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Type IIA Dual of the Six-Dimensional CHL Compactification

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Abstract

We propose a candidate for the dual (in the weak/strong coupling sense) of the six-dimensional heterotic string compactification constructed recently by Chaudhuri, Hockney and Lykken. It is a type IIA string theory compactified on an orbifold $K3/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ action involves an involution of $K3$ with fixed points, and also has an embedding in the U(1) gauge group associated with the Ramond-Ramond sector of the type IIA string theory. This introduces flux of the U(1) gauge field concentrated at the orbifold points. This construction provides an explicit example where the dual of a super-conformal field theory background of the heterotic string theory is not a standard super-conformal field theory background of the type IIA string theory.

It has been conjectured recently that various heterotic string compactifications are nonperturbatively equivalent to type IIA string compactifications. The first example of this kind is the equivalence between heterotic string theory compactified on a four-dimensional torus and type IIA string theory compactified on the $K3$ surface[1, 2, 3, 4, 5, 6, 7, 8]. The massless sectors of both of these theories correspond to non-chiral $N = 2$ supergravity in six dimensions with a gauge group of rank 24. Toroidal compactification to four dimensions then gives dual theories with $N = 4$ supersymmetry. More recently, examples of dual pairs of theories in four dimensions with $N = 2$ supersymmetry have been found[9, 10]. The conjectured equivalences of these theories require some highly non-trivial identities to be satisfied, many of which have been verified explicitly[11, 12, 13].

In another interesting development, Chaudhuri, Hockney and Lykken (CHL) have constructed new examples of heterotic string compactification with $N = 4$ supersymmetry in four dimensions or $N = 2$ supersymmetry in six dimensions[14]. The original construction was formulated in terms of fermionic world sheet variables, but recently Chaudhuri and Polchinski have constructed one of these models as an asymmetric orbifold[15]. In simple terms, this model corresponds to a $\mathbb{Z}_2$ orbifold of the toroidally compactified heterotic string theory, where the $\mathbb{Z}_2$ acts by exchanging the two E_8 components of the momentum lattice, together with a shift by half a period along one of the compact directions. The effect of this $\mathbb{Z}_2$ modding is to remove eight of the $U(1)$ gauge bosons from the spectrum, thereby reducing the rank of the gauge group by eight. This construction does not require that the $E_8 \times E_8$ gauge group is unbroken before the $\mathbb{Z}_2$ modding, but only that the two E_8's are broken in an identical manner so that the resulting lattice still has a $\mathbb{Z}_2$ automorphism. This allows us to work at a point in the moduli space where the unbroken gauge group is just a product of the $U(1)$ factors.

The question that we wish to address is whether there are type IIA string compactifications that are non-perturbatively equivalent to the CHL compactification of the heterotic string. Since the CHL construction can be understood as an orbifold of the toroidal compactification, and since we already know the equivalent type IIA theory in the latter case, one might imagine that the answer to this question lies in taking an appropriate orbifold of the type IIA string theory compactified on $K3$[16, 17]. While this is precisely the procedure that was used in ref.[10] to construct a dual pair of theories with $N = 2$ supersymmetry in four dimensions, the situation is more subtle in our case.

For $D \leq 5$ the construction is simple[18]. For example, in five dimensions the dual theory can be constructed by taking a type IIA theory compactified on $K3 \times S^1$, and then modding it by a $\mathbb{Z}_2$ action which includes a shift on the S^1 by π, and whose action on $K3$ is such that the $(0,0)$, $(2,2)$, $(0,2)$ and $(2,0)$ forms as well as twelve out of the twenty $(1,1)$ forms are invariant under this $\mathbb{Z}_2$. (The other eight $(1,1)$ forms are mapped to their negatives.) This reduces the rank of the resulting gauge group by eight. This $\mathbb{Z}_2$ involution of K_3, which we call σ, will play an important role in our subsequent discussion. An explicit example of such an involution is given in the appendix. Some of these models in four dimensions were constructed by Ferrara and Kounnas[19] in the fermionic description.

In six dimensions this construction breaks down. In fact, at first sight it would seem to be impossible to construct a type IIA string compactification in six dimensions that is dual to the CHL compactification for the following reason. There are three ways to get $N = 2$ supersymmetry in six dimensions from compactification of a type II string. The first two possibilities are that all the supersymmetries come from the left (or right) sector of the world-sheet theory; such theories are usually referred to as of type $(4_L, 0_R)$ or $(0_L, 4_R)$, with the number 4 representing the number of different spin fields in the left or the right sector of the world sheet theory. The remaining possibility is that half the supersymmetries come from the left sector of the world sheet and the other half come from the right sector. These theories are called $(2_L, 2_R)$ type. If the theory is of $(2_L, 2_R)$ type, then using the same conformal field theory as a background for the type IIB theory gives a chiral theory in six dimensions. The massless field content of a chiral $N = 2$ theory in six dimensions is completely determined by the requirement of anomaly cancellation[20]. This implies that we cannot a get a theory with a reduced rank gauge group by using such conformal field theories for compactification of the type IIA theory. This argument does not rule out the possibility of obtaining reduced rank theories from $(4_L, 0_R)$ or $(0_L, 4_R)$ compactification, since for such backgrounds both the type IIA and type IIB theories give non-chiral theories. However, if the theory is of $(4_L, 0_R)$ type (say), then according to an argument of Banks and Dixon[21] the left sector of the world-sheet theory representing compact dimensions is described by four free superfields. This, in particular, implies that the only possible way to get vertex operators representing massless states from the Ramond-Ramond (RR) sector is to take the product of the dimension $(1/4, 0)$ spin field from the left sector of the internal conformal field theory and multiply it by a dimension $(0, 1/4)$ spin field from the right sector of the internal conformal field theory. (Recall that in six dimensions, a massless state corresponds to a dimension (1/4,1/4) operator in the RR sector of the internal conformal field theory.) But since there is no supersymmetry coming from the right sector, there is no dimension $(0, 1/4)$ spin field in this theory. This shows that there are no massless states arising from the RR sector of this theory. Since under the duality transformation that maps heterotic string theory to type IIA string theory, the gauge fields in the heterotic string theory get mapped to the RR gauge fields in the type IIA theory, we see that $(4_L, 0_R)$ or $(0_L, 4_R)$ compactifications of the type IIA theory cannot give the required dual of the CHL compactification in six dimensions.

Even though conventional compactification of the type IIA string cannot give rise to the dual of the CHL compactification in six dimensions, we can obtain the dual theory by means of a type IIA compactification that involves non-trivial RR background fields. To explain how this works, let us return to the construction of the CHL compactification in six dimensions. It consists of compactifying the heterotic string theory on a four-dimensional torus and modding out the resulting theory by a $\mathbb{Z}_2$ action that exchanges the two E_8 lattices and simultaneously shifts one of the circles of T^4 by half a period. Let us examine the action of this $\mathbb{Z}_2$ for the type IIA string theory compactified on $K3$ following a procedure similar to the one adopted in ref.[10]. As already argued, the effect of exchanging the two E_8's can be represented by a $\mathbb{Z}_2$ involution on $K3$, which we have denoted by σ. The result of a half period shift along one of the circles can be represented in the type IIA theory by a $\mathbb{Z}_2$ subgroup of the $U(1)$ gauge group associated with the ten-dimensional gauge field A_μ originating in the RR sector[4, 10].[1] Thus we conclude that if we consider the $\mathbb{Z}_2$ orbifold of the type IIA theory compactified on $K3$ with $\mathbb{Z}_2$ acting on $K3$ as the involution σ, and also having a non-trivial embedding in the U(1) gauge group involving the RR gauge field, the resulting theory is a good candidate for the dual of the CHL compactification of the heterotic string in six dimensions. From now on we shall refer to this new type IIA string compactification as the twisted IIA theory.

The embedding of $\mathbb{Z}_2$ in the U(1) gauge group implies that all fields carrying even U(1) charge are even under the action of σ, and all fields carrying odd U(1) charges are odd under the action of σ. By a suitable gauge transformation that is singular on $K3/\sigma$ we can make all the fields even under σ; but this introduces background $U(1)$ gauge fields on $K3/\sigma$ whose effect is to give a vev of -1 to any Wilson loop on $K3/\sigma$ that cannot be contracted to a point without going through one of the orbifold points. This implies that flux of the U(1) gauge field is concentrated at the orbifold points. Embedding of the orbifold group in the gauge group has been used many times before, notably in usual orbifold compactification[23], where one must embed the spin connection in the gauge connection. In the usual discussion of orbifolds the gauge group originates in the NS sector, but in our case it originates in the RR sector.

As is well known, if we had not embedded $\mathbb{Z}_2$ in the U(1) gauge group, and modded out the type IIA theory just by the action of σ, the result would have been type IIA theory compactified on another $K3$. The point is that the modding out by σ removes eight massless vector multiplets from the untwisted sector corresponding to the eight (1,1) forms that are odd under σ, but the twisted sector gives back eight massless vector multiplets.[2] The effect of introducing the twist involving the RR sector gauge group is precisely to remove these eight would-be massless multiplets from the twisted sector, thereby reducing the rank of the gauge group by eight. The perturbative spectrum in the untwisted sector is not affected by the embedding of $\mathbb{Z}_2$ into the gauge group, since all of these states are neutral under U(1). Note that

[1] One way to see this is to note that the U(1) gauge field in the heterotic theory, associated with translation along the internal circle, gets mapped to the RR gauge field A_μ of the type IIA theory for a suitable choice of basis.

[2] This is also reflected in the fact that if we blow up the orbifold points, we recover another $K3$ surface which has twenty (1,1) forms.

a Wilson line background for the RR gauge fields was also introduced in ref.[10], but there the complete perturbative spectrum in the type IIA theory was insensitive to it. In the present case, since the involution σ acts on $K3$ with fixed points, the *perturbative spectrum of the type IIA theory in the twisted sector is sensitive to the RR background gauge field.*

Unfortunately, we cannot directly compute the spectrum of massless states of the type IIA theory in the presence of such background RR fields and demonstrate that the twisted sector states disappear as claimed. We cannot even give an independent proof that such a compactification of the type IIA string theory is consistent. The problem, of course, is the lack of a suitable formalism for describing RR backgrounds in terms of conformally invariant world-sheet theories. In this sense the consistency of the CHL compactification is the best proof of the consistency of such a type IIA string compactification. However, as we shall now show, there is an appropriate region in the moduli space of both these compactifications where the effective low-energy theory is described by an eleven-dimensional supergravity theory compactified on a five manifold of large dimensions. The counting of massless states in this effective theory provides an 'independent' way of counting the number of massless states in both the theories. This is done by exploiting another pair of duality conjectures $-$ the duality between type IIA supergravity in ten dimensions and eleven-dimensional supergravity on a circle[22, 4] and the duality between heterotic string theory on T^3 and eleven-dimensional supergravity compactified on $K3$[4]. Both of these dualities must be regarded as dualities between effective theories, *i.e.* in the appropriate limit the effective theory describing the string theory is given by the corresponding supergravity theory. This implies that, in the appropriate limits, both the heterotic string theory compactified on $T^3 \times S^1 = T^4$ and type IIA theory compactified on $K3$ can be described by eleven-dimensional supergravity on $K3 \times S^1$, with both $K3$ and S^1 having large size. In the supergravity theory, one of the vector fields in the resulting six-dimensional theory comes from the component $G_{10,\mu}$ of the metric, where 10 denotes the coordinate on S^1, and one comes from dualizing the antisymmetric tensor field $C_{\mu\nu\rho}$ in six dimensions; the other 22 vector fields, which arise from the components $C_{mn\mu}$ of the antisymmetric tensor field with m, n denoting coordinates on $K3$, are in one-to-one correspondence with the harmonic two-forms on $K3$.

Let us now consider a $\mathbb{Z}_2$ modding of the supergravity theory that corresponds to the involution σ on $K3$ and a shift on S^1 by a half period. This $\mathbb{Z}_2$ action corresponds to the same $\mathbb{Z}_2$ action that was used for constructing the CHL model from the toroidally compactified heterotic string theory, and the $\mathbb{Z}_2$ that we used earlier in our construction of the twisted IIA string compactification. Thus we claim that in some region of moduli space both the CHL compactification of the heterotic string and the twisted IIA string compactification that we have proposed are described at low energy by an effective theory consisting of eleven-dimensional supergravity compactified on $(K3 \times S^1)/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ acts as the involution

σ on $K3$ and a half-period shift on S^1. Since this $\mathbb{Z}_2$ action on $K3 \times S^1$ has no fixed points, the supergravity theory is well-defined on the quotient manifold as an effective theory, and we can reliably count the total number of massless states in the theory by simply counting the massless states in this effective supergravity theory. Since there are no twisted states in the supergravity theory, the result of the $\mathbb{Z}_2$ action is simply to eliminate the vector multiplets corresponding to (1,1) forms that are odd under σ. As already mentioned, there are eight such (1,1) forms, and hence the resulting theory has the rank of its gauge group reduced by eight.

We can learn more about the twisted IIA theory by considering compactification to five dimensions. Before the $\mathbb{Z}_2$ modding, we can consider four equivalent descriptions of the theory: 1) the eleven-dimensional theory compactified on $K3 \times S^1_a \times S^1_b$, where we have added the labels a and b to distinguish the two circles; 2) type IIA theory on $K3 \times S^1_a$ (we shall denote this theory by IIAb); 3) type IIA theory on $K3 \times S^1_b$ (this theory will be denoted by IIAa); and 4) heterotic string theory on $T^3 \times S^1_a \times S^1_b$. There are also equivalent type IIB string compactifications, which we shall not discuss. Let us denote by the superscripts (SG), $(IIAa)$, $(IIAb)$ and (het) the fields in the supergravity, IIAa, IIAb, and heterotic string theory, respectively. We also denote by V_M the volume of a manifold M, by R_i the radius of the circle S^1_i, and by Φ_n the n-dimensional dilaton. Then the relevant moduli fields setting the scales in these four theories are as follows:

- Supergravity theory: $V^{(SG)}_{K3}$, $R^{(SG)}_a$, $R^{(SG)}_b$.
- IIAa theory: $\Phi^{(IIAa)}_{10}$, $V^{(IIAa)}_{K3}$, $R^{(IIAa)}_b$.
- IIAb theory: $\Phi^{(IIAb)}_{10}$, $V^{(IIAb)}_{K3}$, $R^{(IIAb)}_a$.
- Heterotic theory: $\Phi^{(het)}_7$, $R^{(het)}_a$, $R^{(het)}_b$.

Following ref.[4] we can find the relationship between the moduli of different theories. They are given by

$$\begin{aligned} V^{(SG)}_{K3} &= \exp(\frac{4}{3}\Phi^{(het)}_7) \\ &= V^{(IIAa)}_{K3} \exp(-\frac{4}{3}\Phi^{(IIAa)}_{10}) \\ &= V^{(IIAb)}_{K3} \exp(-\frac{4}{3}\Phi^{(IIAb)}_{10})\,, \end{aligned} \tag{1}$$

$$\begin{aligned} R^{(SG)}_a &= \exp(-\frac{2}{3}\Phi^{(het)}_7) R^{(het)}_a \\ &= \exp(\frac{2}{3}\Phi^{(IIAa)}_{10}) \\ &= R^{(IIAb)}_a \exp(-\frac{1}{3}\Phi^{(IIAb)}_{10})\,, \end{aligned} \tag{2}$$

$$
\begin{aligned}
R_b^{(SG)} &= \exp(-\frac{2}{3}\Phi_7^{(het)})R_b^{(het)} \\
&= R_b^{(IIAa)}\exp(-\frac{1}{3}\Phi_{10}^{(IIAa)}) \\
&= \exp(\frac{2}{3}\Phi_{10}^{(IIAb)})\,. \qquad (3)
\end{aligned}
$$

If $G_{\mu\nu}$ denotes the five-dimensional metric ($0 \le \mu, \nu \le 4$) then the metrics of the different theories are related as follows:

$$
G_{\mu\nu}^{(SG)} = e^{-\frac{4}{3}\Phi_7^{(het)}} G_{\mu\nu}^{(het)} = e^{-\frac{2}{3}\Phi_{10}^{(IIAa)}} G_{\mu\nu}^{(IIAa)} = e^{-\frac{2}{3}\Phi_{10}^{(IIAb)}} G_{\mu\nu}^{(IIAb)}\,. \qquad (4)
$$

Let C_{MNP} denote the three form field of the eleven-dimensional supergravity theory, A_M denote the U(1) gauge field arising in the RR sector of the type IIA string theory, and B_{MN} be the usual antisymmetric tensor field of the IIA theory. Then the relationship between different gauge fields in five dimensions is given by,

$$
\begin{aligned}
G_{a\mu}^{(SG)} &\sim G_{a\mu}^{(het)} \sim G_{a\mu}^{(IIAb)} \sim A_\mu^{(IIAa)} \\
G_{b\mu}^{(SG)} &\sim G_{b\mu}^{(het)} \sim A_\mu^{(IIAb)} \sim G_{b\mu}^{(IIAa)} \\
C_{ab\mu}^{(SG)} &\sim \widetilde{B}_\mu^{(het)} \sim B_{a\mu}^{(IIAb)} \sim B_{b\mu}^{(IIAa)}\,, \qquad (5)
\end{aligned}
$$

where $\widetilde{B}_\mu$ denotes the dual of the antisymmetric tensor field in five dimensions. The symbol $\sim$ in the above relations signifies that we have ignored normalizations. The subscripts a and b refer to the circles S_a^1 and S_b^1.

A fundamental type IIAa string wrapping around the circle S_b^1 corresponds to a state with a non-zero $B_{b\mu}^{(IIAa)}$ charge. Let us normalize $B_{b\mu}^{(IIAa)}$ so that this state carries unit charge. Similarly, we shall normalize $B_{a\mu}^{(IIAb)}$ in such a way that a fundamental type IIAb string wrapped around the circle S_a^1 carries unit $B_{a\mu}^{(IIAb)}$ charge. From eq.(5) we see that the field $B_{a\mu}^{(IIAb)}$ in the IIAb theory gets mapped to the field $B_{b\mu}^{(IIAa)}$ in the type IIAa theory. Due to the symmetry of the exchange $a \leftrightarrow b$, the relative normalization between these two fields must be ± 1. Thus we learn that the fundamental type IIAb string wrapped around the circle S_a represents the same state as the fundamental type IIAa string wrapped around the circle S_b. This can be further checked by comparing the masses of these states; the fundamental IIAa (IIAb) string, wrapped around S_b^1 (S_a^1) has mass $R_b^{(IIAa)}$ ($R_a^{(IIAb)}$) measured in the metric $G_{\mu\nu}^{(IIAa)}$ ($G_{\mu\nu}^{(IIAb)}$). Using eqs.(1)–(4) one can easily check that these correspond to identical masses.[3]

So far we have only collected the relevant results of ref.[4]. Let us now consider modding out each of the four theories by $\mathbb{Z}_2$, where $\mathbb{Z}_2$ acts as the involution σ on $K3$ and a shift by π on S_a^1. On the heterotic side this corresponds to CHL

[3]Another way to understand this phenomenon is to note that both these states can be regarded as the 11-dimensional supermembrane wrapped around $S_a^1 \times S_b^1$.

compactification. For the IIAb theory, this corresponds to the compactification of an usual type IIA theory on $(K3\times S^1_a)/\mathbb{Z}_2$. For the IIAa theory, on the other hand, it corresponds to the twisted IIA theory compactified on the circle S^1_b. Thus the twisted IIAa theory compactified on the circle S^1_b must be (non-perturbatively) equivalent to the IIAb theory compactified on the manifold $(K3\times S^1_a)/\mathbb{Z}_2$. In particular, the spectrum of these two theories must agree. Now, the IIAb theory compactified on $(K3\times S^1_a)/\mathbb{Z}_2$ has states with half-integer $B^{(IIAb)}_{a\mu}$ charge[24], since there can be winding states of the string that wind half way around S^1_a, and at the same time start and end on points of $K3$ related by the involution σ. Thus the twisted IIAa theory compactified on the circle S^1_b must also contain states with half integer $B^{(IIAa)}_{b\mu}$ charge. As has been stated before, in this theory states with integer $B^{(IIAa)}_{b\mu}$ charge correspond to fundamental strings wound around the circle S^1_b. But now we have a puzzle: since the string cannot have half a winding around S^1_b, there is no way to get states with half a unit of $B^{(IIAa)}_{b\mu}$ charge by winding fundamental strings around S^1_b. The only resolution to this puzzle is that the original twisted IIAa theory in six dimensions has a solitonic string that *carries half the density of $B_{\mu\nu}$ charge of a fundamental string.*[4] Winding of this string along S^1_b would then produce a state with half a unit of $B^{(IIAa)}_{b\mu}$ charge.

To summarize, we have shown that the dual of the six-dimensional CHL compactification of the heterotic string theory is given by the compactification of the type IIA theory on a $K3/\mathbb{Z}_2$ orbifold, where the $\mathbb{Z}_2$ action involves an involution of $K3$, and also has an embedding in the U(1) gauge group arising from the RR sector of the ten-dimensional string theory. Geometrically this represents flux associated with this U(1) gauge field concentrated at the orbifold points. This construction shows that it is possible to construct non-trivial background for the propagation of type IIA string with non-vanishing background fields associated with the RR sector. This new six-dimensional string theory contains solitonic strings which carry half the density of $B_{\mu\nu}$ charge as the fundamental string.

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[4]From the eleven-dimensional viewpoint such a string would correspond to a solitonic membrane labelled by surface coordinates (σ_1,σ_2), with σ_1 pointing towards a particular direction in the 5-dimensional non-compact space, and σ_2 lying along a closed curve C in $(K3\times S^1_a)/\mathbb{Z}_2$, with two ends of of C being related by the $\mathbb{Z}_2$ transformation in $K3\times S^1_a$. Classically the minimum energy configuration for such a membrane would correspond to the choice of C which lies along S^1_a spanning a length π, and sits at one of the eight fixed points of σ on $K3$. Viewed as a string in six dimensions, this will have half the tension of a fundamental string which is obtained by wrapping the membrane fully around S^1_a.

A Example of Z_2 Involution on $K3$

In this appendix we shall construct an example of the involution σ that eliminates eight of the $(1,1)$ forms. For this purpose let us consider the $K3$ surface described by the quartic polynomial equation in $\mathbb{C}\mathrm{P}^3$:

$$\sum_{i=1}^{4}(z^i)^4 = 0, \tag{A.1}$$

where z^i are the homogeneous coordinates in $\mathbb{C}\mathrm{P}^3$. Let us denote by σ the involution:

$$z^1 \to -z^1, \quad z^2 \to -z^2, \quad z^3 \to z^3, \quad z^4 \to z^4\,. \tag{A.2}$$

It can be easily verified that it preserves the $(2,0)$ and $(0,2)$ forms on $K3$. There are eight fixed points on $K3$ under the action of σ, described by the points

$$\begin{aligned} z^1 = z^2 = 0, \qquad & \frac{z^3}{z^4} = e^{(2k+1)i\pi/4}, \quad k = 0,1,2,3, \\ z^3 = z^4 = 0, \qquad & \frac{z^1}{z^2} = e^{(2k+1)i\pi/4}, \quad k = 0,1,2,3. \end{aligned} \tag{A.3}$$

If n_+ and n_- denote the number of $(1,1)$ forms on $K3$ that are even and odd under σ, respectively, then by Lefschetz fixed-point theorem[25], we have

$$8 = 4 + n_+ - n_-\,, \tag{A.4}$$

where the number 4 on the right-hand side represents the contribution from the (0,0), (2,2), (2,0) and (0,2) forms, all of which are even under σ. Since,

$$n_+ + n_- = 20\,, \tag{A.5}$$

we get

$$n_+ = 12, \qquad n_- = 8\,. \tag{A.6}$$

This shows that eight out of the twenty harmonic (1,1) forms are odd under σ, as required.

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