

STRING STRING DUALITY CONJECTURE IN SIX DIMENSIONS AND CHARGED SOLITONIC STRINGS

Ashoke Sen

International Centre for Theoretical Physics
P.O. Box 586, I-34100 Trieste, ITALY

and

Tata Institute of Fundamental Research
Homi Bhabha Road, Bombay 400005, INDIA
sen@theory.tifr.res.in, sen@tifrvax.bitnet

February 1, 2008

TIFR-TH-95-16
hep-th/9504027
April, 1995

Abstract

It has recently been conjectured that the type IIA string theory compactified on K3 and the heterotic string theory compactified on a four dimensional torus describe identical string theories. The fundamental heterotic string can be regarded as a non-singular soliton solution of the type IIA string theory with a semi-infinite throat. We show that this solution admits 24 parameter non-singular deformation describing a fundamental heterotic string carrying electric charge and current. The charge is generated due to the coupling of the gauge fields to the anti-symmetric tensor field, and not to an explicit source term. This clarifies how soliton solutions carrying charge under the Ramond-Ramond fields can be constructed in the type IIA theory, and provides further support to the string string duality conjecture. Similarly, the fundamental type IIA string can be regarded as a non-singular solution of the heterotic string theory with a semi-infinite throat, but this solution does not admit any deformation representing charged string. This is also consistent with the expectation that a fundamental type IIA string does not carry any charge that couples to the fields originating in the Ramond-Ramond sector.

1 Introduction and Summary

It has recently been conjectured[1, 2, 3] that the type IIA string theory compactified on the K3 surface and heterotic string theory compactified on a four dimensional torus describe identical six dimensional string theories. Some of the compelling evidences for this conjecture come from the fact that the two theories give rise to identical low energy effective field theories, and have identical moduli spaces[4, 5]. If this conjecture is correct, then the S-duality of the heterotic string theory compactified on a six dimensional torus[6]-[14] follows as a consequence of the T-duality of the type IIA string theory[2, 3]. This duality has been called string-string duality conjecture[2, 3] and we shall refer to it by this name in this paper. Other aspects of duality symmetries in string theory relating strong and weak coupling limits have been discussed in refs.[15, 16, 17].

In order to test this conjecture, we must show that the spectrum of Bogomol'nyi saturated states[18] in the two theories are identical. Here we encounter a puzzle. The spectrum of the heterotic string theory contains charged Bogomol'nyi saturated states. On the other hand, since in the type IIA string theory all the gauge fields arise from the Ramond-Ramond sector, and the elementary string states in the theory do not carry any charge under the gauge fields in the Ramond-Ramond sector, all the elementary string states in the type IIA string theory are charge neutral. This seems to contradict the conjecture that the two theories are equivalent. However, an analysis of the Bogomol'nyi bound formula shows that in the type IIA string theory, the states that are charged under the gauge field have masses inversely proportional to the coupling constant of the type IIA theory[3]. This shows that we do not expect these states to arise in the elementary string spectrum, instead they must arise as solitons in this theory. Extremal charged black holes have been proposed as possible candidates[1].

The situation however still remains a bit unsatisfactory, since even for extremal black holes, in order for it to carry electric charge, we need to put a source of electric field (electric charge) at the center of the black hole. In other words, if we were not motivated by the string-string duality conjecture, and were just analyzing the spectrum of the type IIA string theory compactified on K3, there would be no reason to include the extremal charged black holes as solutions in the theory, since they require using charged particles as sources which are not present in the theory to start with. It would be much

more compelling if we could construct solitons in the type IIA theory which carry electric charge even if we do not use an explicit source for the electric field. These solutions will be analogous to the 't Hooft - Polyakov monopole solutions in Yang-Mills theories, which give rise to magnetically charged solitons without the necessity of putting an external source of magnetic field.

One might think that it is impossible to construct a charged soliton in type IIA string theory without putting a source of electric charge somewhere. Since all the fields appearing in the low energy effective action are neutral, the standard Gauss' law will tell us that the total flux of electric field through a closed surface vanishes unless there is a source of electric charge somewhere inside the surface. The reason that we can get around this argument is that the standard Gauss' law does not apply to the field equations of the supergravity theory describing the string theory under consideration. Due to the non-standard coupling of the gauge fields to the rank two anti-symmetric field in the theory, the divergence of the electric field is not zero, but is proportional to the three form field strength contracted with the electromagnetic field. This allows us to construct charged solitons in this theory without the necessity of introducing a source of electric field.

The charged solitons that we shall construct are not particle like states, but string like states. To this end, we note that the low energy supergravity theory admits two neutral string like solutions[2]. One of them describes the fundamental heterotic string solution of ref.[19], whereas the other describes the fundamental type IIA string solution. When expressed in the natural variables of the type IIA string theory, the fundamental heterotic string solution is non-singular[20, 21], and describes an extremal black string with a semi-infinite throat, with the type IIA string coupling becoming strong as we go down the throat. On the other hand, when expressed in the natural variables of the heterotic string theory, the fundamental type IIA string solution is non-singular, and describes an extremal black string with a semi-infinite throat, with the heterotic string coupling becoming strong as we go down the throat. We shall show that the first solution has a 24 parameter deformation describing a fundamental heterotic string carrying electric charge and electric current. 20 of these parameters describe deformations for which electric charge per unit length of the string is equal to the electric current, whereas the rest 4 parameters describe deformations for which the electric charge per unit length of the string is equal to the negative of the electric current. This is consistent with the known properties of the heterotic string,

i.e. that it carries 20 left moving and 4 right moving currents on the world sheet. But more importantly, we shall see that this electric charge is not generated by any source term. In fact, as we go down the semi-infinite throat, the total electric flux approaches zero, showing that there is no source of the electric charge at the far end of the throat. The charge is generated solely due to the coupling to the anti-symmetric tensor field as mentioned before. On the other hand, the second solution, representing the fundamental type IIA string, does not admit any deformation that converts it to an electrically charged string. This is again consistent with our general understanding that the fundamental type IIA string does not carry any world sheet current which couples to the gauge fields originating from the Ramond-Ramond sector of the theory.

The situation may be summarized as follows:

- The type IIA string theory contains a non-singular soliton solution carrying the quantum numbers of a fundamental heterotic string. This solution admits non-singular 24 parameter deformations which has the quantum numbers of a fundamental heterotic string carrying 20 left moving currents and 4 right moving currents on the world sheet. This is consistent with the known properties of the fundamental heterotic string.
- The heterotic string theory contains a non-singular soliton solution carrying the quantum numbers of a fundamental type IIA string. This solution does not admit any deformation which might represent fundamental type IIA string carrying world sheet currents that couple to the gauge fields. This is consistent with the known properties of the fundamental type IIA string.

These results provide further support to the string-string duality conjecture in six dimensions.

We now give the plan of the paper. In section 2 we shall write down the action of the six dimensional supergravity theory in two different sets of variables, – one natural to the heterotic string and the other natural to the type IIA string. We shall also give the map between these two sets of variables. Finally we shall write down the solutions describing the fundamental heterotic string and the fundamental type IIA string in both sets of variables

and discuss the geometrical properties of these solutions. In section 3 we shall use the by now standard method of $O(d, d)$ transformation[22, 23, 24] to generate the deformed heterotic string solution describing the charged string. We show that the deformed solutions are non-singular, and do not have any source of electric charge at the far end of the throat. We also show that the $O(d, d)$ transformations *cannot* be used to generate a charged type IIA string solution. We end in section 4 with a few remarks about the possibility of constructing solvable conformal field theories that might describe the fundamental heterotic string as a solution of the type II string theory, and vice versa.

We end this section by stating some of the notations that we shall be using. The unprimed fields will denote the fields that couple naturally to the heterotic string, whereas the primed fields will denote the fields which couple naturally to the type IIA string. The subscript (*het*) will be used to denote the solution that represents a fundamental heterotic string, whereas the subscript (*IIA*) will denote the solution representing a fundamental type IIA string. Finally, the subscript (*chet*) will denote the solution representing a heterotic string carrying electric charge and current.

2 Low Energy Effective Action, and Solitonic Strings

The material in this section will be a review of the results contained in refs.[4, 1, 2, 3]. The low energy effective action describing the heterotic string compactified on a four dimensional torus (T^4) is given by,

$$S \propto \int d^6x \sqrt{-G} e^{-\Phi} [R_G + G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{12} G^{\mu\mu'} G^{\nu\nu'} G^{\rho\rho'} H_{\mu\nu\rho} H_{\mu'\nu'\rho'} - G^{\mu\mu'} G^{\nu\nu'} F_{\mu\nu}^{(a)} (LML)_{ab} F_{\mu'\nu'}^{(b)} + \frac{1}{8} G^{\mu\nu} \text{Tr}(\partial_\mu ML \partial_\nu ML)], \quad (2.1)$$

where $G_{\mu\nu}$, $B_{\mu\nu}$, $A_\mu^{(a)}$ ($0 \leq \mu \leq 5$, $1 \leq a \leq 24$), Φ and M denote respectively the string metric, the antisymmetric tensor field, the 24 abelian gauge fields, the dilaton field, and the 24×24 matrix valued scalar field representing an element of $O(4, 20)/(O(4) \times O(20))$, satisfying

$$M^T = M, \quad MLM^T = L, \quad (2.2)$$

where,

$$L = \begin{pmatrix} -I_{20} & \\ & I_4 \end{pmatrix}. \quad (2.3)$$

R_G denotes the scalar curvature associated with the metric $G_{\mu\nu}$, and,

$$\begin{aligned} F_{\mu\nu}^{(a)} &= \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} \\ H_{\mu\nu\rho} &= (\partial_\mu B_{\nu\rho} + 2A_\mu^{(a)} L_{ab} F_{\nu\rho}^{(b)}) + \text{cyclic permutations of } \mu, \nu, \rho. \end{aligned} \quad (2.4)$$

The matrix valued scalar field M originate from the internal components of the ten dimensional metric, antisymmetric tensor field and the gauge fields. Four of the 24 gauge fields originate from the components of the ten dimensional metric, four of them originate from the components of the ten dimensional anti-symmetric tensor field, and the remaining sixteen gauge fields can be identified to the gauge fields associated with the Cartan subalgebra of $E_8 \times E_8$ or $SO(32)$.

On the other hand, the low energy effective action describing the type IIA string compactified on $K3$ is given by,

$$\begin{aligned} S' \propto \int d^6x & \left(\sqrt{-G'} [e^{-\Phi'} \{ R'_G + G'^{\mu\nu} \partial_\mu \Phi' \partial_\nu \Phi' - \frac{1}{12} G'^{\mu\mu'} G'^{\nu\nu'} G'^{\rho\rho'} H'_{\mu\nu\rho} H'_{\mu'\nu'\rho'} \right. \\ & + \frac{1}{8} G'^{\mu\nu} \text{Tr}(\partial_\mu M' L \partial_\nu M' L) \} - G'^{\mu\mu'} G'^{\nu\nu'} F'^{(a)}_{\mu\nu} (L M' L)_{ab} F'^{(b)}_{\mu'\nu'}] \\ & \left. - \frac{1}{4} \varepsilon^{\mu\nu\rho\sigma\tau\epsilon} B'_{\mu\nu} F'^{(a)}_{\rho\sigma} L_{ab} F'^{(b)}_{\tau\epsilon} \right), \end{aligned} \quad (2.5)$$

where $G'_{\mu\nu}$, $B'_{\mu\nu}$, $A'^{(a)}_\mu$, Φ' and M' denote respectively the string metric, the antisymmetric tensor field, the 24 abelian gauge fields, the dilaton field, and the 24×24 matrix valued scalar field representing an element of $O(4, 20)/(O(4) \times O(20))$. M' satisfies equations identical to those satisfied by M :

$$M' L M'^T = L, \quad M'^T = M'. \quad (2.6)$$

Here

$$\begin{aligned} F'^{(a)}_{\mu\nu} &= \partial_\mu A'_\nu^{(a)} - \partial_\nu A'_\mu^{(a)} \\ H'_{\mu\nu\rho} &= \partial_\mu B'_{\nu\rho} + \text{cyclic permutations of } \mu, \nu, \rho. \end{aligned} \quad (2.7)$$

Note that there is no Chern-Simons term involving the gauge fields in the expression for H' . In this case the scalar field degrees of freedom contained

in M' come from the internal components (along the tangent space of $K3$) of the metric and the anti-symmetric tensor field. On the other hand, the gauge fields come from various components of the anti-symmetric tensor field C_{MNP} and the vector field E_M ($0 \leq M \leq 9$) in the ten dimensional theory which originate in the Ramond-Ramond sector of the type IIA superstring theory. In particular E_μ gives one gauge field, the components $C_{mn\mu}$, where m, n denote the tangent space directions on $K3$, give 22 gauge fields, and dualization of the three form $C_{\mu\nu\rho}$ in six dimensions give another gauge field.

It can be easily verified that the equations of motion derived from the actions (2.1) and (2.5) are identical, provided one makes the following identification of fields:

$$\begin{aligned} \Phi' &= -\Phi, & G'_{\mu\nu} &= e^{-\Phi} G_{\mu\nu}, & M' &= M, & A'^{(a)}_\mu &= A^{(a)}_\mu, \\ \sqrt{-G} e^{-\Phi} H^{\mu\nu\rho} &= \frac{1}{6} \varepsilon^{\mu\nu\rho\sigma\tau\epsilon} H'_{\sigma\tau\epsilon}. \end{aligned} \quad (2.8)$$

Let us now describe solutions of the equations of motion derived from the above effective actions, describing fundamental heterotic string, and fundamental type IIA string respectively. To this end, we note that a fundamental heterotic string will carry electric type $B_{\mu\nu}$ charge, and hence magnetic type $B'_{\mu\nu}$ charge, since $B_{\mu\nu}$ and $B'_{\mu\nu}$ are related by duality transformations. On the other hand, the fundamental type IIA string will carry electric type $B'_{\mu\nu}$ charge and magnetic type $B_{\mu\nu}$ charge. Thus they must be represented by different solutions of the field equations. Both of these solutions are given in refs.[2, 20], and are related to the solitonic string solutions of ref.[19] in appropriate variables. The fundamental heterotic string solution is given in the unprimed field variables by

$$\begin{aligned} ds^2_{(het)} &= \left(1 + \frac{C}{r^2}\right)^{-1} (-dt^2 + (dx^5)^2) + dr^2 + r^2 d\Omega_3^2, \\ e^{-\Phi_{(het)}} &= 1 + \frac{C}{r^2}, \\ B_{(het)5t} &= \frac{C}{C + r^2}, \\ A^{(a)}_{(het)\mu} &= 0, \quad M_{(het)} = I_{24}, \end{aligned} \quad (2.9)$$

where $d\Omega_3$ denotes the line element on a three sphere S^3 , and C is a constant determined by the heterotic string tension. This solution is identical to the

one constructed by Dabholkar et. al.[19]. The metric has a singularity at $r = 0$ in these variables. However, the same solution expressed in terms of the primed variables is given by,

$$\begin{aligned}
ds_{(het)}'^2 &= -dt^2 + (dx^5)^2 + \left(1 + \frac{C}{r^2}\right)dr^2 + (C + r^2)d\Omega_3^2, \\
e^{-\Phi'_{(het)}} &= \left(1 + \frac{C}{r^2}\right)^{-1}, \\
H'_{(het)ijk} &= -2C\varepsilon_{ijk}, \\
A'_{(het)\mu}{}^{(a)} &= 0, \quad M'_{(het)} = I_{24},
\end{aligned} \tag{2.10}$$

where ε_{ijk} denotes the volume form on the three sphere S^3 . The apparent singularity of the metric at $r = 0$ is only a coordinate singularity[20], as can be seen by defining new coordinate ρ near $r = 0$:

$$\rho = \ln r. \tag{2.11}$$

In this coordinate, the point $r = 0$ is mapped to the point $\rho = -\infty$, and the metric near $r = 0$ takes the form:

$$ds_{(het)}'^2 \simeq -dt^2 + (dx^5)^2 + Cd\rho^2 + Cd\Omega_3^2 \tag{2.12}$$

which represents the geometry of a semi-infinite line labelled by ρ tensored with a three sphere of constant radius, and a two dimensional Minkowski space labelled by t and x^5 . The dilaton near $r \simeq 0$ takes the form:

$$e^{-\Phi'_{(het)}} \simeq Ce^{2\rho} \tag{2.13}$$

showing that the string coupling grows as we go down the semi-infinite throat labelled by ρ towards $\rho = -\infty$. This shows that the fundamental heterotic string can be represented as a non-singular soliton solution of the type IIA string theory compactified on $K3$.

Similarly, we can construct a solution of the equations of motion of the six dimensional effective field theory which represents the fundamental type IIA string. In the primed variables this solution is again identical to the one constructed in ref.[19], and is given by,

$$ds_{(IIA)}'^2 = \left(1 + \frac{C'}{r^2}\right)^{-1}(-dt^2 + (dx^5)^2) + dr^2 + r^2d\Omega_3^2,$$

$$\begin{aligned}
e^{-\Phi'_{(IIA)}} &= 1 + \frac{C'}{r^2}, \\
B'_{(IIA)5t} &= \frac{C'}{C' + r^2}, \\
A'^{(a)}_{(IIA)\mu} &= 0, \quad M'_{(IIA)} = I_{24},
\end{aligned} \tag{2.14}$$

where the constant C' is now determined by the string tension of the type IIA theory. The metric is again singular at the origin. But in terms of the unprimed variables, the solution looks like,

$$\begin{aligned}
ds^2_{(IIA)} &= -dt^2 + (dx^5)^2 + \left(1 + \frac{C'}{r^2}\right) dr^2 + (C' + r^2) d\Omega_3^2, \\
e^{-\Phi_{(IIA)}} &= \left(1 + \frac{C'}{r^2}\right)^{-1}, \\
H_{(IIA)ijk} &= -2C' \varepsilon_{ijk}, \\
A^{(a)}_{(IIA)\mu} &= 0, \quad M_{(IIA)} = I_{24},
\end{aligned} \tag{2.15}$$

which can again be seen to be non-singular. Thus we see that the type IIA string can be regarded as a non-singular soliton solution of the heterotic string theory.

3 Charged Solitonic Strings

In the last section we saw that the type IIA string theory compactified on $K3$ contains a non-singular soliton solution which has the quantum numbers of the heterotic string. We shall now study in more detail whether this solution has the right properties expected of a heterotic string. In particular, we know that the heterotic string can carry 20 left moving and 4 right moving world-sheet currents that couple to the 24 gauge fields. So in order to interpret this soliton solution as the fundamental heterotic string, we must show that the solution admits a 24 parameter deformation which represents a heterotic string carrying 20 left moving and 4 right moving currents on the world sheet. (Since we are considering a static string extending in the x^5 direction, in this case the world sheet coordinates can be identified to the space-time coordinates x^5 and t .)

But here we encounter a puzzle. It is well known that the type II string does not have any elementary excitations that carry electric charge associated with the Ramond-Ramond fields. Since all the gauge fields in the

compactified theory arise from the Ramond-Ramond sector, this means that the theory does not have any state (field) that carries gauge charges. Thus it would seem impossible to construct a soliton solution in the theory that carries gauge charges. On the other hand, this is precisely what we need to do if we have to construct solitonic strings representing heterotic strings.

A resolution to this puzzle comes from looking at the equations of motion for the gauge fields. In terms of the primed variables, these equations take the form:

$$D'_\nu[(M'L)_{ab}F'^{(b)\mu\nu}] + \frac{1}{12}(\sqrt{-G})^{-1}\varepsilon^{\tau\epsilon\mu\sigma\rho\nu}H'_{\sigma\rho\nu}F'_{\tau\epsilon}{}^{(a)} = 0 \quad (3.1)$$

This shows that although the theory does not contain any charged field, the second term in the above equation can act as a source term in the gauge field equations of motion, and give rise to solitons carrying net electric charge. As we shall see this is precisely the mechanism that allows us to construct a charged heterotic string solution.

The explicit construction of the solution is done using the solution generating transformations[22]-[25]. The relevant group of transformations belong to the coset $(O(20,1)/O(20)) \times (O(4,1)/O(4))$. Since the transformations used here are identical to the ones used in ref.[25], we shall not give the details here, but only quote the final result. The final solution is characterized by 24 extra parameters, consisting of two boost angles α and β , a 20 dimensional unit vector \vec{n} , and a 4 dimensional unit vector \vec{p} . In the unprimed variables (where the solution generating transformations act naturally) the transformed solution is given by,

$$ds_{(chet)}^2 = r^2\Delta^{-1}[-(r^2 + C)dt^2 + C(\cosh\alpha - \cosh\beta)dt dx^5 + (r^2 + C \cosh\alpha \cosh\beta)(dx^5)^2] + (dr^2 + r^2 d\Omega_3^2), \quad (3.2)$$

$$B_{(chet)5t} = \frac{C}{2\Delta}(\cosh\alpha + \cosh\beta)\{r^2 + \frac{1}{2}C(1 + \cosh\alpha \cosh\beta)\}, \quad (3.3)$$

$$e^{-\Phi_{(chet)}} = \frac{\Delta^{1/2}}{r^2}, \quad (3.4)$$

$$A_{(chet)t}^{(a)} = -\frac{n^{(a)}}{2\sqrt{2}\Delta}C \sinh\alpha\{r^2 \cosh\beta + \frac{1}{2}C(\cosh\alpha + \cosh\beta)\}$$

$$\begin{aligned}
& \text{for } 1 \leq a \leq 20, \\
& = -\frac{p^{(a-20)}}{2\sqrt{2}\Delta} C \sinh \beta \{r^2 \cosh \alpha + \frac{1}{2}C(\cosh \alpha + \cosh \beta)\} \\
& \text{for } 21 \leq a \leq 24,
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
A_{(chet)5}^{(a)} & = -\frac{n^{(a)}}{2\sqrt{2}\Delta} C \sinh \alpha \{r^2 + \frac{1}{2}C \cosh \beta (\cosh \alpha + \cosh \beta)\} \\
& \text{for } 1 \leq a \leq 20, \\
& = \frac{p^{(a-20)}}{2\sqrt{2}\Delta} C \sinh \beta \{r^2 + \frac{1}{2}C \cosh \alpha (\cosh \alpha + \cosh \beta)\} \\
& \text{for } 21 \leq a \leq 24,
\end{aligned} \tag{3.6}$$

$$M = I_{28} + \begin{pmatrix} Pnn^T & Qnp^T \\ Qpn^T & Ppp^T \end{pmatrix}, \tag{3.7}$$

where,

$$\Delta = r^4 + Cr^2(1 + \cosh \alpha \cosh \beta) + \frac{C^2}{4}(\cosh \alpha + \cosh \beta)^2, \tag{3.8}$$

$$P = \frac{C^2}{2\Delta} \sinh^2 \alpha \sinh^2 \beta, \tag{3.9}$$

$$Q = -C\Delta^{-1} \sinh \alpha \sinh \beta \{r^2 + \frac{1}{2}C(1 + \cosh \alpha \cosh \beta)\}, \tag{3.10}$$

For $\beta = 0$, these solutions agree with the charged string solutions found in ref.[24].

By examining the solution we see that the string tension of the heterotic string is now proportional to $C(\cosh \alpha + \cosh \beta)$. Also the solution is as usual singular at $r = 0$. We shall now verify that the solution is non-singular in the primed variables. In these variables, the solution takes the form:

$$\begin{aligned}
ds_{(chet)}'^2 & = \Delta^{-1/2}[-(r^2 + C)dt^2 + C(\cosh \alpha - \cosh \beta)dt dx^5 \\
& + (r^2 + C \cosh \alpha \cosh \beta)(dx^5)^2] \\
& + \Delta^{1/2} \left(\frac{dr^2}{r^2} + d\Omega_3^2 \right),
\end{aligned} \tag{3.11}$$

$$e^{-\Phi'_{(chet)}} = \frac{r^2}{\Delta^{1/2}}, \quad (3.12)$$

$$H'_{(chet)ijk} = -C(\cosh \alpha + \cosh \beta)\varepsilon_{ijk}, \quad (3.13)$$

$$M'_{(chet)} = M_{(chet)}, \quad (3.14)$$

$$A'_{(chet)\mu}{}^{(a)} = A_{(chet)\mu}{}^{(a)}. \quad (3.15)$$

Near $r = 0$, we again use the coordinate $\rho = \ln r$. In this coordinate system the metric near $r = 0$ takes the form:

$$\begin{aligned} ds'^2_{(chet)} \simeq & \frac{2}{\cosh \alpha + \cosh \beta} \left(-dt^2 + \cosh \alpha \cosh \beta (dx^5)^2 \right. \\ & \left. + (\cosh \alpha - \cosh \beta) dt dx^5 \right) + \frac{C}{2} (\cosh \alpha + \cosh \beta) (d\rho^2 + d\Omega_3^2), \end{aligned} \quad (3.16)$$

and describes a completely non-singular geometry. We also see from eqs.(3.5), (3.6) that the gauge fields are non-singular as $r \rightarrow 0$.

We define the electric charge per unit length ($q^{(a)}$) and electric current ($j^{(a)}$) carried by the solution in terms of the asymptotic values of the gauge fields ($r \rightarrow \infty$):

$$F'_{rt}{}^{(a)} \simeq \frac{q^{(a)}}{r^3}, \quad F'_{r5}{}^{(a)} \simeq \frac{j^{(a)}}{r^3}. \quad (3.17)$$

Comparing eq.(3.17) with eqs.(3.5), (3.6) and (3.15) we get

$$\begin{aligned} q^{(a)} &= \frac{n^{(a)}}{\sqrt{2}} C \sinh \alpha \cosh \beta, \quad \text{for } 1 \leq a \leq 20, \\ &= \frac{p^{(a-20)}}{\sqrt{2}} C \sinh \beta \cosh \alpha, \quad \text{for } 21 \leq a \leq 24, \end{aligned} \quad (3.18)$$

and,

$$\begin{aligned} j^{(a)} &= \frac{n^{(a)}}{\sqrt{2}} C \sinh \alpha, \quad \text{for } 1 \leq a \leq 20, \\ &= -\frac{p^{(a-20)}}{\sqrt{2}} C \sinh \beta, \quad \text{for } 21 \leq a \leq 24. \end{aligned} \quad (3.19)$$

Thus, for small α, β ,

$$\begin{aligned} q^{(a)} = j^{(a)} &\simeq \frac{n^{(a)}}{\sqrt{2}} C \alpha & \text{for } 1 \leq a \leq 20, \\ q^{(a)} = -j^{(a)} &\simeq \frac{p^{(a-20)}}{\sqrt{2}} C \beta & \text{for } 21 \leq a \leq 24. \end{aligned} \quad (3.20)$$

This shows that the deformed solution does represent a fundamental heterotic string carrying 20 left-moving and 4 right moving currents on its world-sheet.

Let us briefly examine the source of the electric charge. From eq.(3.5), (3.6) we can easily verify that the total electric flux per unit length of the string through a surface of constant r (or ρ) vanishes as $r \rightarrow 0$ ($\rho \rightarrow -\infty$). This shows that there is no source of electric field hidden at the far end of the semi-infinite geometry ($\rho \rightarrow -\infty$). In fact, a more detailed examination of the solution shows that the total electric charge carried by the solution is indeed given by the integral of the second term in eq. (3.1) over the whole space, thereby proving that this is the only source responsible for the electric charge of the soliton.

Let us now ask the opposite question: do we get charged type IIA string soliton by starting with the neutral type IIA string soliton, and applying the solution generating transformations on this solution? If the answer is yes, it would imply that the type IIA string soliton *does not* satisfy the properties of a fundamental type IIA string, since it is known that the fundamental type IIA string does not carry any world-sheet current that couples to the gauge fields originating from the Ramond-Ramond sector. We again start from the type IIA soliton expressed in the unprimed variables, since the solution generating transformations act naturally on the unprimed variables. This solution has been given in eq.(2.15). Notice that the the part of the metric involving $(-dt^2 + (dx^5)^2)$ does not have any conformal factor. This, in turn, implies that this solution is left invariant under the solution generating transformations, and we do not generate any new solutions! This is precisely what we want, since if we had gotten a new solution using the solution generating transformation, it would have almost certainly corresponded to a charge carrying solution, thereby showing that the soliton does not have the right properties for being identified as the fundamental type IIA string. When expressed in the primed variables, the $(-dt^2 + (dx^5)^2)$ term in the metric does have a conformal factor, but no solution generating transformation is

known that acts naturally on the primed variables, and most likely no such transformation exists. Thus we conclude that the type IIA string soliton, which is a non-singular solution of the heterotic string theory, is rigid, in that it does not allow deformations which correspond to charge carrying strings.

4 String Solitons as Exact Conformal Field Theories

We have seen in the previous two sections that the fundamental heterotic string can be regarded as a non-singular solution of the type IIA string theory and vice versa. It is natural to ask if it is possible to represent them as exact conformal field theories. Unfortunately the complete answer to this question is not known. However by examining the (uncharged) solutions near the semi-infinite geometry ($r \simeq 0$) we see that it can be represented as SO(3) WZW model with a linear dilaton background[26]. The level of the WZW theory will depend on the precise relationship between the coupling constants of the two string theories. In particular, the product of the coupling constants of the two theories satisfy a quantization condition[2, 20], and in the case where the quantized product takes its minimal value we would expect both the solutions to be described by a level 1 SO(3) WZW model.

References

- [1] C. Hull and P. Townsend, preprint QMW-94-30 (hep-th/9410167).
- [2] M. Duff, preprint CTP-TAMU-49/94 (hep-th/9501030);
M. Duff and R. Khuri, Nucl. Phys. **B411** (1994) 473 (hep-th/9305142).
- [3] E. Witten, preprint IASSNS-HEP-95-18 (hep-th/9503124).
- [4] N. Seiberg, Nucl. Phys. **B303** (1988) 286.
- [5] P. Aspinwall and D. Morrison, preprint DUK-TH-94-68 (hep-th/9404151).

- [6] C. Montonen and D. Olive, Phys. Lett. **B72** (1977) 117;
P. Goddard, J. Nyuts and D. Olive, Nucl. Phys. **B125** (1977) 1;
H. Osborn, Phys. Lett. **B83** (1979) 321.
- [7] A. Font, L. Ibanez, D. Lust and F. Quevedo, Phys. Lett. **B249** (1990) 35;
S.J. Rey, Phys. Rev. **D43** (1991) 526;
S. Kalara and D. Nanopoulos, Phys. Lett. **B267** (1991) 343.
- [8] A. Sen, Int. J. Mod. Phys. **A9** (1994) 3707 (hep-th/9402002), and references therein.
- [9] J. Schwarz, preprint CALT-68-1965 (hep-th/9411178).
- [10] A. Sen, Phys. Lett. **B329** (1994) 217 (hep-th/9402032).
- [11] J. Gauntlett and J. Harvey, preprint EFI-94-36 (hep-th/9407111).
- [12] C. Vafa and E. Witten, preprint HUTP-94-A017 (hep-th/9408074).
- [13] G. Segal, to appear.
- [14] L. Girardello, A. Giveon, M. Porrati and A. Zaffaroni, preprint NYU-TH-94/06/02 (hep-th/9406128); preprint NYU-TH-94/12/01 (hep-th/9502057).
- [15] P. Townsend, preprint DAMTP-R/95/2 (hep-th/9501068).
- [16] S. Gates and V. Rodgers, UMD EPP 95-76 (hep-th/9503237).
- [17] I. Bars, preprint USC-95/HEP-82 (hep-th/9503228).
- [18] E. Witten and D. Olive, Phys. Lett. **B78** (1978) 97.
- [19] A. Dabholkar, G. Gibbons, J. Harvey and F. Ruiz, Nucl. Phys. **B340** (1990) 33.
- [20] M. Duff, R. Khuri and J. Lu, preprint CTP/TAMU-67/92 (hep-th/9412184), and references therein.
- [21] M. Duff, G. Gibbons and P. Townsend, preprint DAMTP/R-93/5 (hep-th/9405124).

- [22] G. Veneziano, Phys. Lett. **B265** (1991) 287;
K. Meissner and G. Veneziano, Phys. Lett. **B267** (1991) 33; Mod. Phys. Lett. **A6** (1991) 3397;
A. Sen, Phys. Lett. **B271** (1991) 295; **B274** (1991) 34;
M. Gasperini, J. Maharana and G. Veneziano, Phys. Lett. **B272** (1991) 272.
- [23] S. Hassan and A. Sen, Nucl. Phys. **B375** (1992) 103 (hep-th/9109038);
S. Hassan, preprint TIFR-TH-94-26 (hep-th/9408060).
- [24] A. Sen, Nucl. Phys. **B388** (1992) 457.
- [25] A. Sen, preprint TIFR-TH-94-47 (hep-th/9411187).
- [26] C. Callan, J. Harvey and A. Strominger, Nucl. Phys. **B359** (1991) 611;
B367 (1991) 60.