

Phenomenology of the Neutrino-Mass-Giving Higgs Triplet and the Low-Energy Seesaw Violation of Lepton Number

Ernest Ma¹, Martti Raidal^{1,2}, and Utpal Sarkar³

¹ *Physics Department, University of California, Riverside, California 92521, USA*

² *National Institute of Chemical Physics and Biophysics, Tallinn 10143, Estonia*

³ *Physical Research Laboratory, Ahmedabad 380 009, India*

Abstract

Small realistic Majorana neutrino masses can be generated via a Higgs triplet (ξ^{++}, ξ^+, ξ^0) without having energy scales larger than $M_* = \mathcal{O}(1)$ TeV in the theory. The large effective mass scale Λ in the well-known seesaw neutrino-mass operator $\Lambda^{-1}(LL\Phi\Phi)$ is naturally obtained with $\Lambda \sim M_*^2/\mu$, where μ is a *small* scale of lepton-number violation. In theories with large extra dimensions, the smallness of μ is naturally obtained by the mechanism of “shining” if the number of extra dimensions $n \geq 3$. We study here the Higgs phenomenology of this model, where the spontaneous violation of lepton number is treated as an external source from extra dimensions. The observable decays $\xi^{++} \rightarrow l_i^+ l_j^+$ will determine directly the magnitudes of the $\{ij\}$ elements of the neutrino mass matrix. The decays $\xi^+ \rightarrow W^+ J^0$ and $\xi^0 \rightarrow Z J^0$, where J^0 is the massless Goldstone boson (Majoron), are also possible, but of special importance is the decay $\xi^0 \rightarrow J^0 J^0$ which provides stringent constraints on the allowed parameter space of this model. Based on the current neutrino data, we also predict observable rates of $\mu - e$ conversion in nuclei.

1 Introduction

The idea that standard-model (SM) fields can be localized on a brane has greatly changed our approaches to quantum gravity and extra dimensions. It allows the fundamental scale of quantum gravity to be as low as $\mathcal{O}(1)$ TeV [1], thus providing distinctive experimental signatures of extra dimensions at future colliders [2]. These include the radiation of gravitons into extra dimensions and the exchange of graviton Kaluza-Klein towers which modify SM neutral-current processes.

An important development in these TeV-scale extra-dimensional theories is their implication on new approaches to low-energy model building. They allow us to consider new possibilities in addressing current topics of interest in particle physics. Perhaps the most important of them is the question of nonzero neutrino masses, as indicated by the atmospheric and solar neutrino anomalies [3]. In extra-dimensional theories, there is a possibility that neutral gauge singlet particles are not confined to our 3-brane and propagate in the full volume of the theory, similarly to the gravitons. The first idea of explaining the smallness of neutrino masses in extra-dimensional theories was to introduce singlet neutrinos in the bulk [4, 5]. These become the right-handed partners of the observed left-handed neutrinos, but with small Dirac masses, being suppressed by the large volume of the extra dimensions.

More recently, we have proposed an alternative scenario [6, 7] which obtains small *Majorana* neutrino masses instead. Given the SM particle content, such neutrino masses are induced by a single effective operator [8]

$$\frac{1}{\Lambda} LL\Phi\Phi, \tag{1}$$

where L and Φ are the SM lepton and Higgs-boson doublets respectively, and Λ is the *effective* mass scale of the new physics; any extension of the SM to generate small Majorana neutrino masses is merely a particular realization of this effective operator provided the

particle content of the model below the electroweak scale remains the SM one [9]. The scale Λ must be very large, $\mathcal{O}(10^{12} - 10^{15})$ GeV, to account for realistically small neutrino masses. It is usually associated with the scale of lepton-number violation, as in all models with the canonical seesaw mechanism [10] in which the small neutrino masses are inversely proportional to the very large right-handed Majorana neutrino masses, $\Lambda \sim M_N$. However, this is just one specific solution which cannot be preferred over any other possible realization of Eq. (1) from the viewpoint of low-energy phenomenology. For example, if

$$\frac{1}{\Lambda} \sim \frac{\mu}{M^2}, \quad (2)$$

where M is the fundamental scale of new physics ($M \sim \mathcal{O}(1)$ TeV in our case) and μ is the *small* scale associated with the breaking of lepton number, we still achieve the same very large effective scale Λ in Eq. (1) *without actually having the very large scale in the theory*. Similar possibilities of generating small neutrino masses with a low-scale seesaw mechanism have also been considered recently in Ref. [11]. The crucial advantage of the low-scale seesaw mechanism (2) over the canonical scenario is that the former allows unambiguous tests of the neutrino-mass-giving mechanism at future colliders while the latter cannot be directly probed at terrestrial experiments.

In our recently proposed scenario [6] (for first similar proposals see [4]), we use the distant breaking [12] of lepton number to generate small Majorana neutrino masses through a scalar Higgs triplet [13] localized in our 3-brane. The trilinear interaction of the triplet with the SM Higgs doublets required for this mechanism is induced by the “shining” of a scalar singlet which lives in the bulk and communicates the breaking of lepton number from another brane to our world. The smallness of neutrino masses, or equivalently the smallness of the lepton-number breaking scale μ in Eq. (2), comes from the suppression of the Yukawa potential of the bulk singlet by the large separation of the two branes in the extra dimensions. To achieve the desired suppression, the number of extra dimensions n should be at least 3. Notice that

the structure of TeV-scale quantum gravity itself suggests naturally such a mechanism of breaking global quantum numbers because it contains “our world” on a 3-brane, the “hidden sector” on another brane not necessarily identical to our brane, and the “messenger sector” with particles living in the bulk. The analogous mechanism for breaking the Peccei-Quinn symmetry has been considered in Ref. [14].

The aim of the present paper is to work out details of our model proposed in Ref. [6], concentrating on the collider and low-energy phenomenology predicted by this model. In doing that, we first have to address the question of how to treat the field theory in our 3-brane consistently if n extra dimensions are present at scales $\mathcal{O}(1)$ TeV. We show that this can be achieved by considering the 4-dimensional field theory in the presence of an external source. In this case the singlet bulk field decouples from the fields in our world, except for the massless Majoron which propagates also in the bulk and provides the only connection between these two sectors.

We first work out the Higgs phenomenology of the model and show that the decay of the neutral component of the Higgs triplet into two Majorons, i.e. $\xi^0 \rightarrow J^0 J^0$, stringently constrains the pattern of its vacuum expectation values (VEVs). Since the neutrino mass matrix in this model is uniquely fixed by the Yukawa couplings of the Higgs triplet to the leptons, the existing neutrino data may be used to determine the entries of this Yukawa matrix up to a normalization scale. We can thus predict the rates of rare unobserved lepton-flavour violating processes both at low-energy experiments as well as at colliders. Conversely, if the Higgs triplet is kinematically accessible at future colliders, the decay branching fractions of its doubly charged component into charged leptons, i.e. $\xi^{++} \rightarrow l_i^+ l_j^+$, will determine uniquely the relative magnitude of each element of the neutrino mass matrix. Our neutrino-mass scenario is thus directly and unambiguously testable at colliders and can foretell the results of future neutrino factories and long-baseline neutrino-oscillation experiments.

This paper is organized as follows. In Section 2 we introduce the model. In Section 3 we study the Higgs potential of our model. In Section 4 we work out the details of the Higgs-boson phenomenology at colliders. In Section 5 we study the connection between neutrino-oscillation data and lepton-flavour violating processes. Conclusions are in Section 6.

2 The Triplet Model in the Presence of Extra Dimensions

The fermion sector of our model is identical to that of the SM, containing the left-handed lepton doublets and right-handed charged-lepton singlets with the following $SU(2)_L \times U(1)_Y$ quantum numbers:

$$L_i = \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_L \sim (2, -1/2), \quad l_{iR} \sim (1, -1), \quad (3)$$

where $i = e, \mu, \tau$. There are no right-handed neutrinos in the model.

The Higgs sector consists of the usual SM doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (2, 1/2), \quad (4)$$

and two additional scalar fields, a triplet ξ and a singlet χ :

$$\xi = \begin{pmatrix} \xi^+/\sqrt{2} & \xi^{++} \\ -\xi^0 & -\xi^+/\sqrt{2} \end{pmatrix} \sim (3, 1), \quad \chi = \chi^0 \sim (1, 0). \quad (5)$$

The latter two fields carry lepton number $L = -2$. The Higgs triplet ξ is presented in the form of a 2×2 matrix transforming under $SU(2)$ as $\xi \rightarrow U\xi U^\dagger$. The triplet couples to leptons via the Yukawa interaction

$$\mathcal{L}_Y = f_{ij} L_i^T C^{-1} i\tau_2 \xi L_j + h.c.. \quad (6)$$

If neutrinos obtain Majorana masses via Eq. (6), lepton number is broken by two units. It is important to notice that the neutrino mass matrix in this case is proportional to a single Yukawa matrix f_{ij} . Thus any inference from neutrino-oscillation data regarding the actual form of the neutrino mass matrix may now be tested in low-energy lepton-flavour violating processes as well as in collider experiments where ξ may be produced and its decays observed. The results of future neutrino factories and long-baseline neutrino-oscillation experiments may also be predicted.

If the lepton-number violation occurs spontaneously [15] via the VEV of the triplet in Eq. (6), a massless Majoron will appear. Based on searches for it at LEP via the invisible width of the Z , this model is ruled out. However, if lepton number is violated explicitly [16] by the trilinear coupling $\mu\Phi^\dagger\xi\tilde{\Phi}$ as in Ref. [13], there is no (triplet) Majoron and no contradiction with present experimental data (for allowed Majoron models see, e.g., Ref. [17]).

In our model as proposed in Ref. [6], the SM fields together with ξ are localized in our world (a 3-brane \mathcal{P} at $y = 0$) and are blind to the extra space dimensions. Lepton number is assumed to be conserved as far as these fields are concerned. The Higgs singlet χ which carries lepton number is special because

- it propagates also in the bulk;
- it serves as a “messenger” which communicates the violation of lepton number from another brane to our world through the large extra dimensions.

We assume the existence of a field η which is localized in a distant 3-brane (\mathcal{P}') situated at a point $y = y_*$ in the extra dimensions. It is a singlet under the standard model but has $L = 2$ and couples to χ (with $L = -2$). When the field η acquires a VEV, lepton number is broken maximally in the other brane. It will then act as a point source for L violation, and

the field χ is the messenger which carries it to our wall (the interface between our brane and the bulk). The “shining” of χ at all points in our world is the mechanism [12] which breaks lepton number and gives mass to the neutrinos.

At energies much below the fundamental scale M_* , the lepton-number violating effect will be suppressed by the distance between the source brane at \mathcal{P}' and our brane at \mathcal{P} . We assume that the source brane is situated at the farthest point in the extra dimensions so that $|y_*| = r$ is the radius of compactification and it is related to the fundamental scale M_* and the reduced Planck scale ($M_P = 2.4 \times 10^{18}$ GeV) by the relation

$$r^n M_*^{n+2} \sim M_P^2. \quad (7)$$

This explains why lepton number is only violated weakly in our world.

We assume here that the source brane has the same dimensional structure as our world and there are n extra dimensions. In our world (\mathcal{P}) the field χ has only the lepton-number conserving interaction of the form $\Phi(x)^\dagger \xi(x) \tilde{\Phi}(x) \chi^\dagger(x, y = 0)$. In the other brane (\mathcal{P}') the field χ couples to the field η through the interaction

$$\mathcal{S}_{other} = \int_{\mathcal{P}'} d^4 x' \mu^2 \eta(x') \chi(x', y = y_*), \quad (8)$$

where μ is a mass parameter. Lepton-number violation from $\langle \eta \rangle$ is carried by χ to our world through its “shined” value $\langle \chi \rangle$:

$$\langle \chi(x, y = 0) \rangle = \Delta_n(r) \langle \eta(x, y = y_*) \rangle, \quad (9)$$

where $\langle \eta \rangle$ acts as a point source, and $\Delta_n(r)$ is the Yukawa potential in n transverse dimensions, i.e. [12]

$$\Delta_n(r) = \frac{1}{(2\pi)^{\frac{n}{2}} M_*^{n-2}} \left(\frac{m_\chi}{r} \right)^{\frac{n-2}{2}} K_{\frac{n-2}{2}}(m_\chi r), \quad (10)$$

K being the modified Bessel function. If the mass of the carrier field χ is large ($m_\chi r \gg 1$),

it has the profile

$$\langle\chi\rangle\approx\frac{m_\chi^{\frac{n-3}{2}}}{2(2\pi)^{\frac{n-1}{2}}M_*^{n-2}}\frac{e^{-m_\chi r}}{r^{\frac{n-1}{2}}}. \quad (11)$$

The suppression here is exponential, hence the amount of lepton-number violation in our world is very small, but its precise value depends sensitively on m_χ . An interesting alternative is to have a light carrier field with a mass less than $1/r$. However, it should be larger than about $(0.1\text{ mm})^{-1}$, to be consistent with the present experimental data on gravitational interactions.

If $m_\chi r \ll 1$, $\Delta_n(r)$ is logarithmic for $n = 2$ and $\langle\chi\rangle$ is not suppressed. For $n > 2$, the asymptotic form of the profile of χ is

$$\langle\chi\rangle\approx\frac{\Gamma(\frac{n-2}{2})}{4\pi^{\frac{n}{2}}}\frac{M_*}{(M_*r)^{n-2}}, \quad (12)$$

which is suitably small for large r . Because of the interaction $\Phi^\dagger\xi\tilde{\Phi}\chi^\dagger$ in our brane, the triplet ξ will acquire a small effective VEV via tadpole diagrams as Φ and χ acquire VEVs. Thus small Majorana neutrino masses are generated via Eq. (6). Because the breakdown of lepton number in the distant brane occurs spontaneously, there exists a (singlet) Majoron which propagates also in the bulk and plays a very special role in our model.

To realize the above-described idea of neutrino masses (or any other in which some particles propagate both on the 3-brane and also in the bulk), one has to answer a nontrivial question: i.e. *how to describe the four-dimensional field theory in our brane consistently if the large extra dimensions are present?* Clearly, in the full $4 + n$ -dimensional theory, the precise answer should be derived from a consistent theory of quantum gravity which is, however, not yet available. On the other hand, the SM particles are confined on a 3-brane, and we do know how to treat them in the context of a field theory in four dimensions. To take into account possible new physics effects from extra dimensions, we propose in this work three simple *ansätze*. We assume that:

- (i) independently of the origin of lepton-number violation, it is communicated to our world solely via the small nonzero VEV of the field χ ; details of the physics in extra dimensions giving rise to it are irrelevant for our phenomenological approach;
- (ii) the value of the VEV of the singlet χ does not depend on the parameters in our 3-brane *i.e.*, on other parameters of the model;
- (iii) the Majoron is the massless field (in analogy to the graviton) which may cross from the bulk to the brane.

It follows from these assumptions that the model dynamics in our world should be determined using the four-dimensional field theory in the presence of *an external source* (provided in our model by χ via the “shining” mechanism.) As we show in the next section, the way χ is being treated in our model is very different from what it would be if it were an ordinary singlet confined to our brane.

3 Consistent Treatment of the Higgs Potential

For our purpose it is convenient to express the bulk field χ as

$$\chi = \frac{1}{\sqrt{2}}(\rho + z)e^{i\varphi}, \quad (13)$$

where $z/\sqrt{2} \equiv \langle \chi \rangle$ denotes the VEV of the field in our brane. According to our assumptions (i) and (ii) in Section 2, the VEV z should be regarded as a boundary condition, which is not altered by any other parameter of the model; all such effects are already included in z . In theories of extra dimensions, z is induced by the “shining” mechanism and its numerical value is small, $z \ll M_Z$. The lepton-number transformations of ρ and φ under $U(1)_L$ are given by

$$\rho \rightarrow \rho, \quad \varphi \rightarrow \varphi - 2x, \quad (14)$$

while the $U(1)_L$ transformations for neutrinos and the Higgs triplet read as usual:

$$\nu \rightarrow e^{ix}\nu, \quad \xi \rightarrow e^{-2ix}\xi. \quad (15)$$

The self-interaction terms for the bulk scalar ρ can now be expressed as

$$V(\chi) = \lambda_\chi z^2 \rho^2 + \lambda_\chi z \rho^3 + \frac{1}{4} \lambda_\chi \rho^4 \quad (16)$$

and the lepton-number conserving Higgs potential of the other fields as

$$\begin{aligned} V = & m_0^2 \Phi^\dagger \Phi + m_\xi^2 \text{Tr}[\xi^\dagger \xi] + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 \text{Tr}[\xi^\dagger \xi]^2 + \lambda_3 (\Phi^\dagger \Phi) \text{Tr}[\xi^\dagger \xi] \\ & + \lambda_4 \text{Tr}[\xi^\dagger \xi^\dagger] \text{Tr}[\xi \xi] + \lambda_5 \Phi^\dagger \xi^\dagger \xi \Phi + \left(\frac{\lambda_0 z e^{-i\varphi}}{\sqrt{2}} \Phi^\dagger \xi \tilde{\Phi} + h.c. \right), \end{aligned} \quad (17)$$

where $m_0^2 < 0$, but $m_\xi^2 > 0$. Notice the presence of the VEV z in the last term of Eq. (17), which gives rise to the desired trilinear coupling of the Higgs doublets to the triplet.

In a similar fashion, let us express

$$\phi^0 = \frac{1}{\sqrt{2}}(H + v)e^{i\theta}, \quad \xi^0 = \frac{1}{\sqrt{2}}(\zeta + u)e^{in}, \quad (18)$$

where $v/\sqrt{2}$ and $u/\sqrt{2}$ are the vacuum expectation values of ϕ^0 and ξ^0 respectively. This way of writing allows a simple and consistent treatment of the massless Goldstone modes of the model. Consider now only the neutral scalar fields H , ζ , and the correctly normalized fields $v\theta$, $u\eta$, and $z\varphi$, then

$$\begin{aligned} V_0 = & \frac{1}{2} m_0^2 (H + v)^2 + \frac{1}{2} m_\xi^2 (\zeta + u)^2 + \frac{1}{8} \lambda_1 (H + v)^4 + \frac{1}{8} \lambda_2 (\zeta + u)^4 + \frac{1}{4} \lambda_3 (H + v)^2 (\zeta + u)^2 \\ & - \frac{\lambda_0 z}{2} (H + v)^2 (\zeta + u) \left[1 - \frac{1}{2} (\varphi - \eta + 2\theta)^2 + \dots \right]. \end{aligned} \quad (19)$$

The minimum of V_0 is determined by the first-derivative conditions

$$\begin{aligned} m_0^2 + \frac{1}{2} \lambda_1 v^2 + \frac{1}{2} \lambda_3 u^2 - \lambda_0 z u &= 0, \\ u \left(m_\xi^2 + \frac{1}{2} \lambda_2 u^2 + \frac{1}{2} \lambda_3 v^2 \right) - \frac{1}{2} \lambda_0 z v^2 &= 0. \end{aligned} \quad (20)$$

Therefore, $v^2 \simeq -2m_0^2/\lambda_1$ as usual, but $u \simeq \lambda_0 z v^2 / 2m_\xi^2$, with $u, z \ll v$. The small VEV u of the triplet, which gives masses to the neutrinos via Eq. (6), is proportional to the value of z and inversely proportional to the square of the Higgs triplet mass, i.e. m_ξ^2 . Thus the smallness of the singlet VEV z together with the possible suppression by other free parameters of the model should ensure the correct order of magnitude for u as determined by the scale of the neutrino masses.

Solving Eq. (20) for the parameters m_0^2 and m_ξ^2 , the mass-squared matrix of the neutral scalar fields in the (H, ζ) basis is given by

$$\mathcal{M}_S^2 = \begin{bmatrix} \lambda_1 v^2 & \lambda_3 u v - \lambda_0 z v \\ \lambda_3 u v - \lambda_0 z v & \frac{1}{2} \lambda_0 v^2 z / u + \lambda_2 u^2 \end{bmatrix}. \quad (21)$$

Since $u, z \ll v$, the fields H and ζ are almost exact mass eigenstates. Thus H behaves just like the SM Higgs boson and ζ is a heavy neutral scalar boson of mass $\simeq m_\xi$.

For the pseudoscalar fields, the mass-squared matrix in the basis $(z\varphi, u\eta, v\theta)$ is given by

$$\mathcal{M}_{PS}^2 = \frac{1}{2} \lambda_0 z v^2 u \begin{bmatrix} 1/z^2 & -1/zu & 2/zv \\ -1/zu & 1/u^2 & -2/uv \\ 2/zv & -2/uv & 4/v^2 \end{bmatrix}. \quad (22)$$

This mass matrix can be diagonalized by the orthogonal matrix

$$U = U_J U_G, \quad (23)$$

where

$$U_G = \frac{1}{\sqrt{v^2 + 4u^2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2u & v \\ 0 & -v & 2u \end{pmatrix}, \quad (24)$$

and

$$U_J = \frac{1}{\sqrt{u^2 v^2 / (v^2 + 4u^2) + z^2}} \begin{pmatrix} z & 0 & -uv/\sqrt{v^2 + 4u^2} \\ 0 & 1 & 0 \\ uv/\sqrt{v^2 + 4u^2} & 0 & z \end{pmatrix}. \quad (25)$$

Thus the physical mass eigenstates can be found from

$$\begin{pmatrix} J^0 \\ G^0 \\ \Omega^0 \end{pmatrix} = U \begin{pmatrix} z\varphi \\ u\eta \\ v\theta \end{pmatrix}, \quad (26)$$

where G is the Goldstone mode giving mass to the Z boson, J^0 is the physical massless Majoron, and Ω^0 is the physical massive pseudoscalar boson which is mostly triplet and is the partner of ζ . The factorization of U in Eq. (23) is particularly useful since it allows the immediate recognition of the Z -boson longitudinal component given explicitly by

$$G^0 = \frac{v^2\theta + 2u^2\eta}{\sqrt{v^2 + 4u^2}}, \quad (27)$$

as well as the physical Majoron

$$J^0 = \frac{(v^2 + 4u^2)z^2\varphi + v^2u^2\eta - 2u^2v^2\theta}{\sqrt{z^2(v^2 + 4u^2)^2 + u^2v^4 + 4v^2u^4}}. \quad (28)$$

The massive combination of $(z\varphi, u\eta, v\theta)$ is of course

$$\Omega^0 = \frac{\varphi - \eta + 2\theta}{\sqrt{z^{-2} + u^{-2} + 4v^{-2}}}, \quad (29)$$

with its mass-squared given by

$$M_\Omega^2 = \frac{1}{2}\lambda_0 \left(v^2\frac{z}{u} + 4uz + v^2\frac{u}{z} \right). \quad (30)$$

Notice that Ω^0 is almost degenerate in mass with ζ as expected.

At this point a comment is in order. Notice that the massive Higgs singlet propagating in the bulk, i.e. ρ , is completely decoupled from the Higgs fields living in our 3-brane. The only connection between these two sectors is due to their couplings to the Majoron J^0 .

Similarly we find the masses of the charged Higgs bosons. The singly-charged Higgs mass-squared matrix in the basis (ϕ^+, ξ^+) is found to be

$$\mathcal{M}_+^2 = \left(\lambda_0 z + \frac{1}{2}\lambda_5 u \right) \begin{bmatrix} u & v/\sqrt{2} \\ v/\sqrt{2} & v^2/(2u) \end{bmatrix}. \quad (31)$$

The longitudinal component of W^+ is easily found to be

$$G^+ = \frac{v \phi^+ - \sqrt{2}u \xi^+}{\sqrt{v^2 + 2u^2}}, \quad (32)$$

while the massive physical charged Higgs boson is orthogonal to that,

$$h^+ = \frac{\sqrt{2}u \phi^+ + v \xi^+}{\sqrt{v^2 + 2u^2}}, \quad (33)$$

with mass squared

$$M_{h^+}^2 = \frac{1}{2} \left(\lambda_0 \frac{z}{u} + \frac{1}{2} \lambda_5 \right) (v^2 + 2u^2). \quad (34)$$

The would-be Goldstone boson is predominantly doublet while the physical charged Higgs boson is predominantly triplet.

Finally, the mass of the doubly-charged Higgs boson ξ^{++} is given by

$$M_{\xi^{++}}^2 = \frac{1}{2} \left(\lambda_0 \frac{z}{u} + \lambda_5 \right) v^2 + 2\lambda_4 u^2. \quad (35)$$

Therefore $M_{\xi^{++}}^2 - M_{h^+}^2 \approx M_{h^+}^2 - M_{\zeta^0}^2 \approx \lambda_5 v^2 / 4$ as expected.

4 Higgs Phenomenology at Colliders

4.1 Neutral sector

In hadron and lepton colliders, the neutral Higgs bosons can be produced via the gauge interactions in the Drell-Yan and Bjorken processes mediated only by the Z boson. The production follows by the kinematically allowed decays

$$\zeta^0, \Omega^0 \rightarrow \nu\nu, \bar{\nu}\bar{\nu}, \quad (36)$$

$$\zeta^0, H^0 \rightarrow ZJ^0, \quad (37)$$

$$\zeta^0, H^0 \rightarrow J^0J^0, \quad (38)$$

$$\zeta^0 \rightarrow H^0H^0, \quad (39)$$

$$\Omega^0 \rightarrow H^0J^0. \quad (40)$$

Here the first decays (36) to neutrinos come from the Yukawa interaction of Eq. (6) and their widths depend on the magnitudes of the Yukawa couplings. The others follow from the scalar self-interactions. The decays (39) and (40) are suppressed because $u, z \ll v$. There are of course also the well-known SM decays of H^0 , but we do not discuss them here. Thus the processes involving the new neutral scalar bosons ζ^0 and Ω^0 are practically invisible and are not of great phenomenological interest at collider experiments. On the other hand, as we show below, they do constrain the allowed parameter space of our model.

The couplings of the Majoron to the other Higgs bosons follow from the kinetic-energy terms involving ϕ^0 and ξ^0 . These are given by

$$\partial_\mu \bar{\phi}^0 \partial^\mu \phi^0 + \partial_\mu \bar{\xi}^0 \partial^\mu \xi^0 = \frac{1}{2}(\partial_\mu H)^2 + \frac{1}{2}(H + v)^2(\partial_\mu \theta)^2 + \frac{1}{2}(\partial_\mu \zeta)^2 + \frac{1}{2}(\zeta + u)^2(\partial_\mu \eta)^2. \quad (41)$$

Reversing Eq. (26) and substituting the fields into the interaction terms above, we find a term involving $(\partial_\mu J)^2$ given by

$$\frac{(4vu^4H + uv^4\zeta)(\partial_\mu J)^2}{z^2(v^2 + 4u^2)^2 + u^2v^4 + 4v^2u^4}. \quad (42)$$

Because $u, z \ll v$, the coupling $H(\partial_\mu J)^2$ is suppressed by the small factor $(u/v)^3$ so the decay $H \rightarrow JJ$ is completely negligible. However, the coupling $\zeta(\partial_\mu J)^2$ is not suppressed. To the contrary, the decay of ζ into two massless Majorons in (38) is enhanced by the large mass of ζ ($\partial_\mu \rightarrow M_\zeta$ in the calculation). Indeed

$$\Gamma(\zeta \rightarrow JJ) \approx \frac{1}{64\pi} \frac{M_\zeta^3 u^2}{(u^2 + z^2)^2}, \quad (43)$$

which implies that in order for the ζ width not to exceed its mass, z is required to be at least of order MeV, i.e. much larger than the scale of the neutrino masses. Therefore we must have $u \ll z \ll v$.

As for (37), these processes come from the terms $2g_Z \zeta Z_\mu \partial^\mu (u\eta)$ and $g_Z H Z_\mu \partial^\mu (v\theta)$. They are suppressed by u/z and $2u^2/zv$ respectively and are thus negligible.

4.2 Singly-charged sector

In addition to its coupling to the Z boson, the charged physical Higgs boson h^+ (which is a triplet up to the negligible u/v component of the doublet) couples also to the photon. If kinematically accessible, it can be pair-produced via the Drell-Yan process at hadron and lepton colliders. Its kinematically allowed decays are

$$h^+ \rightarrow l^+ \bar{\nu}, \quad (44)$$

$$h^+ \rightarrow W^+ J^0, \quad (45)$$

$$h^+ \rightarrow W^+ \Omega^0, \quad (46)$$

$$h^+ \rightarrow W^+ \zeta^0. \quad (47)$$

Here the last two decays may not be kinematically allowed (see the previous section). Even if allowed, the decays (46) and (47) will be suppressed by phase space. The decays (44) and (45) are always allowed kinematically. However, because the triplet component in the Majoron is suppressed by the factor u/z (see Eq. (28)), the decay (45) may not compete with (44) unless the Yukawa coupling of the latter is very small. Therefore, the best candidate for the h^+ decay channel is likely to be the decay (44) induced by the Yukawa Lagrangian of Eq. (6).

The expected experimental signature of the process $pp \rightarrow h^+ h^-$ at the LHC is two hard oppositely charged leptons plus large missing energy carried away by the neutrinos in the decay (44). If the decay (44) is suppressed by very small Yukawa couplings, the other decays (45), (46), (47) may also be relevant. However, the leptons coming from the W^+ decays are softer and may be discriminated from the decay products of (44).

4.3 Doubly-charged sector

The production of doubly charged Higgs bosons at future colliders offers background-free and complete experimental tests of our model of neutrino masses. The only pair-production mechanism of ξ^{++} at the LHC and Tevatron is the Drell-Yan process mediated by s-channel photon and Z exchange [18]. Thus the production rate is enhanced by the double charge of ξ^{++} and is uniquely determined by the gauge couplings. At the parton level, the differential cross section of the process

$$\bar{f}f \rightarrow \xi^{++}\xi^{--}, \quad (48)$$

where $f = u, d$ quarks, is given by

$$\frac{d\sigma}{dt} = \frac{e^4}{48\pi s^2} \mathcal{M}^2, \quad (49)$$

where s, t are the kinematical invariants and the squared amplitudes $\mathcal{M}^2 = \mathcal{M}_1^2 + \mathcal{M}_2^2 + \mathcal{M}_{12}^2$ read

$$\mathcal{M}_1^2 = -\frac{8Q_f^2}{s^2} [(t - M_{\xi^{++}}^2)^2 + st], \quad (50)$$

$$\mathcal{M}_2^2 = -\frac{(1 + X_f^2)}{8(s - M_Z^2)^2} \left(\frac{1 - 2\sin^2\theta_W}{\sin^2\theta_W \cos^2\theta_W} \right)^2 [(t - M_{\xi^{++}}^2)^2 + st], \quad (51)$$

$$\mathcal{M}_{12}^2 = -\frac{2Q_f X_f}{s(s - M_Z^2)} \left(\frac{1 - 2\sin^2\theta_W}{\sin^2\theta_W \cos^2\theta_W} \right) [(t - M_{\xi^{++}}^2)^2 + st]. \quad (52)$$

Here $Q_f = 2/3$, $X_f = 1 - (8/3)\sin^2\theta_W$ for $f = u$, and $Q_f = -1/3$, $X_f = -1 + (4/3)\sin^2\theta_W$ for $f = d$. To obtain the cross section of $pp, p\bar{p} \rightarrow \xi^{++}\xi^{--}$ at the LHC and Tevatron, we have calculated the subprocesses involving the u, \bar{u} and d, \bar{d} collisions and convoluted them over the parton distributions given by the default set of the CERN library package PDFLIB [19]. The total cross section as a function of the triplet mass $M_{\xi^{++}}$ is plotted in Fig. 1.

Once produced, ξ^{++} will decay via one of the following channels

$$\xi^{++} \rightarrow l^+l^+, \quad (53)$$

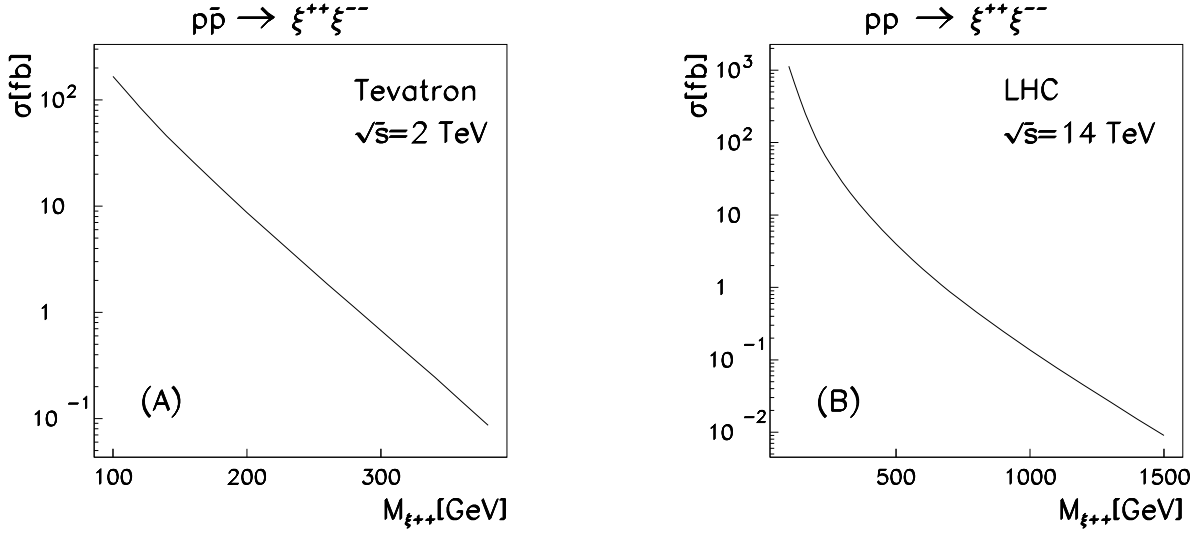


Figure 1: Cross section of $\xi^{++}\xi^{--}$ Drell-Yan pair production at Tevatron (A) and LHC (B).

$$\xi^{++} \rightarrow W^+W^+, \quad (54)$$

$$\xi^{++} \rightarrow h^+W^+. \quad (55)$$

Because $\langle \xi \rangle = u$ is tiny, the decay branching fraction of $\xi^{++} \rightarrow W^+W^+$ is negligible. The decay (55) may not be allowed kinematically and is suppressed by phase space in any case. Thus the only unsuppressed decay channels in our scenario are $\xi^{++} \rightarrow l_i^+ l_j^+$ with the partial rates

$$\Gamma_{ij} = |f_{ij}|^2 \frac{M_{\xi^{++}}}{4\pi}, \quad (56)$$

for $i \neq j$, and 1/2 smaller for $i = j$. This same-sign dilepton signal at the invariant mass of ξ is very distinctive at the LHC or Tevatron because it is completely background-free. Assuming the total integrated luminosity of the LHC to be 1000 fb^{-1} (10 fb^{-1} at the Tevatron), the reconstruction efficiency of the event to be 10%, and the predicted average of $N = -\ln(1-p)$ Poisson distributed events to provide a discovery, the cross sections in Fig. 1 imply at $p = 95\%$ confidence level that $M_{\xi^{++}} \lesssim 1.2 \text{ TeV}$ ($M_{\xi^{++}} \lesssim 300 \text{ GeV}$) can be probed at the LHC (Tevatron). Its decay branching fractions will then determine $|f_{ij}|$, i.e.

the magnitude of each element of the neutrino mass matrix up to an overall scale factor. This is the only model of neutrino masses which has the promise of being verified directly [20] from collider experiments without involving other theoretical assumptions which must be tested elsewhere.

Complementary measurements of $|f_{ij}|$ are also provided by the resonant processes $e^-e^- (\mu^-\mu^-) \rightarrow l_i^-l_j^-$ at a future Linear Collider and/or Muon Collider. The $M_{\xi^{++}}$ reach in these colliders extends up to the collision energies, which may be as high as 4 TeV. The sensitivity to $|f_{ij}|$ depends on the beam properties of the machines. The detailed estimate in Ref. [21] implies that $|(f \cdot f^*)_{ij}| \gtrsim 10^{-8}$ can be probed in these processes.

5 Neutrino Masses and Predictions for Lepton-Flavour Violating Processes

In our model the Majorana neutrino mass matrix follows from Eq. (6) and is given by

$$(\mathcal{M}_\nu)_{ij} = 2f_{ij}\langle\xi\rangle. \quad (57)$$

The VEV of the triplet $\langle\xi\rangle = u/\sqrt{2}$ should be derived from the minimization conditions of Eq. (20) which are nonlinear in u . Notice, however, that u identically vanishes if $\lambda_0 z \rightarrow 0$. Because of the hierarchy among the VEVs, $u \ll z \ll v$, we are allowed to make good approximations to relate the value of u to other VEVs and to physical Higgs-boson masses. Let us choose the ξ^{++} mass to be the physical parameter. Then it follows from Eq. (35) that

$$u \approx \frac{1}{2}\lambda_0 z \frac{v^2}{M_{\xi^{++}}^2}, \quad (58)$$

and the neutrino mass matrix takes the form

$$(\mathcal{M}_\nu)_{ij} \approx \frac{1}{\sqrt{2}}f_{ij}\lambda_0 z \frac{v^2}{M_{\xi^{++}}^2}. \quad (59)$$

For our phenomenological study of neutrino masses we treat the combination $\lambda_0 z$ as a small free parameter; the correct order of magnitude of the VEV z is given by Eq. (12). As mentioned before, because the neutrino mass matrix (59) is proportional to the Yukawa matrix f_{ij} , the branching fractions of the ξ^{++} decays determine the neutrino masses up to the overall scale which should be fixed from other experiments. On the other hand, the existing neutrino data may already be used to make predictions for the rare unobserved lepton flavour violating processes induced by f_{ij} in our model.

Consider now a phenomenological hierarchical neutrino mass matrix consistent with the atmospheric and solar neutrino results [3, 22]:

$$\mathcal{M}_\nu = m \begin{pmatrix} 0 & b & -bx \\ b & x^2 + a & x - ax \\ -bx & x - ax & 1 + ax^2 \end{pmatrix}, \quad (60)$$

where m is the normalization mass and $0.67 < x < 1$ determines the $\nu_\mu \rightarrow \nu_\tau$ mixing as required by the atmospheric neutrinos. The three solutions of the solar neutrino problem correspond to

- (i) large-angle matter-enhanced oscillations: $a = 0.02$, $b = 0.4$;
- (ii) small-angle matter-enhanced oscillations: $a = 0.04$; $b = 0.003$; and
- (iii) vacuum oscillations: $a = 0.002$, $b = 0.012$.

While all these solutions to neutrino anomalies are still allowed, the new global fits of neutrino data [22], which include the recent Super-Kamiokande data [23], clearly prefer the large angle matter-enhanced solution (i).

Given the pattern of f_{ij} via the neutrino mass matrix of Eq. (60), lepton-flavour violation through ξ exchange may be observable at low energies. The processes most sensitive to the new flavour-violating physics are the decay $\mu \rightarrow e\gamma$ and $\mu - e$ conversion in nuclei. Planned

experiments will reach the sensitivity of 10^{-16} for $\mu-e$ conversion in aluminium [24] and 10^{-14} for $\mu \rightarrow e\gamma$ [25]. Because of the off-shell photon exchange, the amplitude of $\mu-e$ conversion in nuclei is enhanced by $\ln(M_{\xi^{++}}^2/m_\mu^2)$ compared to that of $\mu \rightarrow e\gamma$ [26]. Therefore we expect that the former process is more sensitive to the existence of our neutrino-mass-giving triplet than the latter.

The matrix element of photonic conversion is given by

$$\mathcal{M} = (4\pi\alpha/q^2)j^\mu J_\mu, \quad (61)$$

where q is the momentum transfer with $q^2 \approx -m_\mu^2$, J is the hadronic current, and

$$j^\lambda = \bar{u}(p_e) \left[(f_{E0} + \gamma_5 f_{M0}) \gamma_\nu \left(g^{\lambda\nu} - \frac{q^\lambda q^\nu}{q^2} \right) + (f_{M1} + \gamma_5 f_{E1}) i \sigma^{\lambda\nu} \frac{q_\nu}{m_\mu} \right] u(p_\mu) \quad (62)$$

is the usual leptonic current. The coherent $\mu-e$ conversion ratio in nuclei is given by

$$R_{\mu e} = \frac{8\alpha^5 m_\mu^5 Z_{eff}^4 Z |\overline{F}_p(p_e)|^2 \xi_0^2}{\Gamma_{capt} q^4}, \quad (63)$$

where $\xi_0^2 = |f_{E0} + f_{M1}|^2 + |f_{E1} + f_{M0}|^2$, and for ^{13}Al , $Z_{eff}^{Al} = 11.62$, $\overline{F}_p^{Al}(q) = 0.66$, and $\Gamma_{capt}^{Al} = 7.1 \times 10^5 \text{ s}^{-1}$ [27]. We calculate the form factors induced by the one-loop diagrams involving ξ^{++} and obtain

$$f_{E0} = f_{M0} = \sum_l \frac{f_{\mu l} f_{l e}^*}{24\pi^2} [4s_l + rF(s_l)], \quad (64)$$

$$f_{M1} = -f_{E1} = \sum_l \frac{f_{\mu l} f_{l e}^*}{24\pi^2} s_\mu, \quad (65)$$

where

$$F(s_l) = \ln s_l + \left(1 - \frac{2s_l}{r} \right) \sqrt{1 + \frac{4s_l}{r}} \ln \left[\frac{\sqrt{r + 4s_l} + \sqrt{r}}{\sqrt{r + 4s_l} - \sqrt{r}} \right],$$

and $r = -q^2/M_{\xi^{++}}^2$, $s_l = m_l^2/M_{\xi^{++}}^2$, $l = e, \mu, \tau$. In the interesting limit $s_l \rightarrow 0$, we get $F(s_l) \rightarrow \ln r$ which implies the logarithmic enhancement of the form-factors f_{E0} and f_{M0} . Notice that all the form factors in Eq. (62) contribute to the $\mu-e$ conversion rate (63).

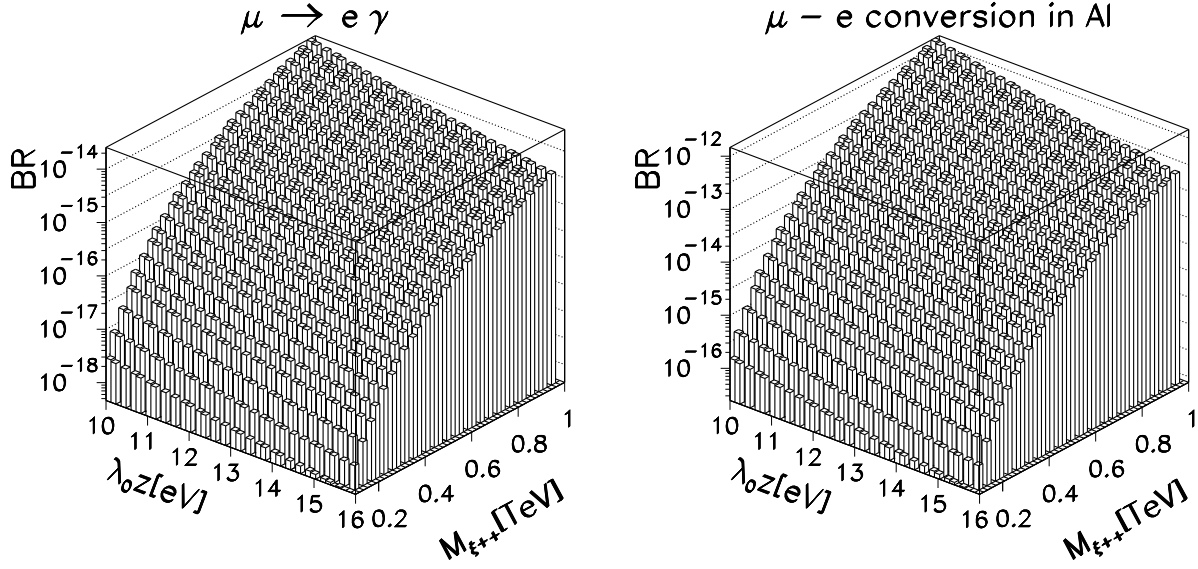


Figure 2: Rates of $\mu \rightarrow e\gamma$ and $\mu - e$ conversion in ^{13}Al against the ξ^{++} mass and the free parameter $\lambda_0 z$, assuming large-angle matter-enhanced solution to the solar neutrino problem.

However, it is well known that the decay $\mu \rightarrow e\gamma$ is induced only by the form-factors f_{E1} and f_{M1} . Its branching ratio is given by

$$R_{\mu \rightarrow e\gamma} = \frac{96\pi^3\alpha}{G_F^2 m_\mu^4} (|f_{M1}|^2 + |f_{E1}|^2), \quad (66)$$

where $\alpha = 1/137$ and G_F is the Fermi constant.

For numerical estimates we assume $m = 0.03$ eV and $x = 0.9$ in Eq. (60) and the currently most-favoured large-angle matter-enhanced oscillation solution (i) to the solar neutrino problem. In Fig. 2 we plot the branching ratio of the decay $\mu \rightarrow e\gamma$ and the ratio of $\mu - e$ conversion in aluminium as a function of the mass $M_{\xi^{++}}$ and the free parameter $\lambda_0 z$. The behaviour of these ratios can be understood from Eq. (59): a fixed neutrino mass implies $f \propto M_{\xi^{++}}^2$ and $f \propto 1/(\lambda_0 z)$. Notice the complementarity of collider and $\mu - e$ conversion experiments. For small $M_{\xi^{++}}$, $R_{\mu e}$ is suppressed while the collider cross section is kinematically enhanced, and vice versa.

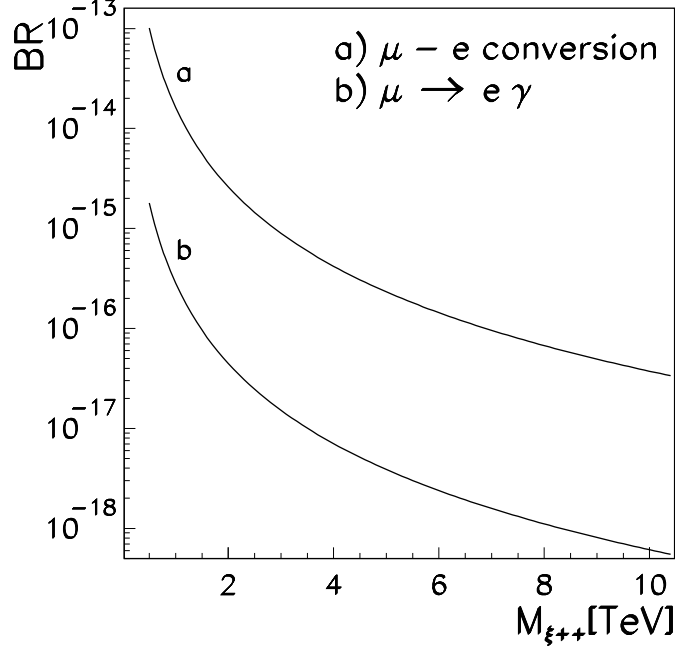


Figure 3: Rates of $\mu \rightarrow e\gamma$ and $\mu - e$ conversion in ^{13}Al against the ξ^{++} mass. We assume that at $M_{\xi^{++}} = 0.5$ TeV $\lambda_0 z = 10$ eV, and it scales according to Eq. (12) for $n = 3$ extra dimensions.

For the same parameters $\lambda_0 z$ and $M_{\xi^{++}}$, the numerical values of $R_{\mu e}$ are almost two orders of magnitude larger than the ones of $R_{\mu \rightarrow e\gamma}$. There are two reasons. First, $R_{\mu e}$ is enhanced by the large logarithm in Eq. (64). Second, there is a deep cancellation between the triplet Yukawa couplings in Eq. (65) which follows from the structure of the mass matrix Eq. (60) [the minus sign in the (13), (31) entries]. In Eq. (64), however, the cancellation does not occur because the flavour-dependent multiplicative function gives different weights to different contributions in the sum. Hence $R_{\mu \rightarrow e\gamma}$ (which depends only on f_{M1} , f_{E1}) is further diminished compared to $R_{\mu e}$. One should remember that the sensitivity of the planned $\mu - e$ conversion experiments is at least two orders of magnitude higher than the sensitivity of the planned $\mu \rightarrow e\gamma$ experiments. Therefore the $\mu - e$ conversion experiments

will probe the existence of the neutrino-mass-giving triplet to high mass scales while the $\mu \rightarrow e\gamma$ experiments will have just a marginal chance to test this scenario.

This can also be seen in Fig. 3 where we plot the branching ratios of $\mu \rightarrow e\gamma$ and $\mu - e$ conversion in Al against $M_{\xi^{++}}$. In doing so, we assume that the triplet mass is equal to the fundamental scale of the theory, $M_{\xi^{++}} = M_*$, and the VEV of the singlet χ (and thus $\lambda_0 z$) evolves according to Eq. (12) for $n = 3$ extra dimensions. We assume the initial value $\lambda_0 z = 10$ eV for $M_{\xi^{++}} = M_* = 0.5$ TeV. For this choice of parameters, Fig. 3 indicates that MECO will test our model up to the scale of 7 TeV while the $\mu \rightarrow e\gamma$ rate is always below the sensitivity of the currently planned experiments.

6 Conclusions

The neutrino-mass-giving Higgs triplet (ξ^{++}, ξ^+, ξ^0) is proposed to be observable at future colliders with m_ξ of order 1 TeV in a model where spontaneous lepton-number violation comes from a scalar bulk singlet χ in a theory of large extra dimensions. We show how ξ couples to the massless Majoron J^0 of this model and study the various decay modes of the Higgs triplet components. The backgroundless decays $\xi^{++} \rightarrow l_i^+ l_j^+$ will determine directly the relative magnitudes of the $\{ij\}$ elements of the neutrino mass matrix. The decay $\xi^0 \rightarrow J^0 J^0$ puts a severe constraint on the parameter space of this model, making $\langle \chi \rangle$ of order 1 MeV in our brane. Using present neutrino-oscillation data, we predict observable rates of $\mu - e$ conversion in nuclei while the planned $\mu \rightarrow e\gamma$ experiments will have a much smaller chance for a positive signal.

Acknowledgement. This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837 and ESF under Grant No. 3832. We thank M. Hirsch for pointing out an error in the neutrino mass matrix in the first version of this paper.

References

- [1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. **B429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. **B436**, 257 (1998); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. **D59**, 086004 (1999).
- [2] G. Giudice, R. Rattazzi, and J. Wells, Nucl. Phys. **B544**, 3 (1999); T. Han, J. D. Lykken, and R.-J. Zhang, Phys. Rev. **D59**, 105006 (1999); E. A. Mirabelli, M. Perelstein, and M. E. Peskin, Phys. Rev. Lett. **82**, 2236(1999); J. L. Hewett, Phys. Rev. Lett. **82**, 4765 (1999). For a review and a list of references see, *e.g.*, G. Landsberg, hep-ex/0009038.
- [3] Y. Fukuda *et al.*, Super-Kamiokande Collaboration, Phys. Rev. Lett. **81**, 1158 (1998), **82**, 1810 (1999), **82**, 2430 (1999); Phys. Lett. **B433**, 9 (1998), **B467**, 185 (1999); Phys. Rev. Lett. **82**, 2644 (1999).
- [4] K. R. Dienes, E. Dudas, and T. Gherghetta, Nucl. Phys. **B557**, 25 (1999); N. Arkani-Hamed, S. Dimopoulos, G. Dvali, and J. March-Russell, hep-ph/9811448.
- [5] A. E. Faraggi and M. Pospelov, Phys. Lett. **B458**, 237 (1999); R. N. Mohapatra, S. Nandi, and A. Perez-Lorenzana, Phys. Lett. **B466**, 115 (1999); A. Das and O. C. W. Kong, Phys. Lett. **B470**, 149 (1999); G. Dvali and A. Yu. Smirnov, Nucl. Phys. **B563**, 63 (1999); A. Pilaftsis, Phys. Rev. **D60**, 105023 (1999); G. C. McLaughlin and J. N. Ng, Phys. Lett. **B470**, 157 (1999); nucl-th/0003023; hep-ph/0008209; A. Ioannisian and A. Pilaftsis, Phys. Rev. **D62**, 066001 (2000); Y. Grossman and M. Neubert, Phys. Lett. **B474**, 361 (2000); R. N. Mohapatra and A. Perez-Lorenzana, Nucl. Phys. **B576**, 466 (2000); hep-ph/0006278; R. Barbieri, P. Creminelli, and A. Strumia, Nucl. Phys. **B585**, 28 (2000); K. Agashe, N. G. Deshpande, and G. H. Wu, Phys. Lett. **B489**, 367 (2000); A. Ioannisian and J. W. F. Valle, hep-ph/9911349; A. Lukas and A. Romanino, hep-ph/0004130; A. Lukas, P. Ramond, A. Romanino, and G. G. Ross, hep-ph/0008049,

- hep-ph/0011295; K. R. Dienes and I. Sarcevic, hep-ph/0008144; A. Ioannisian and A. Pilaftsis, hep-ph/0010051; K. Agashe and G.-H. Wu, hep-ph/0010117; N. Cosme *et al.*, hep-ph/0010192; D. Cadwell, R. Mohapatra and S. Yellin, hep-ph/0010353.
- [6] E. Ma, M. Raidal, and U. Sarkar, Phys. Rev. Lett. **85**, 3769 (2000).
- [7] R. N. Mohapatra, A. Perez-Lorenzana, and C. A. de S. Pires, hep-ph/0008158.
- [8] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).
- [9] E. Ma, Phys. Rev. Lett. **81**, 1171 (1998).
- [10] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, ed. P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, ed. O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, Japan, 1979), p. 95; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).
- [11] N. Arkani-Hamed, L. Hall, H. Murayama, D. Smith and N. Weiner, hep-ph/0006312 and hep-ph/0007001; F. Borzumati and Y. Nomura, hep-ph/0007018; K.S. Babu and T. Yanagida, Phys. Lett. B **491** (2000) 148; F. Borzumati, K. Hamaguchi, and T. Yanagida, Phys. Lett. B **497** (2001) 259.
- [12] N. Arkani-Hamed and S. Dimopoulos, hep-ph/9811353; N. Arkani-Hamed, L. Hall, D. Smith, and N. Weiner, Phys. Rev. **D61**, 116003 (2000); Y. Sakamura, hep-ph/9912511.
- [13] E. Ma and U. Sarkar, Phys. Rev. Lett. **80**, 5716 (1998); W. Grimus, R. Pfeiffer, and T. Schwetz, Eur. Phys. J. **C13**, 125 (2000).
- [14] E. Ma, M. Raidal, and U. Sarkar, Phys. Lett. B **504** (2001) 296.
- [15] G. B. Gelmini and M. Roncadelli, Phys. Lett. **99B**, 411 (1981).

- [16] J. Schechter and J. W. F. Valle, Phys. Rev. **D22**, 2227 (1980).
- [17] M. A. Diaz, M. A. Garcia-Jareno, D. A. Restrepo and J. W. Valle, Nucl. Phys. **B527**, 44 (1998).
- [18] K. Huitu, J. Maalampi, A. Pietila, and M. Raidal, Nucl. Phys. **B487**, 27 (1997).
- [19] H. Plochow-Besch, Comp. Phys. Comm. **75**, 396 (1993).
- [20] A less directly testable version of the seesaw mechanism with right-handed neutrinos is proposed by E. Ma, Phys. Rev. Lett. **86** (2001) 2502; E. Ma and M. Raidal, Phys. Rev. Lett. **87** (2001) 011802. Other more involved but testable schemes include W. Porod, M. Hirsch, J. Romao, and J. W. F. Valle, Phys. Rev. D **63** (2001) 115004; and references therein.
- [21] F. Cuypers and M. Raidal, Nucl. Phys. **B501**, 3 (1997); M. Raidal, Phys. Rev. **D57**, 2013 (1998).
- [22] M. C. Gonzalez-Garcia, M. Maltoni, C. Pena-Garay, and J. W. F. Valle, Phys. Rev. D **63** (2001) 033005; see also, G. L. Fogli, E. Lisi, A. Maronne, and G. Scioscia, Phys. Rev. **D59**, 033001 (1999); G. L. Fogli, E. Lisi, D. Montanio, and A. Palazzo, Phys. Rev. **D62**, 013002 (2000); hep-ph/0005261;
- [23] Y. Suzuki, Super-Kamiokande Collaboration, talk given in *XIX International Conference on Neutrino Physics*, June 16, 2000, Sudbury, Canada.
- [24] M. Bachmann *et al.*, MECO Collaboration, Research Proposal E940 at BNL (1997).
- [25] L. M. Barkov *et al.*, Research Proposal at PSI (1999).
- [26] M. Raidal and A. Santamaria, Phys. Lett. **B421**, 250 (1998).
- [27] H. C. Chiang *et al.*, Nucl. Phys. **A559**, 526 (1993).