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The inflationary Universe – From theory to observations

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Abstract. The inflationary Universe resolves some of the most outstanding issues of standard cosmology including the horizon and flatness problems and the origin of density fluctuations in the Universe. Inflationary models also predict the existence of a relic gravity wave background. Both gravity waves and density fluctuations induce fluctuations in the cosmic microwave background (CMB), the discovery of large angle anisotropies in the CMB having the scale invariant spectrum predicted by inflationary models has fuelled the hope that the inflationary scenario may indeed provide the correct description of the very early Universe. Upcoming large scale galaxy surveys (SDSS & 2dF) and CMB missions (MAP, Planck Surveyor) will further probe the inflationary scenario by throwing light on the origin and evolution of large scale structure in the Universe.

Keywords. Cosmology; early Universe; structure formation; Cosmic microwave background.

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1. Horizon and flatness problems in standard cosmology

The Universe is generally believed to be homogeneous and isotropic on scales greater than several hundred Mpc. This assumption is largely based upon the observed homogeneity in the large scale distribution of galaxies, radio galaxies and QSO's and the isotropy of the cosmic microwave background (CMB). A spatially homogeneous and isotropic Universe is described by the Friedmann–Robertson–Walker (FRW) metric

$$ds^2 = c^2 dt^2 - a^2(t)\left(\frac{dr^2}{1-\kappa r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \quad \kappa = 0, \quad \pm 1 \qquad (1)$$

where $a(t)$ is the cosmic expansion factor.

For matter with energy density $\rho = T_0^0$ and pressure $P = -T_\alpha^\alpha$ the Einstein equations $G_i^j = (8\pi G/c^4)T_i^j$ simplify considerably to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2}, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P). \qquad (2)$$

The conservation equation $T_{i;k}^k = 0$ (which can also be derived directly from the Einstein equations) gives

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + P). \qquad (3)$$

For matter with equation of state $P = w\rho$, (2), (3) give

$$\rho = \rho_0 (a_0/a)^{3(1+w)}, \quad a = a_0 (t/t_0)^{2/3(1+w)} \tag{4}$$

so that: $\rho \propto a^{-3}$, $a = a_0 (t/t_0)^{2/3}$ for dust ($w = 0$).

In addition to 'normal' matter with $P > 0$ ($w > 0$) it is interesting to consider matter violating the 'strong energy condition' so that $\rho + 3P < 0$ (equivalently $w < -1/3$). From (3) we see that $\ddot{a} > 0$ i.e. such a Universe accelerates or inflates. A specific example of such an equation of state is a 'cosmological constant' $P = -\rho$. From (3) we immediately find $\rho =$ constant i.e. the density of matter remains fixed to a constant value as the Universe expands. Substituting $\Lambda = 8\pi G\rho$ in (2) and setting $\kappa = 0$ for simplicity, we get

$$a(t) = a_0 \exp Ht, \tag{5}$$

where $H \equiv \dot{a}/a = \sqrt{\Lambda/3}$ is the Hubble parameter. From (2) and (5) we see that the negative pressure of the cosmological constant accelerates the expansion of the Universe and leads to exponentially rapid expansion or 'inflation'.

Interest in the cosmological constant took an interesting turn with the discovery, in the late 1970s and early 1980s, that matter close to Planck scales, could have an effective equation of state $P < -\rho/3$. In the resulting 'inflationary scenario', the Universe rapidly accelerates before entering the radiation dominated era thereby resolving some of the most outstanding problems in standard FRW cosmology known as the 'horizon and flatness problems' which we discuss below.

1.1 *The horizon problem*

The FRW model has an important property – signals propogating at the speed of light can travel only a finite distance since the big bang resulting in a cosmological 'particle horizon' beyond which no causal interaction is possible. Let us examine this more closely by considering rays of light (null geodesics) propogating along the radial coordinate in the direction of the observer located at $r = 0$. Setting $\mathrm{d}s^2 = 0$ and $\mathrm{d}\theta = \mathrm{d}\phi = 0$ in (1) we obtain (for $\kappa = 0$)

$$r_h = c \int_0^t \frac{\mathrm{d}t'}{a(t')} \tag{6}$$

the three sphere with coordinate radius $r = r_h$ is known as the particle horizon at time t. Substituting $a(t) \propto t^{2/3}$, we get $r_h \propto t^{1/3}$ for a matter (dust) dominated Universe. Thus as the Universe expands the particle horizon grows, enveloping new regions of the Universe which now become accessible to observations.

The physical distance to the horizon at time t_0 is $l_{H0} = a(t_0) r_h(t_0) = 3ct_0$ in a matter dominated Universe. In terms of the Hubble parameter $l_H = 2cH^{-1}$. The present value of the cosmological particle horizon is $l_{H0} = 2cH_0^{-1} \simeq 6000\,\mathrm{h}^{-1}$ Mpc $= 1.8 \times 10^{28}\,\mathrm{h}^{-1}$ cm where $h = H_0/100$ is the dimensionless Hubble constant in units of 100 km/sec/Mpc. Since $H(t)$ decreases with time, it is clear that the size of the horizon was smaller in the past. Substituting $l_H(t) = 2cH^{-1}(t)$ with $H(t) = H_0(a_0/a)^{3/2}$ for a matter dominated Universe we get

$$l_H^c = l_{H0}(a_0/a)^{-1/2} \tag{7}$$

for the *comoving* horizon scale at the present epoch. As a result $l_H^c \simeq 180\,\mathrm{h}^{-1}$ Mpc is the radius of the comoving horizon scale at the epoch when CMB photons last scattered off matter (this occured soon after recombination when the Universe was a thousandth of its present size $a_0/a_{ls} \simeq 1,100$, the corresponding angle subtended on the sky today is $\theta \simeq 1.73°$). Regions separated by distances greater than $\sim 180\,\mathrm{h}^{-1}$ Mpc were not in causal contact at a_{ls}, as a result the present observable Universe can be divided into $(l_{H0}/l_H^c)^3 \simeq (1100)^{3/2} > 10,000$ pieces, each of which was causally disconnected from the others until after the epoch of last scattering.

We have seen that the horizon size at an earlier epoch ct is much smaller than the present horizon scale ct_0 scaled to that epoch: $ct_0 \times (a_0/a)^{-1} \gg ct$. The situation becomes rather acute at the Planck time[1] when the entire observable Universe occupied a size $2.10^{28}\,\mathrm{h}^{-1} \times (a_0/a_{pl})^{-1} \simeq 4 \times 10^{-4}\,\mathrm{h}^{-1}$ cm. Small as this size was, it was nevertheless much larger than the Planck length $l_{pl} = ct_{pl} \simeq 1.6 \times 10^{-33}$ cm. Clearly in order for the entire observable Universe to have been causally connected at Planck time the present horizon scale must lie within the Planck radius at t_{pl}! As we shall show this can easily be accomplished if an 'inflationary stage' of exponential expansion preceeded the radiation dominated era. Let us assume that prior to the radiation dominated epoch the expansion of the Universe was determined by a cosmological constant for a short duration $\Delta t = t - t_0$, from (5) we know that the expansion of the Universe will be exponential $a \propto \exp H(t-t_0)$, where $H = \sqrt{\Lambda/3}$. Substituting $a = a_0 \exp H(t-t_0)$ in (6) we find, for the horizon scale during inflation

$$r_h = \int_{t_0}^{t} \frac{c\mathrm{d}t}{a} \underset{t\to\infty}{\simeq} \frac{c}{a_0 H}. \tag{8}$$

We see that a light signal starting from $r = 0$ at $t = t_0$ travels a finite coordinate distance in an infinite time, consequently regions initially separated by distances greater than $a_0 \times r_h \simeq cH^{-1}$ will never be in causal contact no matter how long the inflationary stage lasts! The exponential expansion of the Universe during inflation stretches causally connected regions of size cH^{-1} by an amount $\exp H\Delta t$, consequently regions of size $cH^{-1} \sim l_{pl}$ reach macroscopic scales $cH^{-1} \times \exp H\Delta t \sim 10^{-4}$ cm by the time inflation ends, provided $H\Delta t \simeq 67$. Thereafter conventional expansion of the Universe increases these scales to $\sim 10^{28}$ cm during the radiation and matter dominated era's, thereby resolving the horizon problem in standard cosmology. (The size of causally connected regions can of course be much larger than the present horizon size if $H\Delta t > 67$, thus the present Universe would be causally connected on scales $10^{14} \times l_{H0}$ if inflation lasted for $H\Delta t \simeq 100$ e-foldings.)

The fact that the entire observable Universe today might once have been confined within a region smaller than the Hubble size during inflation has several interesting cosmological implications. In most models inflation occurs close to the Planck scale, it is therefore conceivable for fluctuations of quantum-gravitational origin to have played an

[1] The temperature associated with the Planck energy is $T_{pl} \simeq 1.2 \times 10^{19}$ GeV, substituting T_{pl} and the present CMB temperature $T_0 \simeq 2.73° \simeq 2.35 \times 10^{-13}$ GeV into $a_0/a = T/T_0$, we get $a_0/a_{pl} \simeq T_{pl}/T_0 \simeq 5 \times 10^{31}$. The present horizon size scaled to the Planck epoch is therefore $l_{H0}(a_{pl}/a_0) \simeq 4 \times 10^{-4}\,\mathrm{h}^{-1}$ cm.

important role during inflation. This is indeed true, a detailed study of quantum effects in inflationary models has shown that both tensor and scalar fluctuations are generated during inflation. The former propogate as gravity waves whereas the latter grow, giving rise to galaxies and the large scale structure of the Universe. Both leave behind their imprint in the cosmic microwave background (CMB). The discovery of fluctuations in the CMB by the COBE satellite having precisely the scale-invariant Harrison–Zeldovich form generically predicted by inflation has provided strong observational support to the inflationary Universe scenario.

1.2 *The flatness problem*

The matter density in the Universe can be conveniently expressed in terms of the 'flatness' parameter

$$\Omega = \frac{\rho}{\rho_{cr}} \equiv \frac{8\pi G\rho}{3H^2}. \tag{9}$$

The present value of Ω is constrained by observations to be $0.1 \leq \Omega_0 \leq$ few. The behaviour of Ω can be studied using the Einstein equation (2) which we rewrite as

$$\Omega - 1 = \frac{kc^2}{a^2H^2} \tag{10}$$

from where one gets

$$\frac{1-\Omega}{\Omega} = \frac{1-\Omega_0}{\Omega_0} \times \left(\frac{a_0}{a}\right)^{-(1+3w)}. \tag{11}$$

We see that $\Omega(a) \to 1$ as $a \to 0$, indicating that the density always approaches the critical value at early times. The convergence of Ω to unity in the past is a generic feature of cosmological models filled with matter satisfying the 'strong energy condition' $1 + 3w > 0$ $(\rho + 3P > 0)$ but may be violated for matter with the 'inflationary' equation of state $p < -\rho/3$ as we show below.

Values of $\Omega(t)$ corresponding to a present Universe with $\Omega_0 = 0.1$ are given in table 1.

In Newtonian terms, the Ω parameter is the ratio of the potential energy to the kinetic energy of expansion, $\Omega = \mathrm{PE}/\mathrm{KE}$. Therefore a value of Ω very close to unity at the beginning of expansion $|1-\Omega|/1+\Omega = |\mathrm{KE}-\mathrm{PE}|/\mathrm{KE}+\mathrm{PE} \sim 10^{-59}$, is indicative of the extreme fine tuning of initial conditions which is difficult to account for in the standard FRW cosmological model.

Table 1. Values of $|1-\Omega|$ at early epochs corresponding to $\Omega_0 = 0.1$ today.

Temperature	Epoch	$\|1-\Omega\|$
2.7 K	Present	0.9
4000 K	Recombination	0.005
1 MeV	Nucleosynthesis	10^{-15}
10^2 Gev	Electroweak	10^{-25}
10^{19} Gev	Planck	10^{-59}

Let us now discuss how the inflationary paradigm can resolve the flatness problem. We noted earlier that a necessary condition for inflation is that matter violate the strong energy condition so that $w = P/\rho < -1/3$. Rewriting (11) as

$$\frac{1-\Omega_0}{\Omega_0} = \frac{1-\Omega(a)}{\Omega(a)} \times \left(\frac{a}{a_0}\right)^{|1+3w|} \tag{12}$$

one finds

$$\Omega_0 \to 1 \quad \text{as} \quad a_0 \to \infty. \tag{13}$$

We therefore find that the value of Ω can be driven to unity to any required accuracy provided $1 + 3w < 0$ and the ensuing inflationary stage lasts long enough.

2. The inflationary universe

In the previous section we have shown how the expansion law $a(t) \propto \exp Ht$, $H = \sqrt{\Lambda/3}$ may resolve both the horizon and flatness problems in cosmology. For generic inflationary models Λ is not a constant but depends weakly upon time, so that the expansion law becomes quasi-exponential $a(t) \propto \exp[\int H(t)\mathrm{d}t]$. To see how a time dependent cosmological term may be generated consider the scalar field Lagrangian density $\mathcal{L} = (1/2)\partial_i\phi\partial^i\phi - V(\phi)$. A homogeneous scalar field has energy density ρ and pressure P given by

$$\begin{aligned} \rho &= T_0^0 = \tfrac{1}{2}\dot{\phi}^2 + V(\phi), \\ P &= -T_\alpha^\alpha = \tfrac{1}{2}\dot{\phi}^2 - V(\phi). \end{aligned} \tag{14}$$

Clearly the scalar field has the inflationary equation of state $P \simeq -\rho$ if $\dot{\phi}^2 \ll V(\phi)$. In chaotic inflationary models $V(\phi) = m^2\phi^2, \lambda\phi^4$ and $P \simeq -\rho$ is an attractor, in the sense that models with $P > -\rho$ rapidly approach $P \simeq -\rho$ and inflate [1].

A convenient 'first-order' description of inflation is provided by the parameter pair $\{\epsilon, \eta\}$

$$\begin{aligned} \epsilon &= -\frac{\dot{H}}{H^2} = \frac{3}{2}\frac{\dot{\phi}^2}{V(\phi) + \frac{1}{2}\dot{\phi}^2}, \\ \eta &= -\frac{\ddot{H}}{2H\dot{H}} = -\frac{\ddot{\phi}}{H\dot{\phi}}. \end{aligned} \tag{15}$$

Successful inflation requires $\epsilon, \eta < 1$; quasi-exponential inflation demands a more stringent constraint $\epsilon, \eta \ll 1$; $\epsilon = \eta = 1/p$ during power law inflation $a \propto t^p, p > 1$.

Historically the first inflationary models were not of the chaotic kind but were either built around first order GUT phase transitions [2] or were based on higher derivative theories of gravity [3]. Guth's original model (old inflation) suffered from the graceful exit problem and was soon abandoned for new inflationary models in which GUT phase transitions were weakly first order or second order [4, 5]. Inflation can occur in a wide variety of situations, a summary of some inflationary scenario's together with some of their salient features is provided in table 2. These models illustrate that:

Table 2. Inflationary models.

Type	Features	Perturbation spectrum
Old inflation	1st order SSB	
New inflation	2nd order SSB	Flat: $n_S \simeq 1 - \epsilon$
Chaotic inflation	$V(\phi) = \lambda\phi^n$	Flat: $n_S \simeq 1 - \epsilon$
Power-law inflation (i) $a \propto t^p, p > 1$	$V = V_0 \exp(-\mu\phi)$	Red: $n_S < 1$, large GW
Power-law inflation (ii) $a \propto (t_c - t)^{-q}, q > 1$	Induced gravity action	Blue: $n_S > 1$, small GW
Natural inflation	$V = V_0 \cos^2(\phi/M)$	Red: $n_S < 1$, small GW
Starobinsky model	$\mathcal{L} = c_1\mathcal{R} + c_2\mathcal{R}^2$	Flat: $n_S \simeq 1 - \epsilon$, small GW
Kaluza–Klein inflation	Higher dimensional Einstein action	Blue: $n_S \geq 1$
Extended inflation	1st order SSB, Brans–Dicke action	Red $n_S < 1$
Hybrid inflation	Multiple scalars $V(\phi, \psi)$	Blue: $n_S \geq 1$, red
Superstring inflation	Superstring action	Blue: $n_S \geq 1$

(i) Inflation can arise both with and without spontaneous symmetry breaking (SSB).

(ii) The inflaton field driving inflation need not be a fundamental scalar field, it might be: (a) related to the scalar curvature in a higher derivative theory of gravity (such as $\mathcal{L} = c_1\mathcal{R} + c_2\mathcal{R}^2$), (b) a composite field made out of preonic fermion fields, (c) associated with the compatification of extra spatial dimensions in Kaluza–Klein theories.

(iii) Inflation can take place in theories with a non-Einstein gravity sector including higher derivative gravity, induced gravity, Brans–Dicke models and superstring theories.

3. Gravity waves and density perturbations from inflation

Both gravity waves and adiabatic density perturbations are generically predicted by the inflationary Universe scenario, a consequence of the superadiabatic amplification of zero-point vacuum fluctuations in scalar (density) and tensor (gravity wave) modes. Modes are continuously being pushed outside the Hubble radius during inflation, a mode with a fixed wave number k, which left the Hubble radius during inflation will re-enter it later, during the epoch of radiation or matter domination. On entering our horizon during the matter and radiation dominated epochs, density perturbations as well as gravity waves cause distortions in the cosmic microwave background radiation (CMB) due to the Sachs–Wolfe effect.

On scales larger than the Hubble radius the amplitude A_T of a gravity wave mode freezes to a value determined by the Hubble parameter $H_{\rm HC}$ when that mode left the Hubble radius during inflation

$$A_T^2 = Ak^{n_T} \sim (H_{\rm HC}/m_p)^2, \tag{16}$$

where n_T is the spectral index of gravity waves. For exponential inflation the Hubble radius remains virtually unchanged $H_{\rm HC} \simeq \sqrt{\Lambda/3}$ which gives rise to a scale invariant spectrum $n_T \simeq 0$. (The subscript T in A_T and n_T indicates that the waves are tensorial.) COBE observations of the MBR indicate $A_S, A_T \sim \Delta T/T \sim 10^{-5}$ which leads to $H_{\rm HC}/m_p \sim 10^{-5}$

for the value of the inflationary Hubble parameter at a time when a mode presently entering the particle horizon left the inflationary Hubble radius. (This corresponds to $V(\phi_{\rm HC}) \sim 10^{-10} m_{pl}^4$ for the value of the inflaton potential.)

Of great relevence to us today is the spectral energy density of gravity waves, $\Omega_g(\lambda) = \epsilon_g(\lambda)/\epsilon_{cr}$ and its integrated value

$$\Omega_g = \frac{\epsilon_g}{\epsilon_{cr}} \sim (H_{\rm HC}/m_p)^2$$

($\epsilon_{cr} = 3H^2/8\pi G$ is the critical density). The COBE normalized gravity wave spectral density was first obtained by Souradeep and Sahni [6] who showed that the amplitude of gravity waves was smaller than the sensitivity of the current generation of terrestrial bar and beam detectors such as LIGO. The best prospect for detection appears to lie with the space-interferometer LISA.

In analogy with gravity waves, zero point fluctuations in the inflaton field have an amplitude $\delta\phi \sim (H_{\rm HC}/m_p)$. These fluctuations get transformed into fluctuations in the matter density $\delta\rho/\rho$ after the Universe reheats. For generic inflationary models

$$A_S^2 \equiv (\delta\rho/\rho)^2_{\rm HC} = Bk^{n_s-1}. \tag{17}$$

$n_s = 1$ corresponds to the scale invariant Harrison–Zeldovich spectrum. Values of the spectral index n_S are shown in table 2 for some inflationary models. Most models predict either scale invariant $n_S \simeq 1$ or 'red' spectra $n_S \leq 1$, however blue spectra $n_S \geq 1$ also occasionally arise in some models.

For exponential and power-law inflation the spectral indices n_S, n_T do not evolve with time. In other models some time-dependence in the indices may be present and one may therefore write $A_S^2 \propto k^{n_s(k)-1}$, $A_T^2 \propto k^{n_T(k)}$ (since time-dependence translates into scale-dependence). In the slow roll approximation, scalar and tensor spectra can be directly related to physical parameters during inflation such as H, $\dot{H}$ etc. encoded in the inflationary parameters ϵ and η introduced in (15)

$$\begin{aligned} n_S(k) - 1\Big|_{\rm HC_2} &= \frac{2\eta - 4\epsilon}{1-\epsilon}\Big|_{\rm HC_1} \\ n_T(k)\Big|_{\rm HC_2} &= -\frac{2\epsilon}{1-\epsilon}\Big|_{\rm HC_1}. \end{aligned} \tag{18}$$

HC refers to the 'Hubble crossing' scale, waves left the Hubble radius during inflation $\rm HC_1$ and re-entered it during matter/radiation domination $\rm HC_2$. These relations are valid so long as the spectral indices of the gravity wave and density perturbation spectra do not vary rapidly with scale a requirement which normally translates into $|\epsilon - \eta| \ll 1$. For power law inflation ($a \propto t^p, p > 1$) and $\epsilon = \eta = 1/p$, which results in: $n_S - 1 = n_T = 2/(1-p)$. We therefore find that scalar and tensor spectra are produced with identical spectral indices in power law inflation! Quasi-exponential (chaotic) inflation can be approximated by a large power-law $p \gg 1$ leading to approximately scale invariant spectra $n_S \simeq 1$, $n_T \simeq 0$.

It is sometimes useful to view the scalar field ϕ as the dynamical variable in the Einstein equations and rewrite (2) and (14) as

$$H'^2(\phi) - \frac{12\pi}{m_{pl}^2} H^2(\phi) = -\frac{32\pi^2}{m_{pl}^2} V(\phi) \tag{19}$$

where $H' \equiv \mathrm{d}H/\mathrm{d}\phi = -4\pi G\dot{\phi}$ etc. The scalar and tensor amplitudes now become

$$A_S = \frac{8\pi}{5} c_1 \frac{H^2}{|H'| m_{pl}^2}\bigg|_{\mathrm{HC}} \qquad A_T = \sqrt{16\pi} c_2 \frac{H}{m_{pl}}\bigg|_{\mathrm{HC}}, \tag{20}$$

where c_1, c_2 are model dependent parameters.

It is easy to show that $A_T/A_S \simeq 5\sqrt{\epsilon}$, which demonstrates that the ratio of the gravity wave amplitude to that of density fluctuations can be quite small for models very close to exponential inflation ($\epsilon \ll 1$). On the other hand, $\epsilon = 1/p$ in models of power-law inflation so that $A_T/A_S \sim 1$ if $p \sim 10$, i.e. the gravity wave amplitude can be appreciable in these models. Finally (20) may be used to express the inflationary potential (19) in terms of scalar and tensor amplitudes $A_S\,(k)$ and $A_T(k)$

$$V(\phi) = \left(\frac{m_{pl} A_T}{8}\right)^2 \left[1 - \frac{2}{25}\left(\frac{A_T}{A_S}\right)^2\right] \tag{21}$$

where the expression

$$k(\phi) = a_e H(\phi) \exp\left[-N(\phi)\right] \tag{22}$$

relates the wave number just leaving the Hubble radius during inflation with the value of the scalar field ϕ. a_e is the scale factor at the end of inflation and $N(\phi)$ the number of e-foldings as the scalar field rolls down its potential from ϕ to

$$\phi_e \colon N(\phi, \phi_e) \equiv \int_t^{t_e} H(t)\mathrm{d}t = -(4\pi/m_{pl}^2)\int_\phi^{\phi_e} (H/H')\mathrm{d}\phi.$$

Therefore knowing A_T and A_S – the tensor and scalar amplitudes generated during inflation, it is in principle possible to reconstruct the inflation potential $V(\phi)$ over a limited range of scales [7].

4. Inflationary models and the large scale structure of the Universe

The COBE satellite observed regions on the sky separated by $\theta > 7°$ thus probing angular scales that were causally disjoint at the time of recombination. The COBE detection of anisotropy $\Delta T/T \sim 10^{-5}$ on these scales meant that fluctuations on super-horizon scales were already present at the time of the cosmological recombination of hydrogen about a hundred thousand years after the big bang. As we have seen, the inflationary scenario provides a resolution of this dilemma since, scales originally within the Hubble radius during inflation, would be exponentially stretched and become super-horizon size later, during the radiation and matter dominated epochs. The temperature distribution of the cosmic microwave background (CMB) can be written as a multipole expansion on the celestial sphere

$$T(\theta, \phi) = T_0\left[1 + \frac{\delta T}{T}(\theta, \phi)\right]$$
$$\frac{\delta T}{T}(\theta, \phi) = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_l^m(\theta, \phi), \tag{23}$$

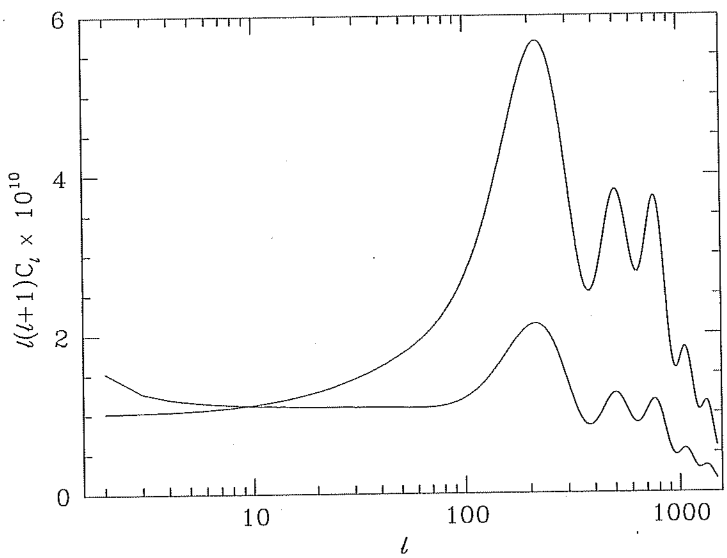

Figure 1. Angular power spectrum for a critical density Universe with cold dark matter, $\Omega_b = 0.05$ and spectral index $n = 1$ (upper panel), $n = 0.85$ (lower panel). (Figure courtesy of M White [11]).

the corresponding 'angular power spectrum' is $C_l \equiv \langle |a_{lm}|^2 \rangle$, (θ, ϕ) are spherical polar coordinates associated with a given direction in the sky, and $T_0 \simeq 2.728 \pm 0.004$ K. As we discussed earlier the scalar/tensor contribution to C_l depends rather sensitively on the parameters of the inflationary model. For power law inflation the contribution to the CMB quadrupole C_2 is [8, 6] $T/S \simeq 7(1 - n_S)$. Therefore the tensor contribution to the large angle anisotropies of the CMB is negligible for $n_S = 1$ but is significant (50%) for $n_S = 0.84$. On scales $\theta < 1°$, smaller than the horizon size at recombination, the situation is quite different. Scalar perturbations on these scales show an increased amplitude due to 'Sakharov oscillations' caused by the strong coupling between baryons and photons prior to recombination whereas the tensor contribution shows a sharp fall for multipoles $l > 60$ corresponding to angular scales $\theta < 1°$ $(l/60 \simeq 1°/\theta)$. As a result the amplitude of the angular spectrum C_l at $l \simeq 100$ is strongly peaked in models having a small gravity wave contribution. This effect is illustrated in figure 1 where we show CMB multipoles C_l for two spectra (both normalised using COBE observations at $l = 10$):

(i) $n_S = 1$, gravity waves contribute negligibly on all angular scales (upper curve),
(ii) $n_S = 0.85$, gravity wave contribution is significant only on large scales (lower curve).

From figure 1 we immediately see that observations of the CMB performed at $l \sim 2-15$ (COBE) and $l \geq 60$ (MAP, Planck surveyor etc.) will go far in separating the scalar and tensor contributions and establishing the value of the spectral index n_S. In figure 2 we show the expected CMB anisotropy in some promising cosmological models, also shown are current observational results and projected sensitivity of future MBR missions [10].

On scales smaller than the horizon size at recombination, the primordial density perturbation spectrum (generated during inflation) gets modified by several astrophysical

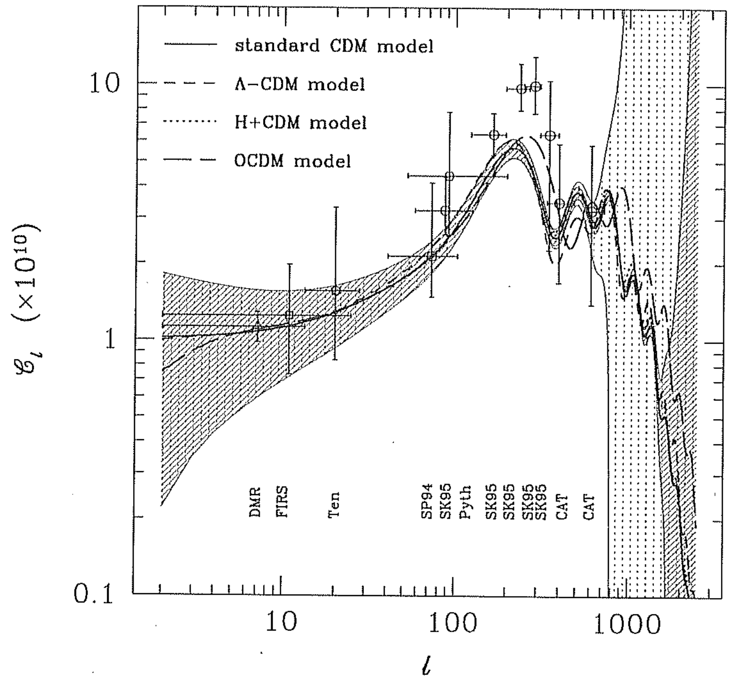

Figure 2. CMB anisotropy expected in several theoretical models CDM, H + CDM, Λ-CDM and open CDM. Also shown are CMB anisotropy measurements from some recent experiments (details are in page 1997). The tilted solid shading shows the expected performance of MAP and Planck Surveyor satellite missions.

processes including: the nature of dark matter (whether 'hot', 'cold' etc.), the fraction of baryonic matter in the Universe, the number of neutrino species, the possibility of reionisation etc. On the largest scales > 1000 Mpc, the initial spectrum of density fluctuations $P(k) = \langle|\delta_k|^2\rangle$ has been probed indirectly by fluctuations in the CMB measured by COBE (the CMB power spectrum C_l and $P(k)$ are linked through the Sachs–Wolfe effect). On smaller scales $\leq$ 200 Mpc the power spectrum has been probed by large scale galaxy surveys both in the optical (CfA, SSRS, LCRS) and in the infrared (IRAS) and by the velocity fields of galaxies. Probing large scale structure using velocity fields has one significant advantage: deviations of the velocity field from the isotropic Hubble flow are caused by fluctuations in both dark and luminous matter, thus **v** linked to the density contrast δ through the continuity equation $\delta = -(1/a\Omega^{0.6}H)\vec{\nabla}\cdot \mathbf{v}$, determines the 'total' matter density in the Universe. Observations of the outflow of galaxies in voids set a lower bound $\Omega \geq 0.3$ [9].

The all-important region 100–1000 Mpc will be probed in the near future by CMB missions MAP and Planck surveyor (which will complement COBE by measuring $\Delta T/T$ down to scales of several arc minutes). In addition large scale galaxy surveys SDSS and 2dF will determine redshifts of over a million galaxies and sample the Universe to a depth of $\sim$600 h^{-1} Mpc. Data coming from CMB missions combined with data from large galaxy surveys is certain to help resolve the following important issues in cosmology:

(i) the nature of dark matter,
(ii) the ratio of scalar and tensor fluctuations and the form of the inflationary potential,
(iii) the scale of homogeneity in the Universe,
(iv) the epoch of structure formation.

Thus the next decade promises much excitement and many new surprises for inflationary cosmology.

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