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Supersymmetric Triplet Higgs Model of Neutrino Masses and Leptogenesis

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Abstract

We construct a supersymmetric version of the triplet Higgs model for neutrino masses, which can generate a baryon asymmetry of the Universe through lepton-number violation and is consistent with the gravitino constraints.

1 Introduction

The first definite evidence for physics beyond the standard model came from the recent evidence for the mass of the neutrinos. The atmospheric neutrino anomaly [1], as observed by the SuperKamiokande experiment, has established that there is a mass-squared difference between the muon neutrino and the tau neutrino. On the other hand, the solar neutrino problem [2] implies a mass-squared difference between the electron neutrino and the other two active neutrinos. Hence it has now been established that at least two neutrinos are massive.

The mass-squared differences between the different generations of neutrinos have to be very small, but the mixing angles large, to explain the atmospheric and solar neutrino anomalies. The required masses for the neutrinos are several orders of magnitude smaller than those of other fermions, which are all Dirac particles. The smallness of the neutrino mass is naturally explained if the neutrinos are Majorana particles [3], hence lepton number is not conserved and that should be due to some physics beyond the standard model. There are several motivations for lepton-number violation in Nature [4]. In addition, the associated lepton-number violation may have the added virtue of accounting for the present observed baryon asymmetry of the Universe. One model of neutrino mass having this virtue is the triplet Higgs model [5]. In the nonsupersymmetric case, this model has been studied in detail and found to share all the interesting features of other models of neutrino mass. Moreover, in theories with large extra dimensions [6], this mechanism happens to be the only one which gives Majorana (rather than Dirac) masses to the neutrinos [7]. In this article we will study the supersymmetric version of this model.

In the supersymmetric version of the triplet Higgs model, there are several new aspects. Similar to the requirement of two Higgs doublets in the supersymmetric extension of the standard model, we now have two Higgs triplets. Only one of them couples to the leptons, but it can acquire a vacuum expectation value (*vev*) only if the other Higgs triplet is present. This is related to the fact that a mass term in the superpotential requires two triplet Higgs superfields, which are of course also necessary for anomaly cancellation. This mass term connecting the two triplet superfields in the superpotential also allows a trilinear coupling to exist between two scalar doublets and the scalar triplet which couples to leptons, which is necessary for neutrino mass as well as leptogenesis [5]. In the present supersymmetric version of the triplet Higgs model, we must consider the decays of both heavy triplets. Note that supersymmetry is not yet broken at this energy scale. There are now also several new diagrams which contribute to the CP violation.

Another important feature of the supersymmetric model comes from the constraints of nucleosynthesis. In supersymmetric models there is a strong bound on the scale of inflation from nucleosynthesis due to the gravitino problem [8]. This means that baryogenesis has to occur at temperatures below about 10^{11} GeV. On the other hand, in the triplet Higgs model, the gauge interactions of the triplet Higgs scalars and fermions bring their number densities to equilibrium at temperatures below $\sim 10^{12}$ GeV. This naive order-of-magnitude estimate thus implies that the supersymmetric triplet Higgs model of leptogenesis is probably not consistent with the gravitino constraints [9]. However, detailed calculations give several possible ways out of this problem. In the following we will consider the cases where this potential problem is first ignored and then taken into account. We point out here that the supersymmetric triplet Higgs model can evade this problem of gravitinos when the masses of the triplet Higgs superfields are moderately degenerate.

In Section 2 we introduce the model and describe its consequences for neutrino masses. Then in Section 3 we calculate the amount of CP violation in the decays of the triplet Higgs scalars and fermions which can generate a lepton asymmetry of the Universe. In Section 4 we solve the Boltzmann equations to calculate the evolution of the lepton asymmetry and present our results. In Section 5 the gravitino problem is discussed. Finally in Section 6 we summarize and conclude.

2 The Model

The Majorana masses of the neutrinos can be generated by extending the standard model to include a triplet Higgs scalar, which acquires a small *vev* and couples to two leptons. If lepton number was spontaneously broken by this *vev* [10], the so-called triplet Majoron (i.e. the resulting massless Goldstone boson) coupling to the Z boson would be predicted. This scenario is now ruled out by the known invisible Z width [11]. Moreover, such models do not explain the present observed baryon asymmetry of the Universe. A new scenario was then proposed in which lepton number is broken explicitly at a very high energy scale [5]. The triplet Higgs scalar would then be extremely heavy. However, it acquires a very tiny *vev* through its lepton-number violating trilinear coupling to the standard-model Higgs doublet, which can then give a small Majorana mass to the neutrinos. The decays of the triplet Higgs scalars also generate a lepton asymmetry of the Universe, which gets converted to a baryon asymmetry of the Universe before the electroweak phase transition.

To implement the triplet Higgs mechanism in a supersymmetric model, we need to extend the supersymmetric standard model to include two triplet Higgs superfields. Since we want these fields to be very heavy, supersymmetry should be unbroken at that stage and the generation of the lepton asymmetry will not depend on the supersymmetry-breaking mechanism. We also assume that R-parity is not violated, so that there is no other source of lepton-number violation except for the Yukawa couplings of the triplet Higgs superfields. We introduce one triplet $\hat{\xi}_1([\hat{\xi}_1^{++}, \hat{\xi}_1^{+}, \hat{\xi}_1^{0}] \equiv [1,3,1]$ under $SU(3)_c \times SU(2)_L \times U(1)_Y)$ and another triplet $\hat{\xi}_2([\hat{\xi}_2^{0}, \hat{\xi}_2^{-}, \hat{\xi}_2^{--}] \equiv [1,3,-1])$ so that a mass term $M\hat{\xi}_1\hat{\xi}_2$ may appear in the superpotential. However, CP violation is not possible with just these two Higgs triplets. For that, we need two of each type of the above Higgs triplets. So, if heavy triplet superfields are used to generate neutrino masses as well as a lepton asymmetry of the Universe, there should be at least four: $\hat{\xi}_1^a([\hat{\xi}_1^{a++}, \hat{\xi}_1^{a+}, \hat{\xi}_1^{a0}] \equiv [1,3,1])$ and $\hat{\xi}_2^a([\hat{\xi}_2^{a0}, \hat{\xi}_2^{a-}, \hat{\xi}_2^{a--}] \equiv [1,3,-1])$, where $a = 1,2$ corresponds to the two scalar superfields, whose mixing gives CP violation for generating the lepton asymmetry of the Universe.

The essential part of the superpotential for the interactions of these scalar superfields with the lepton superfields $\hat{L}_i \equiv (\,\nu_{Li} \quad e^-_{Li}\,) \equiv [1,2,-1/2]$ and the standard Higgs doublets $\hat{H}_1([\phi_1^0, \phi_1^-] \equiv [1,2,-1/2])$ and $\hat{H}_2([\phi_2^+, \phi_2^0] \equiv [1,2,1/2])$ is given by

$$W = M_{ab}\hat{\xi}_1^a\hat{\xi}_2^b + f_{ij}^a\hat{L}_i\hat{L}_j\hat{\xi}_1^a + h_1^a\hat{H}_1\hat{H}_1\hat{\xi}_1^a + h_2^a\hat{H}_2\hat{H}_2\hat{\xi}_2^a + \mu\hat{H}_1\hat{H}_2 + ... \tag{1}$$

where $i = 1,2,3$ is the generation index. The first term gives masses to the triplets. The condition for leptogenesis and neutrino masses would determine this scale M. The next term gives the Yukawa couplings of the triplet Higgs scalar superfield with the left-handed lepton chiral superfields of the three generations. When the scalars ξ_1^a acquire vacuum expectation value (*vev*s), this term gives Majorana masses to the neutrinos. The next two terms give small *vev*s to the triplet Higgs scalars.

The scalars ξ_1^a couple to two leptons, to two Higgsinos $\tilde{H}_1$, to two scalars H_2 and to a H_1H_2 pair. The scalars ξ_2^a couple to two Higgsinos $\tilde{H}_2$, to two sleptons, to two scalars H_1 and to a H_1H_2 pair. This simultaneous decay of the triplets to products with different lepton numbers breaks lepton number explicitly. Thus the scale of lepton-number violation is the same as the mass of the triplet Higgs scalars, which is very heavy, say of the order $\sim O(10^9 - 10^{14})$ GeV. However, since $SU(2)_L$ is unbroken at this scale, these fields do not acquire any *vev*. Only after the electroweak symmetry breaking is there an induced tiny *vev* for these scalars and the neutrinos would acquire mass.

The *vev*s of the triplet Higgs scalars are obtained from the vanishing of the $F-$terms, which corresponds to the minima of the potential. From the conditions $F_{\xi_1^a} = F_{\xi_2^a} = 0$, and assuming that R-parity is conserved (so that the sneutrinos do not acquire any *vev*), we get

$$F_{\xi_1^a} = M_{ab}\xi_2^b + f_{ij}^a \tilde{L}_i \tilde{L}_j + h_1^a H_1 H_1 = 0 \Longrightarrow \langle \xi_2^b \rangle = u_2^b = -M_{ba}^{-1} h_1^a \langle H_1 \rangle^2 = -M_{ba}^{-1} h_1^a v_1^2,$$
$$F_{\xi_2^a} = M_{ab}\xi_1^b + h_2^a H_2 H_2 = 0 \Longrightarrow \langle \xi_1^b \rangle = u_1^b = -M_{ba}^{-1} h_2^a \langle H_2 \rangle^2 = -M_{ba}^{-1} h_2^a v_2^2. \quad (2)$$

Since the masses of the triplet scalar fields are several orders of magnitude higher than the electroweak symmetry breaking scale v, the effective *vev* of the triplet Higgs fields are several orders of magnitude smaller than $v = 246$ GeV.[1] Since these *vev*s give masses to the neutrinos, the smallness of the neutrino mass is now directly related to the large lepton-number violating scale.

The *vev*s of the triplet scalars will give a mass to the neutrinos given by

$$(m_\nu)_{ij} = \sum_a 2 f_{ij}^a u_1^a = \sum_{a,b} -2 f_{ij}^a M_{ab}^{-1} h_2^b v_2^2. \quad (3)$$

Since the leptons do not couple with the other triplet scalar ξ_2, there is no contribution to the neutrino mass from u_2^a. Since the lepton number is now broken at a very large scale explicitly, there is no Majoron in this scenario. There is one would-be Majoron, which becomes too heavy to affect any low-energy phenomenology. This makes it consistent with the measured invisible Z width from LEP (Large Electron Positron Collider) at CERN.

The decay of these scalars to two leptons or two Higgsinos can be read off from the $F-$terms in the superpotential. The decays of these scalars into two sleptons and the standard-model Higgs doublets can be read off from the relevant part of the scalar potential,

$$\begin{aligned} V &= |M_{ab}\xi_2^b + f_{ij}^a \tilde{L}_i \tilde{L}_j + h_1^a H_1 H_1|^2 + |M_{ab}\xi_1^b + h_2^a H_2 H_2|^2 \\ &\quad + |2h_1^a H_1 \xi_1^a + \mu H_2 + ...|^2 + |2h_2^a H_2 \xi_2^a + \mu H_1 + ...|^2 + ... \end{aligned} \quad (4)$$

The various decay modes of the scalar and fermionic components of the triplet scalar superfields are listed below and shown in Figures 1 and 2. The decay modes of the $\hat{\xi}_1^a$ (i.e.

[1] The smallness of these *vev*s makes this triplet model perfectly consistent with the usual constraints on additional triplets coming from the measurement of the ρ parameter at LEP.

the scalars ξ_1^{a++} and the fermions $\tilde{\xi}_1^{a++}$) are

$$\xi_1^{a++} \rightarrow \begin{cases} L_i^+ L_j^+ & (L=-2) \\ H_2^+ H_2^+ & (L=0) \\ \tilde{H}_1^+ \tilde{H}_1^+ & (L=0) \end{cases} \tag{5}$$

and

$$\tilde{\xi}_1^{a++} \rightarrow \begin{cases} \tilde{L}_i^+ L_j^+ & (L=-2) \\ \tilde{H}_2^+ H_2^+ & (L=0) \\ H_1^+ \tilde{H}_1^+ & (L=0) \end{cases} , \tag{6}$$

while the decay modes of $\hat{\xi}_2^a$ are

$$\xi_2^{a++} \rightarrow \begin{cases} \tilde{L}_i^+ \tilde{L}_j^+ & (L=-2) \\ \tilde{H}_2^+ \tilde{H}_2^+ & (L=0) \\ H_1^+ H_1^+ & (L=0) \end{cases} \tag{7}$$

and

$$\tilde{\xi}_2^{a++} \rightarrow \begin{cases} \tilde{L}_i^+ L_j^+ & (L=-2) \\ \tilde{H}_2^+ H_2^+ & (L=0) \\ H_1^+ \tilde{H}_1^+ & (L=0) \end{cases} . \tag{8}$$

The couplings entering in the various decay modes can be read off from the superpotential. Note that we don't consider the decays proportional to μ^2 (i.e. to a $\hat{H}_1 \hat{H}_2$ pair) which are negligible. If there is CP violation and the decays satisfy the out-of-equilibrium condition, then these decays can generate a lepton asymmetry of the Universe [12, 13]. This lepton asymmetry can then get converted to a baryon asymmetry of the Universe [14].

When this lepton asymmetry is generated, the $B+L$ violating (but $B-L$ conserving) sphaleron transitions are taking place at a very fast rate [15]. In fact, during the period

$$10^{12} \text{ GeV } > T > \ 10^2 \text{ GeV}$$

the anomalous $B+L$ violating sphaleron processes remain in equilibrium. During this period, any lepton asymmetry of the Universe would be equivalent to the $B-L$ asymmetry. The sphaleron interactions would then convert this lepton asymmetry to a baryon asymmetry of the Universe within this period [16].

3 CP Asymmetry in Triplet Higgs Decay

The various decay modes of ξ_1^a and $\tilde{\xi}_1^a$ are given in Figure 1 and the decay modes of ξ_2^a and $\tilde{\xi}_2^a$ are given in Figure 2. The simultaneous decay of the triplet Higgs scalars or the triplet Higgsinos into states with lepton number 0 (two scalar Higgs doublets or Higgsinos) and with lepton number 2 (two leptons or sleptons) implies lepton-number violation. For CP violation, the tree-level diagrams by themselves are not enough. Even if the couplings are complex, the probability will be positive definite and hence there will not be any CP violation. However, if there are one-loop diagrams, which interfere with these tree-level diagrams, then the interference may be complex, which gives the CP violation.

In the present case there are one-loop diagrams which are given in Figure 3 (for ξ_1^a and $\tilde{\xi}_1^a$ decays) and in Figure 4 (for ξ_2^a and $\tilde{\xi}_2^a$ decays). As in the nonsupersymmetric case, although some of the tree-level diagrams appear similar to the right-handed neutrino decay diagrams [17], there are no one-loop diagrams which are similar to the vertex diagrams of the right-handed neutrino decays. From this point of view, leptogenesis with the triplet Higgs scalars have this unique feature that CP violation comes only from the self-energy diagrams, which has the interpretation of oscillations of the scalars before they decay [18]. Moreover, it was pointed out that the CP violation coming from the self-energy diagrams has an interesting feature of resonant oscillation. Thus the amount of lepton asymmetry can get highly enhanced when the masses of the triplet Higgs superfields are almost degenerate [18].

In none of the loop diagrams of Fig. 3-4 is there any interference between ξ_1^a and ξ_2^a. So, with one each of ξ_1^a and ξ_2^a, there cannot be any CP violation. In this case, the relative phases between various couplings can be chosen to be real. Only when there are at least two ξ_1^a or ξ_2^a, there can be CP violation. In this case, decays of both ξ_1^a and ξ_2^a will contribute to the amount of CP violation. The relative phases between the couplings of the ξ_1^a to the leptons of different generations cannot generate a lepton asymmetry of the Universe, because they all correspond to final states of the same lepton number. Among the loop diagrams, Figures (c) and (d) are supersymmetric counterparts of Figures (a) and (b), so supersymmetry ensures that the contributions from the first two diagrams are the same as that of the last two diagrams. In the following we will consider explicitly only the decays of the scalar triplets keeping in mind that the decays of their fermionic superpartners give the same lepton asymmetry.

We shall now calculate the amount of CP violation generated from the interference of the tree-level processes and the one-loop diagrams. In the mass-matrix formalism, it is possible to give a physical interpretation to this CP violation. A triplet scalar superfield oscillating into another type before it decays, has a different decay rate compared to its conjugate states. Although the total decay rates are equal by CPT, the partial decay rates now differ, which give rise to CP violation. This CP violation will then lead to a lepton asymmetry due to the fact that (1) the partial decay products do not all have the same lepton number and (2) the interaction rate is not much faster than the expansion rate of the Universe.

Without loss of generality, we shall assume that the mass matrix for the triplet Higgs scalars starts out as real and diagonal,

$$M_{ab} = M_a \delta_{ab},$$

with M_a real. However, in the presence of interactions, they will no longer remain real. Including the interactions, the mass matrix for the left and right chiral superfields gets different contributions from the interference of the tree and loop diagrams. The physical states of the left and right chiral superfields will evolve in a different way and their decays into leptons and antileptons would generate the lepton asymmetry of the Universe. In the following we denote by $\hat{\phi}^m_{1+}$ and $\hat{\phi}^m_{2+}$ with $m = 1, 2$ the physical states which are combinations of the left chiral superfields $\hat{\xi}_1^a$ and $\hat{\xi}_2^a$ respectively, and by $\hat{\phi}_{1-}^m$ and $\hat{\phi}_{2-}^m$ the physical states which are combinations of the conjugates of these superfields $\hat{\xi}_1^{a*}$ and $\hat{\xi}_2^{a*}$ respectively (which are the right chiral superfields).

The effective scalar triplet mass matrix we obtain at one loop is given by

$$\xi_1^{a\dagger}(\mathcal{M}_1^2)_{ab}\xi_1^b + \xi_2^{a\dagger}(\mathcal{M}_2^2)_{ab}\xi_2^b \tag{9}$$

where, for a given value of the squared momemtum p_ξ^2 of the incoming or outgoing particle:

$$\mathcal{M}_k^2 = \begin{pmatrix} M_1^2 - i\Gamma_{11}^k M_1 & -i\Gamma_{12}^k M_2 \\ -i\Gamma_{21}^k M_1 & M_2^2 - i\Gamma_{22}^k M_2 \end{pmatrix}, \tag{10}$$

with $\Gamma_{ab}^k M_b = (\Gamma_{ba}^k)^* M_a$ and

$$\Gamma_{ab}^1 M_b = \frac{1}{8\pi}\left(\sum_{i,j} f_{ij}^{a\,*} f_{ij}^b p_\xi^2 + h_1^{a*}h_1^b p_\xi^2 + M_a M_b h_2^a h_2^{b*}\right),$$

and

$$\Gamma_{ab}^2 M_b = \frac{1}{8\pi}\left(M_a M_b \sum_{i,j} f_{ij}^a f_{ij}^{b*} + h_2^{a*}h_2^b p_\xi^2 + M_a M_b h_1^a h_1^{b*}\right).$$

The decay widths $\Gamma_{\phi_{k\pm}^a}$ of the tree scalars in the triplet $\phi_{k\pm}^a$ are given by $\Gamma_{\phi_{k\pm}^a} = \Gamma_k^{aa} \equiv \Gamma_{\phi_k^a}$. Neglecting terms of order $[\Gamma_{ij}M_j/(M_1^2 - M_2^2)]^2$ the two mass matrices have the eigenvalues $M_{\phi_{k\pm}^a} = M_a$ and the eigenvectors are

$$\phi_{k+}^1 = \xi_k^1 - i\frac{\Gamma_{12}^k M_2}{M_1^2 - M_2^2}\xi_k^2 \tag{11}$$

$$\phi_{k+}^2 = i\frac{\Gamma_{12}^{k*} M_2}{M_1^2 - M_2^2}\xi_k^1 + \xi_k^2 \tag{12}$$

$$\phi_{k-}^1 = \xi_k^{1*} - i\frac{\Gamma_{12}^{k^*} M_2}{M_1^2 - M_2^2}\xi_k^{2*} \tag{13}$$

$$\phi_{k-}^2 = i\frac{\Gamma_{12}^k M_2}{M_1^2 - M_2^2}\xi_k^{1*} + \xi_k^{2*}. \tag{14}$$

Similarly we have

$$\xi_k^1 = \phi_{k+}^1 + i\frac{\Gamma_{12}^k M_2}{M_1^2 - M_2^2}\phi_{k+}^2 \tag{15}$$

$$\xi_k^2 = -i\frac{\Gamma_{12}^{k*} M_2}{M_1^2 - M_2^2}\phi_{k+}^1 + \phi_{k+}^2 \tag{16}$$

$$\xi_k^{1*} = \phi_{k-}^1 + i\frac{\Gamma_{12}^{k*} M_2}{M_1^2 - M_2^2}\phi_{k-}^2 \tag{17}$$

$$\xi_k^{2*} = -i\frac{\Gamma_{12}^{k} M_2}{M_1^2 - M_2^2}\phi_{k-}^1 + \phi_{k-}^2 . \tag{18}$$

Note that, due to CP violation, the ϕ_{k-}^i are not Hermitian conjugates of the ϕ_{k+}^i but the orthonormality relations $\langle \phi_{k+}^i | \phi_{k-}^j \rangle = \langle \phi_{k-}^i | \phi_{k+}^j \rangle = \delta_{ij}$ between the in and out states are satisfied (as they should be) when diagonalizing a non-Hermitian mass matrix (see e.g. Refs. [19, 20]). The resulting lepton asymmetries ε_k^m induced by the decay of the scalar triplet $\phi_{k\pm}^a$ are given by

$$\varepsilon_1^a = 2\,\frac{\Gamma\left(\phi_{1-}^a \to ll\right) - \Gamma\left(\phi_{1+}^a \to l^c l^c\right)}{\Gamma_{\phi_{1-}^a} + \Gamma_{\phi_{1+}^a}}, \tag{19}$$

$$\varepsilon_2^a = 2\,\frac{\Gamma\left(\phi_{2+}^a \to ll\right) - \Gamma\left(\phi_{2-}^a \to l^c l^c\right)}{\Gamma_{\phi_{2+}^a} + \Gamma_{\phi_{2-}^a}}. \tag{20}$$

Putting Eqs. (15)-(18) in Eqs. (1) and (4) we obtain

$$\varepsilon_1^a \simeq \frac{1}{2\pi(M_1^2 - M_2^2)}\frac{\sum_{i,j}\left\{M_a^2\,\mathrm{Im}[h_1^2 h_1^{1*} f_{ij}^1 f_{ij}^{2*}] + \mathrm{Im}[M_2 M_1 h_2^{2*} h_2^1 f_{ij}^1 f_{ij}^{2*}]\right\}}{\sum_{i,j}|f_{ij}^a|^2 + |h_1^a|^2 + |h_2^a|^2} \tag{21}$$

and similarly we have

$$\varepsilon_2^a \simeq \frac{1}{2\pi(M_2^2 - M_1^2)}\frac{M_1 M_2}{M_a^2}\frac{\sum_{i,j}\left\{M_a^2\,\mathrm{Im}[h_2^2 h_2^{1*} f_{ij}^{1*} f_{ij}^2] + \mathrm{Im}[M_1 M_2 h_1^{2*} h_1^1 f_{ij}^{1*} f_{ij}^2]\right\}}{\sum_{i,j}|f_{ij}^a|^2 + |h_2^a|^2 + |h_1^a|^2} \tag{22}$$

As expected the asymmetries come from the interference of the leptonic sector (through the f_{ij}^a's) and the non-leptonic sector (through the h_k^a's). Such asymmetries are obtained from the decay of each one of the tree states in each scalar triplet. Equal asymmetries are also obtained from the decay of the tree fermionic partners of the scalar triplets. Note that for M_1 close to M_2, $\varepsilon_1^a \sim \varepsilon_2^a$.

When the mass difference between the two Higgs scalars is very small and is comparable to the decay width, there is a resonance in the amount of CP asymmetry, hence in the amount of lepton asymmetry. Our present method fails in the limit when the decay width is larger than the mass differences. However as we did already in Eqs. (11)-(18) we will restrict ourselves to a region where the mass squared difference can be small but still larger than the decay widths, so that the formalism we consider can be used safely. Note that from earlier results in the calculation of the resonance conditions, we understand that enhancement of the asymmetry is almost maximal near the resonant condition $M_1^2 - M_2^2 \sim \Gamma_{12}^k M_2$. So, extending the analysis to even smaller mass difference would not improve our result in any case.

4 Boltzmann Equations

We shall now check if the out-of-equilibrium condition is satisfied in this scenario and can generate the required amount of baryon asymmetry of the Universe. The naive consideration for the out-of-equilibrium condition that the decay rates of the triplet Higgs scalars to be less than the expansion rate of the universe is satisfied for a wide range of parameters. This out-of-equilibrium condition reads,

$$K_{\phi_k^a} = \frac{\Gamma_{\phi_k^a}}{H(M_a)} < 1 \tag{23}$$

where the Hubble constant $H(T)$ at the temperature T is given by

$$H(T) = \sqrt{\frac{4\pi^3 g_*}{45}} \frac{T^2}{M_P}, \tag{24}$$

with $g_* \sim 100$ the number of massless degrees of freedom and $M_P \sim 10^{19}$ GeV is the Planck scale. Given any particular temperature, the out-of-equilibrium condition constrains the various coupling constants. If this condition is satisfied and if the various damping terms due to scatterings are negligible, the total amount of lepton asymmetry per comoving volume $X_L \equiv n_L/s = (n_l - n_{\bar{l}})/s$ that will be generated through the decays of the four triplet Higgs superfields will be given by $\sum_k 6(\varepsilon_1^k + \varepsilon_2^k) n_\gamma/(2s) = \sum_k 6(\varepsilon_1^k + \varepsilon_2^k) 45/(2g_*\pi^4)$

where the entropy s and the photon number density n_γ are given by

$$s = g_* \frac{2\pi^2}{45} T^3, \quad (25)$$

$$n_\gamma = \frac{2\,T^3}{\pi^2}. \quad (26)$$

For the out-of-equilibrium condition of $\phi^a_{1,2}$ to be satisfied, we get a bound on the parameters

$$\frac{\sum_{i,j} |f^a_{ij}|^2 + |h^a_1|^2 + |h^a_2|^2}{M_a} < \sqrt{\frac{4\pi^3 g_*}{45}} \frac{8\pi}{M_P} \sim (4 \cdot 10^{-17} \text{ GeV}^{-1}). \quad (27)$$

It is interesting to compare this condition with the condition that a neutrino mass of order $\sim 10^{-3}$ eV is generated from Eq. (3),

$$-\sum_a \frac{f^a_{ij} h^a_2}{M_a} = \frac{(m_\nu)_{ij}}{2v_2^2} \sim (10^{-17} \text{ GeV}^{-1}), \quad (28)$$

where v_2 has to be of order $v = 246$ GeV. A neutrino mass of order 10^{-3} eV can therefore be obtained while the out-of-equilibrium condition is satisfied for any value of M_1 and M_2 provided the couplings $f^a_{i,j}$ and $h^a_{1,2}$ have the appropriate values[2]. This is in general achieved if h^a_2 together with at least one of the f^a_{ij} for $a = 1$ or 2 are of order $\sim [(10^{-17} \cdot \text{GeV}^{-1}) M_{1,2}]^{1/2}$ (with all other couplings taking smaller values). For $M_{1,2} \sim 10^{14}$ GeV this requires $f^a_{ij} \sim h^a_2 \sim 10^{-2}$-$10^{-1}$ while for $M_{1,2} \sim 10^9$ GeV this requires $f^a_{ij} \sim h^a_2 \sim 10^{-3}$-$10^{-4}$. Assuming a maximal CP violating phase, the lepton asymmetry obtained from Eqs. (21)-(22) is then typically of order $X_L \sim 10^{-5}$-10^{-6} in the former case and $X_L \sim 10^{-10}$-10^{-11} in the latter case. A smaller asymmetry can be generated if for example this CP violating phase is not maximal or if in general larger values of the f's and the h's are taken in such a way that Eq. (28) is satisfied but not Eq. (27). In the latter case the damping term of the inverse decay process will suppress the asymmetry. A larger asymmetry can be obtained if M_1 and M_2 are more degenerate. For $M_{1,2} < 10^9$ GeV a certain degree of degeneracy is needed in order to obtain a baryon asymmetry of the order of the one required, i.e. $X_L \sim 10^{-10}$.

[2]For small values of $M_{1,2}$ this would require however very small values of the f's and the h's whose naturalness could be questioned.

The above estimate has however not taken into account possible scattering damping terms. There are for example lepton-number violating scattering processes, which can deplete the generated lepton asymmetry of the Universe. For example, $H_1 + H_1 \to \tilde{L} + \tilde{L}$ and $H_1 + H_1 \to \xi_2 \to \tilde{L} + \tilde{L}$ scattering (which are absent in the nonsupersymmetric case) come from renormalizable terms and may not be suppressed. However, it can be shown that these processes are not really relevant because they go out of equilibrium once we require the decays of the triplets to be slow enough to be away from thermal equilibrium. There is also one lepton-number conserving process which is more of a problem, i.e. the gauge interactions of the triplet Higgs superfields. These induce the very fast $\xi_a^\dagger + \xi_a' \to G_1 + G_2$ scattering process, where ξ_a and ξ_a' are two scalar triplets, and G_1 and G_2 are two $SU(2)_L$ or $U(1)_Y$ gauge bosons, as obtained from the kinetic term of the scalar triplets. This gives a suppression in the generation of the lepton asymmetry of the Universe and implies that the mass of the triplets cannot be too small (except if the two triplets are almost degenerate as shown below). The presence of this damping term requires the explicit calculation of the evolution of the asymmetry using the Boltzmann equations.

Defining the variable $z \equiv M_1/T$ and the various number densities per comoving volume $X_i \equiv n_i/s$, the Boltzmann equations are:

$$\frac{dX_{\phi_k^a}}{dz} = -zK_{\phi_k^a}\frac{K_1(z)}{K_2(z)}\Big(\frac{M_{\phi_k^a}}{M_1}\Big)^2\Big(X_{\phi_k^a} - X^{eq}_{\phi_k^a}\Big) + z\frac{1}{sH(M_1)}\Big(1-\frac{X^2_{\phi_k^a}}{X^{eq2}_{\phi_k^a}}\Big)\gamma^a_{scatt.} \tag{29}$$

$$\frac{dX_L}{dz} = \sum_{a,k} zK_{\phi_k^a}\frac{K_1(z)}{K_2(z)}\Big(\frac{M_{\phi_k^a}}{M_1}\Big)^2\Big[\varepsilon^a_k(X_{\phi_k^a} - X^{eq}_{\phi_k^a}) - \frac{1}{2}\frac{X^{eq}_{\phi_k^a}}{X_\gamma}X_L\Big]. \tag{30}$$

In Eqs. (29)-(30) the equilibrium distributions of the number densities are given by the Maxwell-Boltzmann statistics:

$$n_{\phi_k^a} = g_{\phi_k^a}\frac{M^2_{\phi_k^a}}{2\pi^2}TK_2(M_{\phi_k^a}/T), \tag{31}$$

where $g_{\phi_k^a} = 1$ are the numbers of degrees of freedom of the ϕ_k^a and $K_{1,2}$ are the usual modified Bessel functions. The reaction density for the scattering process $\xi_a^\dagger + \xi_a' \to G_1 + G_2$

is given by

$$\gamma^a_{scatt.} = \frac{T}{64\pi^4} \int_{4M_a^2}^{\infty} ds\, \hat{\sigma}_a(s)\, \sqrt{s}\, K_1(\sqrt{s}/T), \tag{32}$$

where $\hat{\sigma}$ is the reduced cross section which is given by $2(s - 4M_a^2)\sigma_a(s)$. Note that a precise result would require an explicit calculation of all scattering processes involving gauge interactions in all channels.[3] However it can be checked that the dependence of the generated lepton asymmetry on the magnitude of the scattering is much slower than linear. Therefore, considering also the fact that the model allows some freedom in the range of parameters used, this explicit calculation will not add much to our understanding in any case. We will thus make the following estimate:

$$\sigma_a = \frac{1}{\pi\sqrt{s}} \frac{1}{\sqrt{s - 4M_a^2}}\, g^4 \tag{33}$$

where g is the $SU(2)_L$ coupling (which at tree level is given by the relation $m_W^2 = g^2 v^2/4$). Putting Eq. (33) in Eqs. (32) and (29), it turns out that the scattering term has a small effect on the evolution of the lepton asymmetry for values of $M_{1,2}$ above $10^{11} - 10^{12}$ GeV. For smaller values of $M_{1,2}$ the suppression can be very strong due to the fact that the last term of Eq. (29) increases when M_1 decreases and $T \sim M_1$. This will suppress the asymmetry which at some point becomes much smaller than $\sim 10^{-10}$ except if M_1 and M_2 are sufficiently degenerate. Note that the suppression due to these scattering processes is the most effective when the triplet starts decaying. At lower temperatures, the scattering effect is suppressed by the Boltzmann factor due to the higher threshold in Eq. (32). Taking for example f^a_{22}, f^a_{23}, f^a_{32} and f^a_{33} (for both $a = 1$ and 2) equal to the same value f with all other f^a_{ij} equal to zero, i.e. assuming negligible all the f^a_{ij} with $i = 1$ and/or $j = 1$, (which constitutes one of the possible structures leading to a maximal mixing between the second and third generation of neutrinos) and taking all h^a_k couplings equal to the same value h,

[3] There are more than 20 different physical processes of the type $\xi^\dagger_a + \xi'_a \to G_1 + G_2$. There is also scattering of the type $\xi^\dagger_a + \xi'_a \to l + \bar{l}$ with an intermediate gauge boson which is of the same order.

four typical sets of parameters which give an asymmetry of order $\sim 10^{-10}$ together with a neutrino mass of order $10^{-3} - 10^{-2}$ eV are shown below:

$$M_1 = 10^{13} \text{ GeV} \;\; M_2 = 3.0 \cdot 10^{13} \text{ GeV} \;\; h = 1 \cdot 10^{-3} \;\; f = 3 \cdot 10^{-2} \tag{34}$$

$$M_1 = 10^{12} \text{ GeV} \;\; M_2 = 3.0 \cdot 10^{12} \text{ GeV} \;\; h = 1 \cdot 10^{-2} \;\; f = 5 \cdot 10^{-4} \tag{35}$$

$$M_1 = 10^{11} \text{ GeV} \;\; M_2 = 2.0 \cdot 10^{11} \text{ GeV} \;\; h = 8 \cdot 10^{-4} \;\; f = 2 \cdot 10^{-3} \tag{36}$$

$$M_1 = 10^{10} \text{ GeV} \;\; M_2 = 1.1 \cdot 10^{10} \text{ GeV} \;\; h = 1 \cdot 10^{-3} \;\; f = 5 \cdot 10^{-4}. \tag{37}$$

A maximal CP-violating phase has been assumed. Note that the degree of degeneracy which is required for $M_{1,2} \sim 10^{10}$ GeV is relatively small. Note also that smaller values of $M_{1,2}$ are possible if they are even more degenerate. As $M_{1,2}$ decreases, the degree of degeneracy required becomes however very high, due to the damping effects of the scattering processes.

5 Gravitino Problem

So far we have not taken into account the gravitino problem. The main constraint comes from the fact that the lepton asymmetry has to be generated after inflation, which is very important in supersymmetric models [8, 21]. The thermal production of massive gravitinos restricts the beginning of the radiation-dominated era following inflation. The reheating temperature after inflation is constrained by requiring gravitino production to be suppressed so that it will not overpopulate the Universe. Since the gravitinos interact very weakly, they decay very late and modify the abundances of light elements which may become inconsistent with nucleosynthesis. On the other hand, if they are stable, then they overclose the universe. The upper bound on the reheating temperature from the gravitino constraint is [21]

$$T_{RH} \leq 10^{10} \text{ GeV } \times \left(\frac{m_{3/2}}{100 \text{ GeV}}\right) \times \left(\frac{1 \text{ TeV}}{m_{\tilde{g}}(\mu)}\right)^2 \tag{38}$$

where $m_{3/2}$ is the gravitino mass and $m_{\tilde{g}}$ is the running mass of the gluino. This gravitino constraint is satisfied if the lepton asymmetry is generated at temperatures below the reheating temperature $T < T_{RH}$. From this result we can assume that the masses of the triplet Higgs scalars should be around $T_{RH} \sim 10^{10} - 10^{11}$ GeV, so that leptogenesis occurs at a temperature $T < 10^{10} - 10^{11}$ GeV. As shown above, a lepton asymmetry and neutrino masses of the size required can be generated with this value of the mass, but it requires some (moderate) degree of degeneracy between M_1 and M_2 [see Eq. (37)].

6 Summary and Conclusion

We have shown that the supersymmetric triplet Higgs model developed in this article constitutes an interesting and simple alternative for generating neutrino masses and baryogenesis. This requires typically triplet superfields with mass of order $10^9 - 10^{14}$ GeV. We have shown that this mechanism has the interesting property of possible resonant behavior. For masses of order 10^{10} GeV which are consistent with the gravitino problem, this feature of resonant CP violation is necessary (i.e. masses of the triplets need to be moderatly degenerate).

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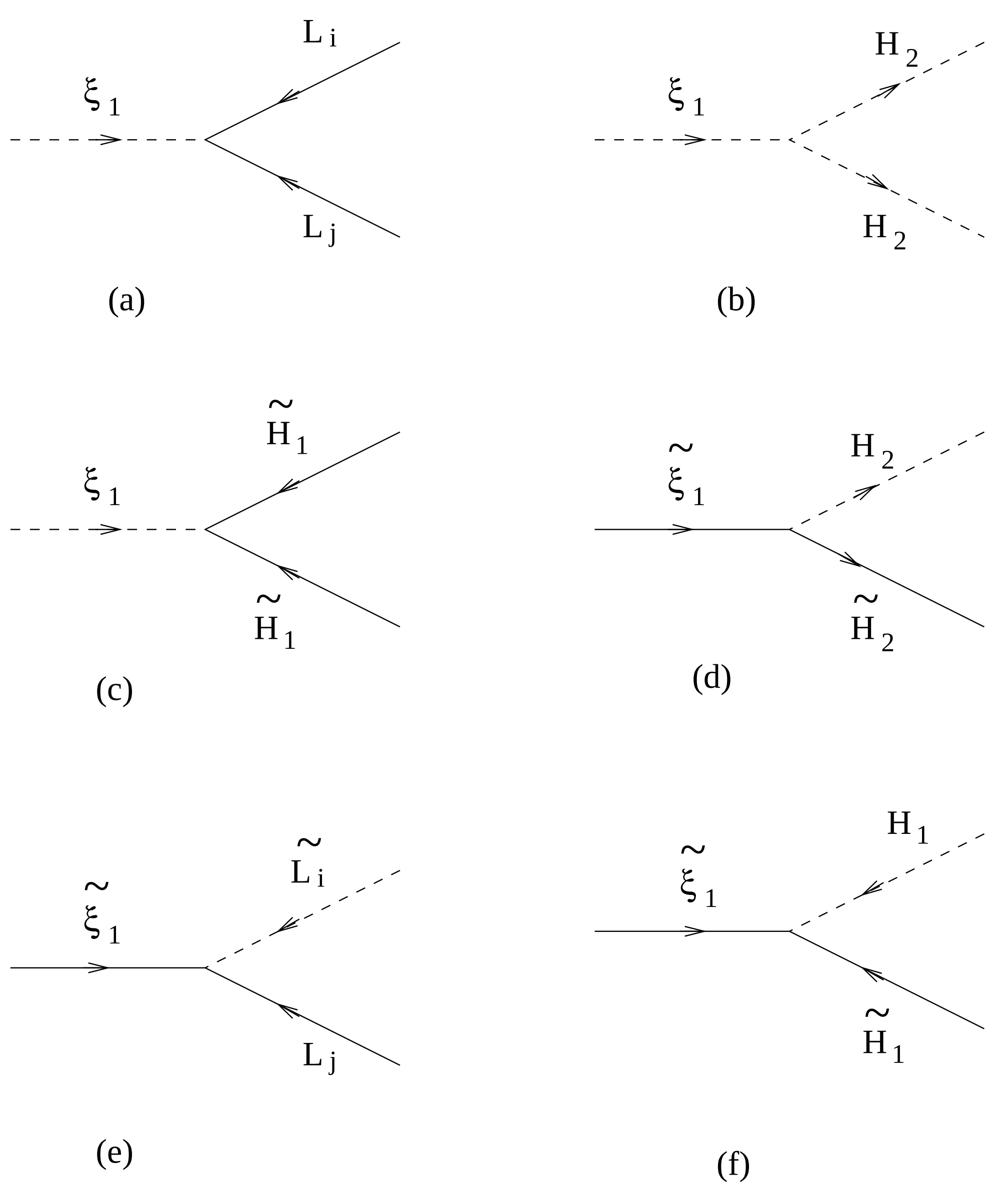

Figure 1: Tree level diagrams for the decay of $\hat{\xi}_1$.

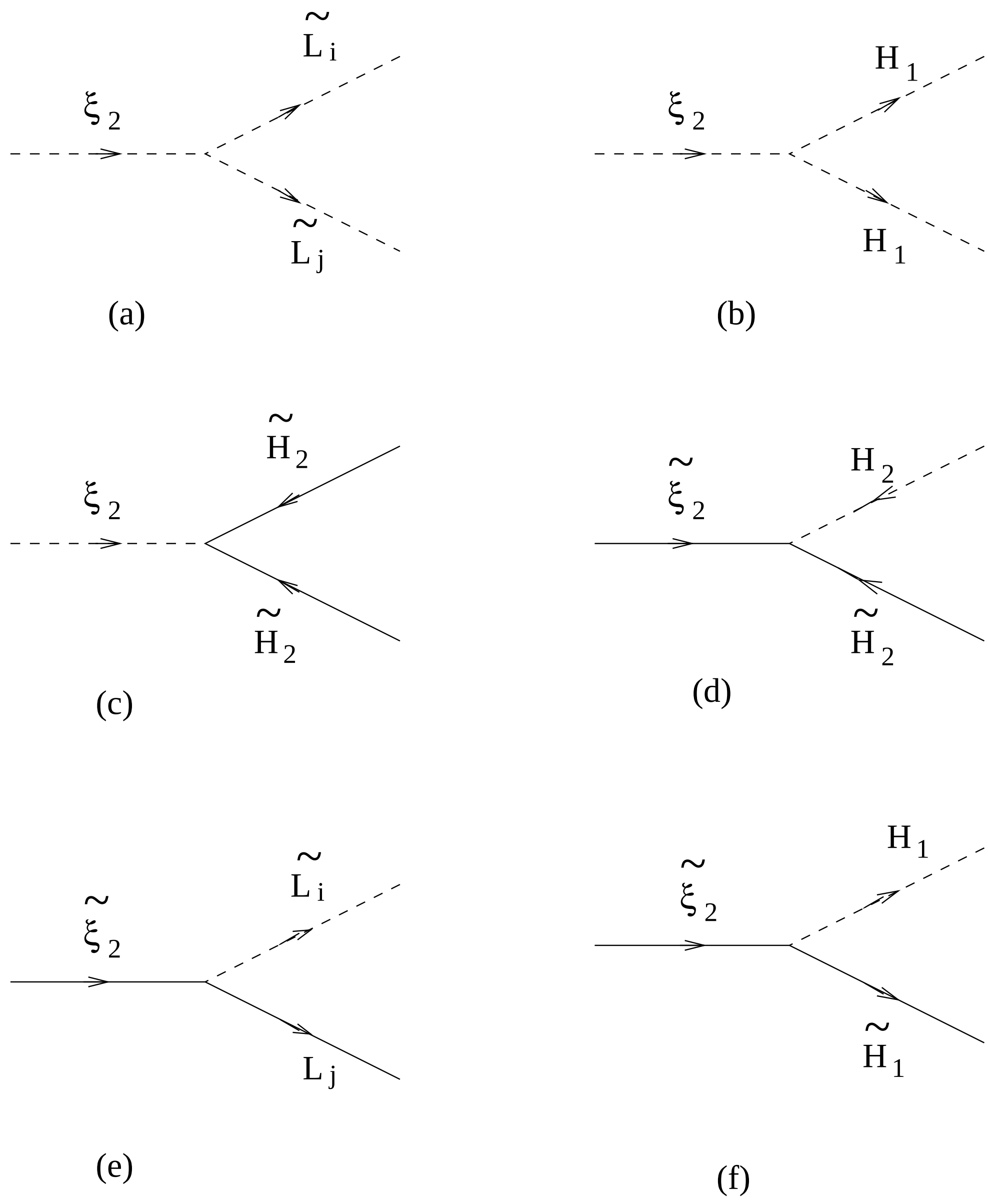

Figure 2: Tree level diagrams for the decay of $\hat{\xi}_2$.

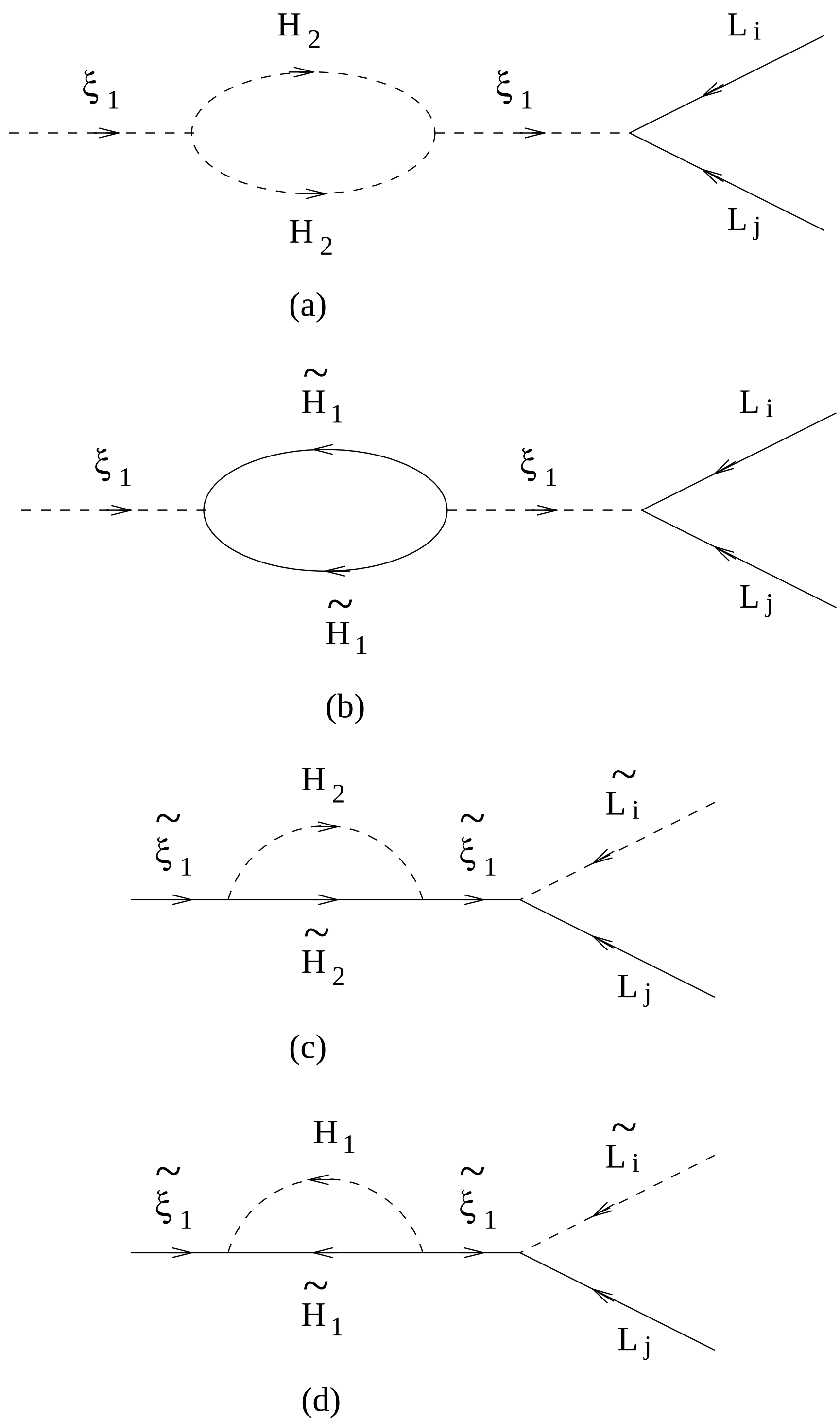

Figure 3: One loop diagrams contributing to CP violation in decays of $\hat{\xi}_1$.

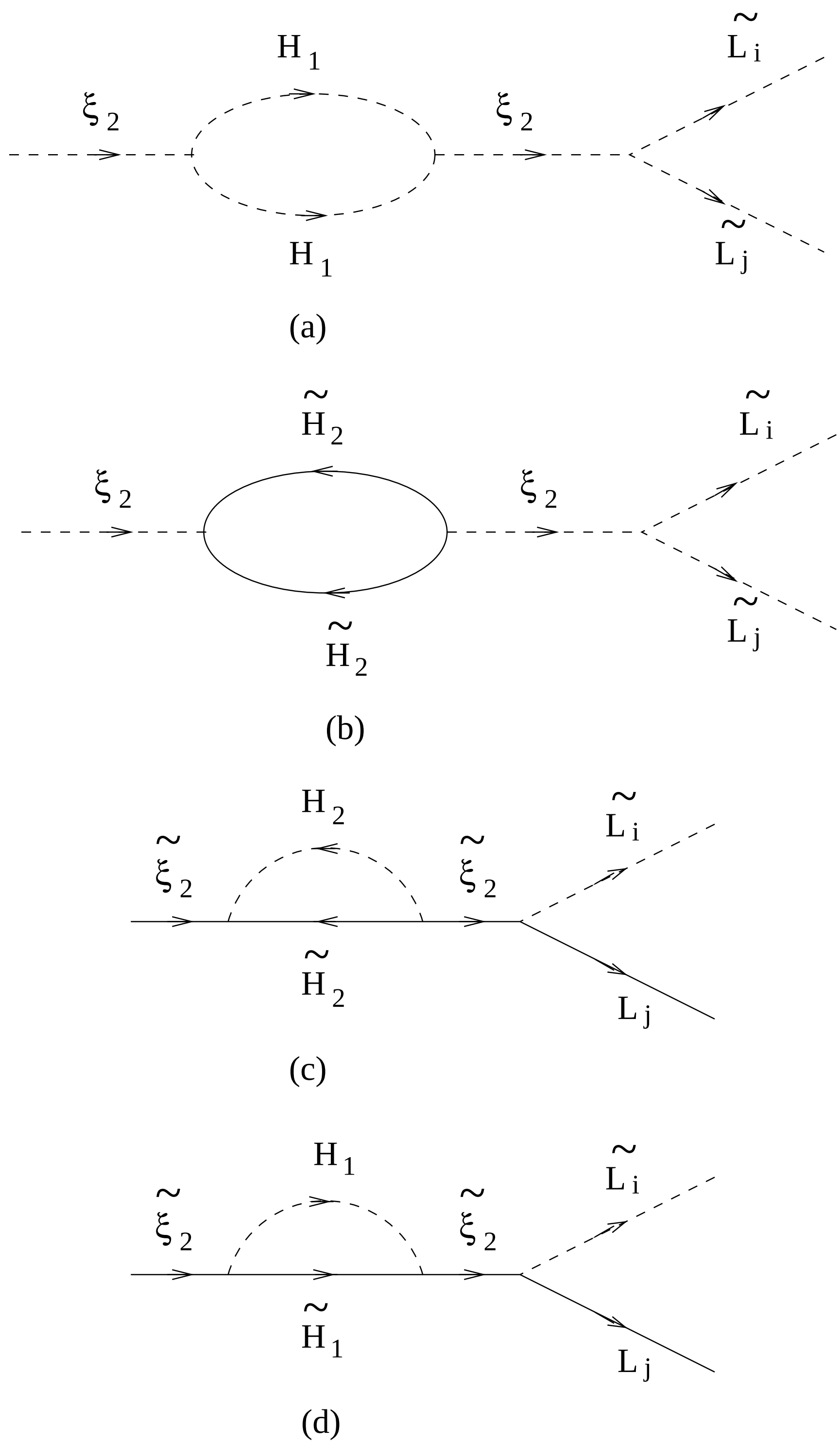

Figure 4: One loop diagrams contributing to CP violation in decays of $\hat{\xi}_2$.