hep-th/9511026

U-DUALITY AND INTERSECTING D-BRANES

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Abstract

Spectrum of elementary string states in type II string theory compactified on a torus contains short multiplets which are invariant under only one quarter of the space-time supersymmetry generators. U-duality transformation converts these states into bound states of Dirichlet branes which wrap around intersecting cycles of the internal torus. We study a class of these bound states that are dual to the elementary string states at the first excited level, and argue that the degeneracy of these bound states is in agreement with the U-duality prediction.

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The spectrum of type IIA or IIB string theory compactified on a torus contains single particle states in the ultra-short (256 dimensional) multiplet. These states carry equal amount of left and right moving charges and are in the ground state of *both*, the left and the right moving oscillators. As a result they always have unit multiplicity. There are also states in the short multiplet (of dimension $(16)^3$) which can carry different amounts of left and right moving charges, and are in the ground state of *either* the left *or* the right moving oscillators. For definiteness, we shall assume from now on that the right moving oscillators are in their ground state. The degeneracy of these states depends on the level of the left moving oscillator state. If $\vec{Q}_R$ and $\vec{Q}_L$ denote the right and the left moving charge vectors, and N_L denotes the level of the left moving part of the state, then, with suitable normalization, the mass of the state (in the Ramond-Ramond (RR) sector) is given by:

$$M^2 = \frac{\vec{Q}_R^2}{2} = \frac{\vec{Q}_L^2}{2} + N_L \, . \tag{1}$$

Similar formula exits in the Neveu-Schwarz (NS) sector, but we do not need to write it down. We shall take eq.(1) to be the defining equation for N_L for given $\vec{Q}_L$, $\vec{Q}_R$. The total degeneracy $d(N_L)$ of such states for a given values of $\vec{Q}_L$ and $\vec{Q}_R$, can be computed from the formula:

$$\sum_{N_L=0}^{\infty} d(N_L) q^{N_L} = 256 \times \prod_{n=1}^{\infty} \left(\frac{1+q^n}{1-q^n}\right)^8 . \tag{2}$$

In order to find the number of short multiplets, one needs to divide the value of $d(N_L)$ obtained this way by $(16)^3$, which is the dimension of the supermultiplet. The degeneracies of some of the low lying states are $d(1) = (16)^3$, $d(2) = 9 \times (16)^3$ etc.

The charges $\vec{Q}_R$ and $\vec{Q}_L$ referred to above couple to the $U(1)$ gauge fields arising in the NS-NS sector of the theory. Since U-duality[1] transforms the states carrying NS-NS charges to states carrying RR charges in general, and since the latter states have been shown to arise from Dirichlet branes (D-branes)[2] wrapped around various internal cycles, we expect that we should be able to reproduce the values of $d(N_L)$ quoted above by working out the degeneracies of the states of D-branes. This analysis has been made possible by the recent discovery of Witten[3] that the dynamics of collective coordinates of n parallel D-branes is described by a supersymmetric $U(n)$ gauge theory.[3] Using this collective coordinate description, we have argued in a previous paper[5] that for ultra-short multiplets, the degeneracy of D-brane states agrees with the prediction of U-duality. In a recent paper[6] Bershadsky, Sadov, and Vafa have generalized Witten's result to the case of intersecting D-branes. In this paper we shall use this result to analyze the degeneracy of short multiplets. In particular, we shall work out the degeneracy of D-brane states dual to the $N_L = 1$ states and argue that the number is indeed equal to $d(1)$, in agreement with the prediction of U-duality.

In order to be more specific, let us consider type IIA string theory compactified on a four torus T^4, with each of the four internal circles having self-dual radius. We shall denote the compact directions by x^m for $(6 \le m \le 9)$, and the non-compact directions by $x^{\bar{\mu}}$ for $(0 \le \bar{\mu} \le 5)$. The index μ will run over all values. This 6-dimensional theory has eight $U(1)$ gauge fields coming from the NS-NS sector, and eight $U(1)$ gauge fields coming from the RR sector. The U-duality group of this theory has a specific Z_2 element, which changes all the NS-NS gauge fields to RR gauge fields and vice versa, and acts as a triality rotation on the T-duality group $SO(4,4;Z)$[7]. We shall use this Z_2 element to convert a state carrying purely NS-NS charge to a state carrying purely RR charge.

[3] Another approach to analyzing bound states of D-brane states has been given in [4].

In order to be more specific about the action of this Z_2 element on various charges, we need to choose a basis for the charge vector. A suitable choice of basis will be as follows. If p_i and w_i denote the momentum and winding number associated with the internal directions x^i, then we represent the charge vector as

$$Q = \begin{pmatrix} p_6 \\ w_6 \\ \cdot \\ \cdot \\ p_9 \\ w_9 \end{pmatrix} . \tag{3}$$

In this basis the inner product of two charge vectors is computed from the metric,

$$L = \begin{pmatrix} \sigma_1 & & & \\ & \sigma_1 & & \\ & & \sigma_1 & \\ & & & \sigma_1 \end{pmatrix}, \qquad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \tag{4}$$

In particular, $\vec{Q}_L$ and $\vec{Q}_R$ will denote the projection of the charge vector to the subspace with L eigenvalues -1 and $+1$ respectively. Also,

$$\vec{Q}_R^2 - \vec{Q}_L^2 = Q^T L Q \,. \tag{5}$$

Consider now the charge vector:

$$\begin{pmatrix} m \\ n \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix} . \tag{6}$$

For this $Q^T LQ = 2mn$. Using eq.(1) we see that for a short multiplet, a state carrying this charge vector has $N_L = Q^T LQ/2 = mn$. One can compute the degeneracy of such states in the elementary string spectrum from eq.(2). Also the mass of this state (which we shall refer to as (m,n) state) is given by:

$$M(m,n) = \frac{1}{\sqrt{2}}|\vec{Q}_R| = \frac{1}{2}(|m| + |n|) \,. \tag{7}$$

This shows that these states are only marginally stable. In particular, an (m,n) state can decay into an $(m,0)$ state and a $(0,n)$ state at rest. We shall come back to this point later.

What will be the dual of these states under the Z_2 transformation mentioned above? The gauge fields in the RR sector come from the components $C_{mn\bar{\mu}}$ of the rank three anti-symmetric tensor fields in the IIA theory, as well as the gauge field $A_{\bar{\mu}}$ and the dual $\widetilde{C}_{\bar{\mu}}$ of $C_{\bar{\mu}\bar{\nu}\bar{\rho}}$. In this basis, the inner product matrix for the RR charges is given as follows. It pairs $A_{\bar{\mu}}$ charge with $\widetilde{C}_{\bar{\mu}}$ charge. For $C_{mn\bar{\mu}}$ charge the inner product matrix is simply the intersection matrix of the corresponding two cycles on the torus. The Z_2 transformation mapping the RR gauge fields to NS-NS gauge fields must preserve the inner product matrix. Thus we can choose the Z_2 to induce the following map between the gauge fields (up to sign)

$$C_{89\bar{\mu}} \leftrightarrow G_{6\bar{\mu}} \qquad\qquad C_{67\bar{\mu}} \leftrightarrow B_{6\bar{\mu}}$$

$$
\begin{array}{ll}
C_{78\bar{\mu}} \leftrightarrow G_{7\bar{\mu}} & C_{96\bar{\mu}} \leftrightarrow B_{7\bar{\mu}} \\
C_{68\bar{\mu}} \leftrightarrow G_{8\bar{\mu}} & C_{79\bar{\mu}} \leftrightarrow B_{8\bar{\mu}} \\
A_{\bar{\mu}} \leftrightarrow G_{9\bar{\mu}} & \widetilde{C}_{\bar{\mu}} \leftrightarrow B_{9\bar{\mu}} \, .
\end{array}
\tag{8}
$$

In particular, the momentum p^6 that couples to $G_{6\bar{\mu}}$ will be mapped to $C_{89\bar{\mu}}$ charge and the winding w^6 that couples to $B_{6\bar{\mu}}$ will be mapped to $C_{67\bar{\mu}}$ charge. Thus the (m, n) state will be mapped to a state with m units of $C_{89\bar{\mu}}$ charge and n units of $C_{67\bar{\mu}}$ charge. As shown in ref.[2], such a state can arise from a state with $(m + n)$ Dirichlet membranes, with m of them wrapping around the 8-9 cycle of the torus, and n of them wrapping around the 6-7 cycle of the torus. In order to prove the invariance of the spectrum under U-duality, one needs to show that this system contains supersymmetric bound states with degeneracy $d(mn)$.

We shall analyze the $(1, 1)$ case in detail and show that the degeneracy of states of the D-membranes agree with $d(1)$ computed from elementary string spectrum. But before we proceed we need to resolve the usual problem with marginally stable states. Since a $(1, 1)$ state can decay into a $(1, 0)$ state and a $(0, 1)$ state at rest, there is no energy barrier against pulling the two D-membranes away from each other. This makes the analysis difficult.[4] To get around this problem, we use the trick used in ref.[5]. Namely, we compactify one more direction (say x^1) so that the momentum along x^1 is quantized, and look at the sector carrying odd units of momentum in this direction. If a marginally stable bound state of two D- membranes of the type discussed above exist in the 6 dimensional theory, then the Kaluza-Klein modes of this state, carrying odd units of momentum along the x^1 direction, will be absolutely stable in the resulting 5 dimensional theory. We shall look for these absolutely stable states in the spectrum.

As in ref.[5] we shall simplify the analysis by performing a T-duality transformation involving the coordinate x^1. Let us denote the coordinates in this new theory by y^μ. This duality transformation will convert the momentum along x^1 into winding number[5] along y^1, and convert a membrane wrapped around m-n plane into a three brane wrapped around the m-n-1 plane. Thus the problem that we have is that of a three brane D_1 wrapped around the 8-9-1 plane interacting with a three brane D_2 wrapped around the 6-7-1 plane. We want to look for supersymmetric ground states of this system carrying k units of winding along the y^1 direction, where k is odd. The dynamics of collective coordinates of this system has been given in ref.[6]. Since the configuration described above has translation invariance only along the y^0 and y^1 direction, it is described by a $(1 + 1)$ dimensional field theory with base space labelled by the coordinates y^0 and y^1. The theory is an $N = 4$ supersymmetric gauge theory (which can be regarded as the dimensional reduction of an $N = 2$ theory in four dimensions) with an $U(1)_1 \times U(1)_2$ gauge group. Besides the $U(1)$ vector multiplets which we shall denote by A_1 and A_2, the theory contains a pair of gauge neutral hypermultiplets Φ_1 and Φ_2, and a hypermultiplet Q of charge $(+1, -1)$ under the $U(1)_1 \times U(1)_2$ gauge group. For our purpose it will be convenient to work with the diagonal sum and difference of the two $U(1)$ groups which we shall denote by $U(1)_c$ and $U(1)_r$. The corresponding vector multiplets will be denoted

[4] Although in principle one can directly study the question of existence of these marginally stable bound states by mapping this problem to a supersymmetric quantum mechanics problem, it has been pointed out by C. Vafa that naive computation of Witten index in these models might not always agree with the predictions of U-duality[8]. This could be related to subtleties that might be present at large separation. For this purpose we shall find it much more convenient to work with the absolutely stable bound states described below.

[5] Throughout this paper, winding number will refer to the winding number of an elementary string that couples to the $B_{\mu\nu}$ field.

by A_c and A_r. (Here c stands for center of mass and r for relative coordinates, for reasons that will be explained soon). Then the gauge group is $(U(1)_c \times U(1)_r)/Z_2$, and the hypermultiplet Q has charge $(0,2)$ under $U(1)_c \times U(1)_r$. Of these set of fields, the gauge multiplet A_c and the hyper-multiplets Φ_1 and Φ_2 have no interactions, the only interaction of the theory comes from the standard gauge coupling between the vector multiplet A_r and the charged hypermultiplet Q.

Before we proceed further, let us give a physical interpretation of the various bosonic fields, at least in the region where the two three branes D_1 and D_2 are well separated in the physical space. We start with the neutral hypermultiplets. Of the four scalars in the hypermultiplet Φ_1, two denote the y^6 and y^7 coordinates of the three brane D_1 lying in the 8-9-1 plane. The other two are coordinates conjugate to the winding number along the 8 and the 9 direction. Similarly, of the four scalars in the hypermultiplet Φ_2, two correspond to the y^8 and y^9 coordinates of the three brane D_2 lying in the 6-7-1 plane, and two are conjugate to the winding number carried by D_2 along y^6 and y^7. Since the winding as well as momenta along the $y^6, \ldots y^9$ directions are quantized, we see that we should interprete the scalar fields in both the hypermultiplets Φ_1 and Φ_2 as compact coordinates in the field space.

A vector multiplet in two dimensions has four scalars and one vector field. The four scalars in the $U(1)_1$ vector multiplet correspond to the coordinates (y^2, y^3, y^4, y^5) of the three brane D_1. The total electric flux associated with the $U(1)_1$ gauge field along y^1 may be interpreted as the winding number carried by D_1 along y^1, – this situation is identical to the one described in ref.[3]. Similarly the four scalars in the $U(1)_2$ vector multiplet correspond to the coordinates (y^2, y^3, y^4, y^5) of the three brane D_2, and the total electric flux associated with the $U(1)_2$ gauge field along y^1 may be interpreted as the winding number carried by D_2 along y^1. In terms of the vector multiplets A_c and A_r this means that the scalar fields in A_c denote the center of mass coordinates of D_1 and D_2 along $y^2, \ldots y^5$, the scalar fields in A_r denote the relative coordinates along $y^2, \ldots y^5$, the $U(1)_c$ electric flux along y^1 denotes the total winding number of the system along y^1, and the electric flux of $U(1)_r$ along y^1 measures the difference between the winding numbers carried by the two Dirichlet three branes.

Finally we turn to the hypermultiplet Q. These fields do not have a simple interpretation as space-time coordinates of the three branes, as can be seen from the fact that due to their coupling to A_r, they become very heavy when the relative separation between D_1 and D_2 is large. These correspond to open string states that start on D_1 and end on D_2 or vice versa.[6]

Let us now turn to the quantization of this system. First we discuss the free part of the theory, the one involving A_c, Φ_1 and Φ_2. Since we are looking for states that do not carry any momentum or winding along any of the directions $y^6, \ldots y^9$, we can take the momenta conjugate to the scalar components of Φ_1 and Φ_2 to be zero. This gives a unique state from this sector. Similarly if we are considering the system at rest we can take the total momenta, which are conjugate to the four scalar components of A_c, to vanish. Finally we can take the system to be in the eigenstate of the electric flux of $U(1)_c$ gauge field with eigenvalue k, where k is the required winding number along y^1. For reasons that has been explained before, we shall take k to be odd. Thus the quantization

[6]Physically, the absence of coupling between the neutral hypermultiplets Φ_1, Φ_2 and the charged hypermultiplet Q may be understood as follows. Mass of Q should vanish whenever the two D-branes intersect. This happens whenever the spatial separation between the two D-branes along $y^2, \ldots y^5$ directions vanish. Since these spatial separations are represented by scalar components of the vector multiplet A_r, the mass of Q cannot depend on the vacuum expectation values of the hypermultiplets Φ_1 and Φ_2.

of the bosonic part of the free system gives a unique state for every value of k.

What about the fermionic part? Each neutral hypermultiplet contains eight real fermionic coordinates, and the $U(1)_c$ vector multiplet also contains eight real fermionic coordinates. This gives rise to 24 fermionic zero modes in total. Quantization of this gives rise to a $2^{12} = (16)^3$ fold degeneracy.

Thus if K denotes the number of supersymmetric ground states of the interacting system, involving the vector multiplet A_r and the charged hypermultiplet Q, then the total degeneracy of this state will be given by:

$$(16)^3 \cdot K \,. \tag{9}$$

This number then needs to be compared with $d(1)$. We have already stated that $d(1) = (16)^3$; thus we see that the prediction of U-duality is

$$K = 1 \,. \tag{10}$$

We shall now proceed to verify this prediction. The approach that we shall use is identical to the one taken in ref.[3]. The interacting system can be viewed as the dimensional reduction of a four dimensional theory, containing an $N = 2$ $U(1)$ vector supermultiplet interacting with an $N = 2$ hypermultiplet of $U(1)$ charge two. We are looking for a supersymmetric ground state in this theory in the sector that carries odd unit of $U(1)_r$ flux along y^1; this requirement comes from the fact that the gauge group is really $(U(1)_c \times U(1)_r)/Z_2$, and hence the presence of an odd unit of $U(1)_c$ flux forces us to have an odd unit of $U(1)_r$ flux as well. Viewed as an $N = 1$ supersymmetric theory, this corresponds to an $N = 1$ supersymmetric $U(1)$ gauge theory, with a $U(1)$ neutral chiral superfield Φ, and a pair of chiral superfields Λ, $\bar{\Lambda}$ with $U(1)$ charge ± 2, interacting through the superpotential:

$$W_0 = \Phi \Lambda \bar{\Lambda} \,. \tag{11}$$

The classical theory has a flat direction, since $\langle \Phi \rangle$ is undetermined. However, for large $\langle \Phi \rangle$ the fields Λ, $\bar{\Lambda}$ become heavy, and the resulting low energy theory is that of a supersymmetric $U(1)$ gauge theory with no matter. The $U(1)_r$ electric flux along the y^1 direction is not screened, and as a result this field configuration has non-zero energy and breaks supersymmetry. This shows that there is a finite energy barrier against taking Φ to be large. As in ref.[3], we shall assume that due to the presence of the energy barrier, we can add mass terms for various fields in the superpotential without changing the number of supersymmetric ground states. Thus we work with the modified superpotential:

$$W = \Phi \Lambda \bar{\Lambda} + \frac{1}{2} \epsilon \Phi^2 + \eta \Lambda \bar{\Lambda} \,. \tag{12}$$

Possible supersymmetric ground states of this system corresponding to the critical points of W are obtained by solving the equations:

$$\epsilon \Phi + \bar{\Lambda} \Lambda = 0 \,, \qquad (\Phi + \eta) \Lambda = 0 = (\Phi + \eta) \bar{\Lambda} \,. \tag{13}$$

For the trivial solution, given by $\Phi = \Lambda = \bar{\Lambda} = 0$, the fields Φ, Λ and $\bar{\Lambda}$ are all massive, and $U(1)$ is unbroken. Thus the background electric field remains unscreened, costing a finite energy, and hence supersymmetry is broken in this vacuum. The non-trivial solution, up to (complexified) $U(1)$ gauge transformation, is given by

$$\Phi = -\eta, \qquad \Lambda = \bar{\Lambda} = \sqrt{\epsilon \eta} \,. \tag{14}$$

In this vacuum the $U(1)$ gauge symmetry is broken, and all fields are massive. Thus the electric flux along y^1 is screened, and we get a supersymmetric ground state. Hence we have a unique supersymmetric ground state, showing that the number K is indeed 1, as predicted by U-duality.

The U-dual of more general (m, n) state described before are given by bound states of m D- membranes lying in the $8-9$ plane, and n D-membranes lying in the $6-7$ plane. The collective coordinate dynamics of this system has been described in ref.[6], and the interacting part of the theory is now given by an $N = 4$ supersymmetric $U(1) \times SU(m) \times SU(n)$ theory in two dimensions, with a hypermultiplet each in the adjoint representations of $SU(m)$ and $SU(n)$, and a charged hypermultiplet that carries $U(1)$ charge, and belongs to the fundamental representations of both, $SU(m)$ and $SU(n)$. It will be extremely interesting to count the number of supersymmetric ground states of this system and see if this agrees with the corresponding number $d(mn)/(16)^3$, which is the number of short multiplets in the elementary string spectrum for $N_L = mn$.

I would like to thank C. Vafa for useful correspondence.

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