi\beta}\right]}. \tag{6}$$

In the limit $L \to \infty$ the end losses become negligible and the efficiency becomes

$$\epsilon_g = \frac{2\phi_w}{UBh} \tag{7}$$

which corresponds to the case when $\rho =$ constant. We note that using an heterogeneous conducting fluid, the total current per unit length in the direction of the magnetic field and power output are increased.

Equivalent Circuit of the Generator

The equivalent circuit of the generator is as shown in Fig. 1.

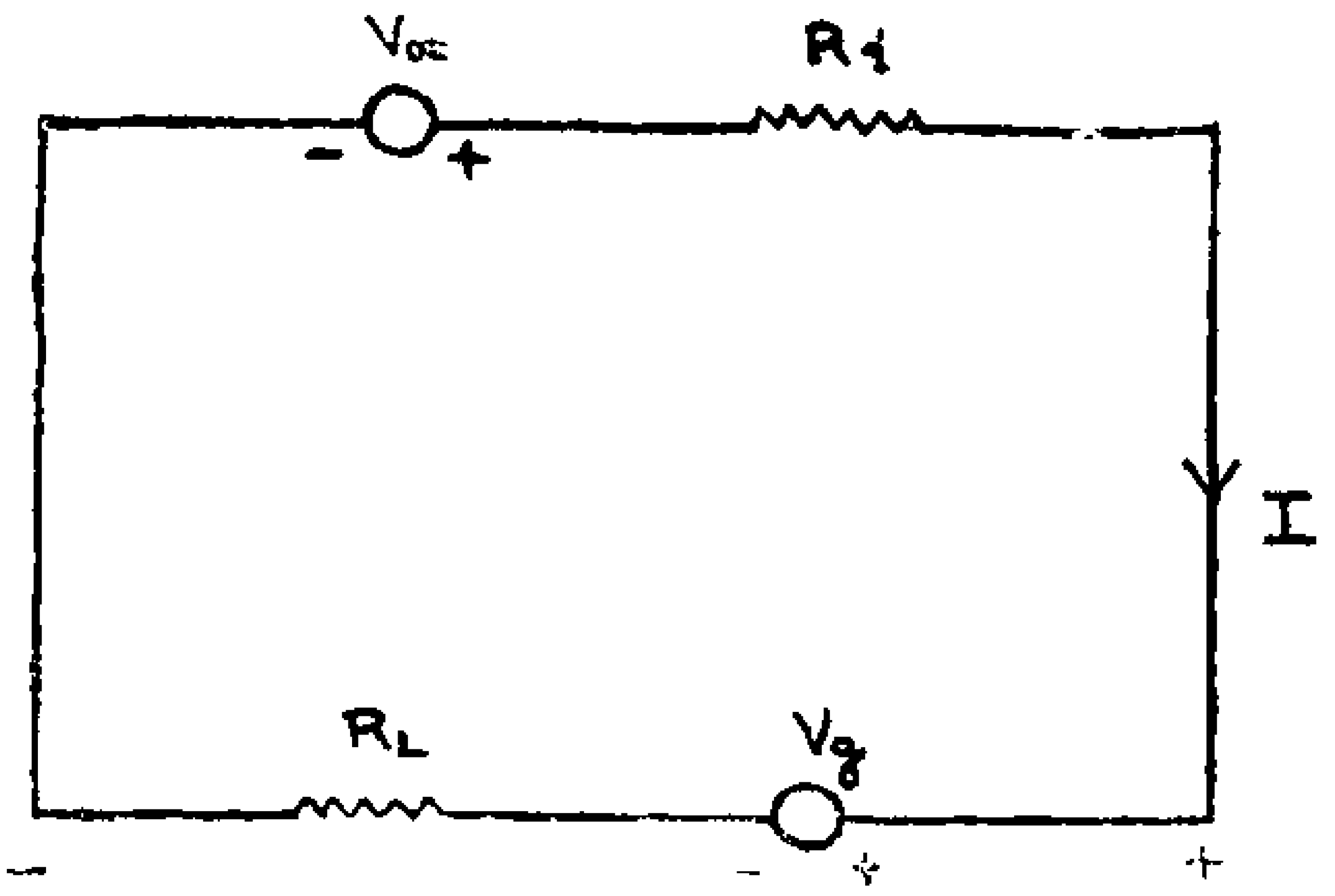

FIG 1

The open circuit voltage V_{oc} is given by setting $J_L = 0$ in Eqn. (3).

$$V_{oc} = -2\phi_w = -\frac{4UBL \sinh \frac{\beta\pi}{4}}{\beta(\pi L + 2h \log 2)} \quad (8)$$

The negative sign indicates that the induced *e.m.f.* is in a direction opposing the applied *e.m.f.* The total short ciruit current, $J_L b = I_{sc}$ where b is the breadth of the electrode, is given by letting $\phi_w = 0$ in Eqn. (3).

$$I_{sc} = -\frac{4\sigma UBLb \sinh \frac{\beta\pi}{4}}{\pi\beta}. \quad (9)$$

Then the internal resistance of the generator is given by

$$R_i = \frac{V_{oc}}{I_{sc}} = \frac{\pi h}{ab(\pi L + 2h \log 2)}. \quad (10)$$

If L is large compared to the height of the channel then

$$(R_i)_{L,\,\text{large}} = \frac{h}{\sigma A} \quad (11)$$

where

$A = Lb.$

Using Kirchoff's law to the equivalent circuit, we obtain

$$V_{oc} - V_g = I(R_i + R_L) \quad (12)$$

where $V_g = 2\phi_w$ is the applied voltage difference between the electrodes, and R_L is the external resistance given by

$$R_L = R_c + \frac{R_{ss} R_w}{R_w + R_{ss}}$$

R_c = Contact resistance
R_w = effective wall resistance
R_{ss} = external load resistance.

Hence, the total current I, from Eqn. (12) using Eqns. (8) and (9) is

$$I = -\frac{2\sigma b}{\beta} \times \left[\frac{2UBLh \sinh \frac{\beta\pi}{4} - \phi_w \beta(\pi L + 2h \log 2)}{\pi h + R_L \sigma b(\pi L + 2h \log 2)}\right] \quad (13)$$

If there is no applied magnetic field, the fluid can no longer generate current as it flows and Eqn. (13) becomes

$$I = +\frac{2\phi_w}{R_i + R_L}. \quad (14)$$

In the limit $L \to \infty$ Eqn. (13) becomes

$$I = -\frac{2}{\beta}\left[\frac{2UBh \sinh \frac{\beta\pi}{4} - \phi_w \beta\pi}{R_L \pi}\right] = -\left[\frac{\phi_i}{R_L} - \frac{2\phi_w}{R_L}\right] \quad (15)$$

where

$$\phi_i = \frac{-4UBh \sinh \frac{\pi\beta}{4}}{\pi\beta}$$

is the induced *e.m.f.* Now, the terminal voltage V_T is given by

$$V_T = V_g + IR_L = -2\phi_w - \frac{2\sigma b R_L}{\beta} \times \left[\frac{2UBLh \sinh \frac{\beta\pi}{4} - \phi_w \beta(\pi L + 2h \log 2)}{\pi Lh + R_L \sigma b(\pi L + 2h \log 2)}\right] \quad (16)$$

The terminal voltage, in the limit $R \to \infty$ tends to open circuit voltage. Therefore, as $R_L \to \infty$ Eqn. (16) becomes

$$V_T = -\frac{4UBLh \sinh \frac{\beta\pi}{4}}{\beta(\pi L + 2h \log 2)} \quad (17)$$

which is same as Eqn. (8).

Finally, we conclude, from Eqn. (3) that when $\beta \geqslant 0$ the induced current will increase and hence the power produced will be more.

1. Rosa, R. J., *Magnetohydrodynamic Energy Conversion*, McGraw-Hill Book Company, 1968.
2. Chandrasekhara, B. C., Kuchela, K. N. and Rudraiah, N., "Liquid metal magnetohydrodynamic power generator," *Curr. Sci.*, 1968, **37**, 688.