

A Note on Enhanced Gauge Symmetries in M - and String Theory

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Abstract

Two different mechanisms exist in non-perturbative String / M - theory for enhanced $SU(N)$ ($SO(2N)$) gauge symmetries. It can appear in type IIA string theory or M -theory near an A_{N-1} (D_N) type singularity where membranes wrapped around two cycles become massless, or it can appear due to coincident D-branes (and orientifold planes) where open strings stretched between D-branes become massless. In this paper we exhibit the relationship between these two mechanisms by displaying a configuration in M -theory, which, in one limit, can be regarded as membranes wrapped around two cycles with A_{N-1} (D_N) type intersection matrix, and in another limit, can be regarded as open strings stretched between N Dirichlet 6-branes (in the presence of an orientifold plane).

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1 Introduction and Conclusion

In non-perturbative string / M - theory, we can get enhanced gauge symmetries at special points in the moduli space of a theory. A particularly interesting case is that of type IIA string / M - theory on a $K3$ surface, where the gauge symmetry enhancement takes places when the $K3$ surface develops singularities[1]. Typically at these singularities, the area of some two cycles vanish, and hence membranes, wrapped around these two cycles, become massless, providing the extra massless states that are required for the gauge symmetry enhancement. The A-D-E classification of these singularities is in one to one correspondence to the A-D-E classification of enhanced gauge symmetries that arise at these points. There is also an apparently unrelated mechanism for gauge symmetry enhancement in special class of string vacua with Dirichlet branes and orientifold planes[2, 3]. When N of these D-branes come on top of each other we get an $SU(N)$ enhanced gauge symmetry[4]. On the other hand if N of the D-branes come on top of an orientifold plane, then we get an enhanced $SO(2N)$ gauge symmetry[2].

The authors of ref.[5] (see also [6]) showed that these two mechanisms are not unrelated, but can be related to each other through a series of T - and S -duality transformations that relates type IIA theory in an ALE space with A_{N-1} type singularity to type IIB theory in the background of N coincident Dirichlet five-branes. In this paper we provide a much more direct map between these two mechanisms by considering a configuration of Kaluza-Klein monopoles in M -theory[7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. The space-time describing N Kaluza-Klein monopoles in M -theory has several independent two cycles whose intersection matrix coincides with the Cartan matrix of the A_{N-1} algebra. In the limit when all monopoles come on top of each other, each of these two cycles acquire vanishing area, and hence membranes wrapped around these two cycles become massless, giving rise to enhanced gauge symmetry. On the other hand, using the identification of M -theory on S^1 with type IIA string theory, the configuration of N Kaluza-Klein monopoles in M -theory can be regarded as a configuration of N parallel Dirichlet six branes in type IIA string theory. We show that under this identification, a membrane wrapped around a two cycle of the Kaluza-Klein monopole space can be regarded as an open string stretched between the two D6-branes in the type IIA string theory. Thus in the language of type IIA theory, the same phenomena of enhanced $SU(N)$ gauge symmetry can be interpreted as due to the vanishing of the mass of the open strings stretched between the D-branes as the D-brane positions coincide.

In type IIA string theory we can define two Z_2 transformations: $(-1)^{F_L}$ which changes the sign of all the Ramond sector states on the left, and Ω which is the world-sheet parity transformation. Whereas $(-1)^{F_L}$ generates a symmetry of the theory by itself, Ω generates a symmetry only when accompanied by a parity transformation in the target space. If we consider type IIA string theory in ten dimensions, and take the quotient of this theory by the Z_2 transformation generated by $(-1)^{F_L} \cdot \Omega \cdot \mathcal{I}_3$, where \mathcal{I}_3 denotes the reversal of sign of three of the nine space-like directions, then the six dimensional plane, left fixed by this transformation, is known as the orientifold plane. If we now consider a configuration where N Dirichlet 6-branes are on top of this orientifold plane, one gets an enhanced $SO(2N)$ gauge symmetry. One can ask if this gauge symmetry enhancement can also be understood from the M -theory view point. It has been shown in ref.[17] that in M -theory, an orientifold 6-plane of type IIA string theory is represented by the Atiyah-Hitchin manifold[18]. Thus a configuration of N D6-branes and an orientifold plane will be represented in M -theory by a configuration of N Kaluza-Klein monopoles and an Atiyah Hitchin space. We identify appropriate two cycles in this space and show that their intersection matrix coincides with the Cartan matrix of D_N algebra. In the limit when the monopoles are on top of the Atiyah-Hitchin space, the two cycles collapse to zero size, and hence the membranes wrapped around these two cycles become massless, giving rise to enhanced $SO(2N)$ gauge symmetry. We show that from the point of view of the type IIA string theory, the membranes wrapped around these various two cycles are simply open strings stretched between various D-branes and their images under the Z_2 transformation. These become massless as the D -branes approach the orientifold plane, thereby giving rise to enhanced $SO(2N)$ gauge symmetry.

In non-perturbative string theory, we also have a novel phenomenon involving the appearance of tensionless strings at special points in the moduli space. This can appear in type IIB string theory in singular background from three branes wrapped around two cycles with vanishing area[19], or in M -theory in the presence of coincident five-branes, from membranes stretched between these five-branes[20, 21]. In the last section we study the relationship between these two phenomena by considering type IIB string theory in the background of Kaluza-Klein monopoles. Using the duality between type IIB on S^1 and M -theory on T^2 , we can relate this to a configuration of five-branes in M -theory[14]. Furthermore, we show that a three brane of type IIB string theory wrapped around a two cycle in the Kaluza-Klein monopole space can be reinterpreted as a membrane of M -theory stretched between the five-branes. Thus when the locations of the Kaluza-

Klein monopoles coincide, we get tensionless strings in type IIB theory from three cycles wrapped on vanishing two cycles. But the same configuration can be reinterpreted in the M -theory description as membranes stretched between coincident five-branes. This shows the equivalence between the two mechanisms for getting tensionless strings.

2 Enhanced $SU(N)$ Gauge Symmetry

The multiple Kaluza-Klein monopole solution in M -theory is described by the metric:

$$ds^2 = -dt^2 + \sum_{m=5}^{10} dy^m dy^m + ds_{TN}^2, \quad (1)$$

where y^m denote the space-like world-volume coordinates on the 6-brane represented by this solution, and ds_{TN} is the metric of the Euclidean multi-centered Taub-NUT space[22]:

$$ds_{TN}^2 = U^{-1}(dx^4 + \vec{\omega} \cdot d\vec{r})^2 + U d\vec{r}^2. \quad (2)$$

Here x^4 denotes the compact direction, and $\vec{r} \equiv (x^1, x^2, x^3)$ denotes the three spatial coordinates transverse to the brane. U and $\vec{\omega}$ are defined as follows:

$$U = 1 + \sum_{I=1}^N \frac{4m}{|\vec{r} - \vec{r}_I|}, \quad (3)$$

and,

$$\vec{\nabla} \times \vec{\omega} = -\vec{\nabla} U. \quad (4)$$

m and \vec{r}_I are parameters labelling the solution. \vec{r}_I can be interpreted as the locations of the Kaluza-Klein monopoles in the transverse space. In order that the solutions are free from conical singularities at $\vec{r} = \vec{r}_I$, x^4 must have periodicity $16\pi m$.

In the multi-centered Taub-NUT space described by the metric (2), one can construct $N - 1$ linearly independent two cycles as follows. Consider a straight line from \vec{r}_i to \vec{r}_j in the three dimensional space labelled by \vec{r} . From this we can construct a two dimensional surface in the Taub-NUT space, by erecting at each point of this line a circle labelled by the periodic coordinate x^4 . Naively, this would seem to have the geometry of a cylinder, but if we take into account the fact that the physical radius of the circle labelled by x^4 in the metric (2) is $16\pi m U^{-1/2}$, which vanishes at $\vec{r} = \vec{r}_i$ and $\vec{r} = \vec{r}_j$, we see that this surface has the topology of a sphere. Let us denote this by S_{ij} . The area of this two cycle as measured in the metric (2), is given by:

$$\int_{S_{ij}} (U^{-1/2}(\vec{r}) dx^4) (U^{1/2}(\vec{r}) |d\vec{r}|) = 16\pi m \int_C |d\vec{r}| = 16\pi m |\vec{r}_i - \vec{r}_j|, \quad (5)$$

where C denotes the straight line curve from \vec{r}_i to \vec{r}_j in the \vec{r} space.³ Eq.(5) also shows that if we consider a deformation of the surface where we replace the straight line from \vec{r}_i to \vec{r}_j by any other curve between the two points, then the area of the surface will be proportional to the length of this curve. Thus the surface that we have chosen, corresponding to the straight line path, is the minimal area surface with this topology. If T_M denotes the tension of a membrane in M -theory, then the mass of a membrane wrapped around the two cycle S_{ij} will be given by

$$m_{ij} = 16\pi m T_M |\vec{r}_i - \vec{r}_j|. \quad (6)$$

We can take $S_{i,i+1}$ for $1 \leq i \leq (N-1)$ as the independent two cycles. In that case the self-intersection number for each of these cycles is 2. To see this, we deform the surface S_{ij} by replacing the straight line path from \vec{r}_i to \vec{r}_j by any other curve between these two points, and note that the resulting surface intersects the original surface at two points, $\vec{r} = \vec{r}_i$ and $\vec{r} = \vec{r}_j$ (with the same sign). $S_{i,i+1}$ intersects $S_{i-1,i}$ once (at the point $\vec{r} = \vec{r}_i$) with negative sign, since for $S_{i-1,i}$ the line is ingoing at $\vec{r} = \vec{r}_i$, whereas for $S_{i,i+1}$ the line is outgoing at $\vec{r} = \vec{r}_i$. Finally, $S_{i,i+1}$ and $S_{j,j+1}$ do not intersect if $j \neq i-1, i, i+1$. This gives the following $(N-1) \times (N-1)$ intersection matrix of the two cycles:

$$I = \begin{pmatrix} 2 & -1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 2 & -1 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & 2 & -1 \\ 0 & 0 & 0 & 0 & \dots & -1 & 2 \end{pmatrix}. \quad (7)$$

This can be easily recognised as the Cartan matrix of the A_{N-1} algebra. When all the \vec{r}_i 's approach each other, the area of all the two cycles S_{ij} go to zero, and we hit an A_{N-1} singularity. For M -theory in such a background, we shall get extra massless states from membranes wrapped around these two cycles, giving rise to enhanced $SU(N)$ gauge symmetry[14].

We can use the correspondence between M -theory on S^1 and type IIA string theory to analyse how a membrane wrapped on S_{ij} is viewed in the type IIA string theory. First of all, note that the configuration of Kaluza-Klein monopoles given in (1) corresponds

³Similar mass formula for three branes wrapped on three cycles of Calabi-Yau manifolds have been derived previously in ref.[23].

to a configuration of Dirichlet 6 branes in type IIA theory located at \vec{r}_i ($1 \leq i \leq N$). Since in the identification of M -theory on S^1 with type IIA string theory the membrane wrapped around S^1 (which in this case is labelled by the coordinate x^4) corresponds to an elementary type IIA string, it is clear from the definition of S_{ij} that a membrane wrapped around S_{ij} will correspond to an elementary type IIA string stretched from \vec{r}_i to \vec{r}_j , *i.e.* starting on the i th D6-brane and ending on the j th D6-brane. Since the type IIA string tension T_S is given by $16\pi m T_M$ – product of the membrane tension and the radius of the compact direction (far away from the D-branes) – we see that the mass formula (6) can be rewritten as

$$m_{ij} = T_S |\vec{r}_i - \vec{r}_j| \quad (8)$$

exactly as we would expect for an open string stretched between two D6-branes situated at \vec{r}_i and \vec{r}_j . When the D -branes come on top of each other, these open strings become massless, giving rise to enhanced gauge symmetries.

This establishes the correspondence between the mechanism of gauge symmetry enhancement in M -theory near an A_{N-1} type singularity, and in type IIA theory from N coincident D6-branes. In the next section we shall analyse similar phenomenon for enhancement of gauge symmetry to $SO(2N)$.

3 Enhanced $SO(2N)$ Gauge Symmetry

In type IIA string theory, enhanced $SO(2N)$ gauge symmetry appears when N D-branes coincide with an orientifold plane. We shall concentrate on the situation where we have a system of N D6-branes in the presence of an orientifold six plane. As has already been mentioned in the introduction, the orientifold six plane is obtained as the fixed point of \mathcal{I}_3 when we mod out the ten dimensional type IIA string theory by $(-1)^{F_L} \cdot \Omega \cdot \mathcal{I}_3$, with \mathcal{I}_3 denoting the transformation that reverses the sign of three spatial coordinates (which we shall take to be $\vec{r} = (x^1, x^2, x^3)$).

We have already seen that the D6-branes of type IIA theory are represented as Kaluza-Klein monopoles in M -theory. We shall now discuss how to describe an orientifold 6-plane of type IIA theory in M -theory, and then combine the two descriptions in order to describe a configuration of orientifold plane and D-branes. This question was addressed in ref.[17] where it was concluded that the orientifold six plane is represented by the Atiyah-Hitchin space[18] (called \mathcal{N} in ref.[17]) in M -theory. From far, this space looks like $(R^3 \times S^1)/\mathcal{I}_4$,

where \mathcal{I}_4 denotes the reversal of sign of all four coordinates labelling R^3 and S^1 . Using the correspondence between the fields in M -theory on S^1 and type IIA string theory, it is easy to see that the transformation \mathcal{I}_4 in M -theory does indeed correspond to $(-1)^{FL} \cdot \Omega \cdot \mathcal{I}_3$ in type IIA theory. Choosing a normalization so that the asymptotic radius of S^1 is given by $16\pi m$, the Atiyah-Hitchin metric can be expressed as[24]:

$$ds^2 = f(\rho)^2 dr^2 + (8m)^2 \left(a(\rho)^2 \sigma_1^2 + b(\rho)^2 \sigma_2^2 + c(\rho)^2 \sigma_3^2 \right), \quad (9)$$

where f , a , b and c are functions defined in ref.[24],

$$\rho = r/8m, \quad (10)$$

and,

$$\begin{aligned} \sigma_1 &= -\sin\left(\frac{x^4}{16m}\right)d\theta + \cos\left(\frac{x^4}{16m}\right)\sin\theta d\phi, \\ \sigma_2 &= \cos\left(\frac{x^4}{16m}\right)d\theta + \sin\left(\frac{x^4}{16m}\right)\sin\theta d\phi, \\ \sigma_3 &= \frac{1}{16m}dx^4 + \cos\theta d\phi. \end{aligned} \quad (11)$$

The coordinate ranges are given by $8m\pi \leq r < \infty$, $0 \leq \theta \leq \pi$, ϕ is periodic with period 2π , and x^4 is periodic with period $16\pi m$. Finally, there is an identification under the transformation I_1 given by:

$$I_1 : (r, \theta, \phi, x^4) \rightarrow (r, \pi - \theta, \pi + \phi, -x^4). \quad (12)$$

It will be convenient to define a new space \mathcal{M} , which is described by the metric (9) before the identification by I_1 given in (12). (Note that this is different from the double cover $\bar{\mathcal{N}}$ discussed in [17], which simply corresponds to doubling the period of x^4 .) As shown in [24], the space \mathcal{M} has a conical singularity at $r = 8\pi m$, which gets removed when we take the quotient of this space by I_1 to recover \mathcal{N} . (Note that the complete metric describing the orientifold 6-plane is obtained by adjoining to (9) the (6+1) dimensional Minkowski space labelled by t and y^m ($5 \leq m \leq 10$), as in eq.(1).)

For large r , the metric (9) can be approximated by (ignoring terms that are exponentially small)[24]:

$$ds^2 \simeq \left(1 - \frac{16m}{r}\right)^{-1} (dx^4 + 16m \cos\theta d\phi)^2 + \left(1 - \frac{16m}{r}\right) (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2), \quad (13)$$

which is the Euclidean Taub-NUT metric with negative mass parameter. Comparing this with the Kaluza-Klein monopole solution (1), (2) we see that this solution corresponds to -4 units of magnetic charge located at $r = 0$. This is consistent with the result that the orientifold 6-plane of type IIA string theory carries -4 units of 6-brane charge (in the covering space R^3 of R^3/\mathcal{I}_3). Upon modding out the space by I_1 (which acts as \mathcal{I}_3 on R^3), this would correspond to -2 units of six brane charge.

We shall now construct the M -theory background that corresponds to N D6-branes in the presence of the orientifold plane. Unfortunately we shall not be able to write down the exact solution, but only an approximate solution that differs from the exact solution by terms that vanish exponentially as we go away from the orientifold plane. The solution is described by the metric:

$$ds^2 \simeq V^{-1}(dx^4 + \vec{\Omega} \cdot d\vec{r})^2 + V d\vec{r}^2, \quad (14)$$

modded out by the transformation:

$$(\vec{r} \rightarrow -\vec{r}, \quad x^4 \rightarrow -x^4), \quad (15)$$

where

$$V = 1 - \frac{16m}{r} + \sum_{i=1}^N \left(\frac{4m}{|\vec{r} - \vec{r}_i|} + \frac{4m}{|\vec{r} + \vec{r}_i|} \right), \quad (16)$$

and,

$$\vec{\nabla} \times \vec{\Omega} = -\vec{\nabla} V. \quad (17)$$

Since $V(r)$ is invariant under $(\vec{r} \rightarrow -\vec{r})$, and $\vec{\Omega} \cdot d\vec{r}$ changes sign under this transformation, the metric (14) is invariant under (15). Thus modding out the space by this transformation is a meaningful concept.

Note that the metric given in (14) is singular at $\vec{r} = 0$ and approaches the metric given in (13). This singularity is removed by replacing the metric near $\vec{r} = 0$ by the Atiyah-Hitchin metric (9), which is completely non-singular (after we mod out by the transformation (15), which acts as I_1 given in eq.(12)). To see that this describes an orientifold plane in the presence of N D6-branes in type IIA string theory, note that near $\vec{r} = 0$ the metric agrees with that given in (9), and hence represents an orientifold 6-plane of type IIA theory. On the other hand near the point $\vec{r} = \vec{r}_i$ or its image $-\vec{r}_i$ for $1 \leq i \leq N$, the metric agrees with the one near a Kaluza-Klein monopole, and hence represents a D6-brane of type IIA theory. (For $N = 1$, the exact metric has been constructed by Dancer[25]).

Let us now examine the two cycles in the space described by the metric (14). First of all there are the usual two cycles corresponding to straight lines from \vec{r}_i to \vec{r}_j as in the case of a configuration of Kaluza-Klein monopoles. Membranes wrapped around these two cycles will have mass given by eq.(6). But now there will be additional two cycles corresponding to straight lines joining \vec{r}_i and $-\vec{r}_j$. Let us denote these two cycles by \bar{S}_{ij} . Membranes wrapped around these two cycles will have mass given by:⁴

$$\bar{m}_{ij} = 16\pi m T_M |\vec{r}_i + \vec{r}_j|. \quad (18)$$

Since under the transformation (15) the two cycle S_{ij} is identified with the two cycle associated with the line joining $-\vec{r}_i$ and $-\vec{r}_j$, the latter do not form an independent set of two cycles. We can then take the independent two cycles to be $S_{i,i+1}$ for $(1 \leq i \leq (N-1))$, and $\bar{S}_{N-1,N}$. The intersection matrix involving S_{ij} 's is as given before. $\bar{S}_{N-1,N}$ has self intersection number 2, as can be seen in the same way as that for the S_{ij} 's. Also $\bar{S}_{N-1,N}$ intersects $S_{N-1,N}$ at two points, \vec{r}_{N-1} and \vec{r}_N (which is identified with its image $-\vec{r}_N$). However, these two points contribute with opposite sign, since the orientation of S^1 labelled by x^4 changes sign under the map (15). Thus the net intersection number of $S_{N-1,N}$ and $\bar{S}_{N-1,N}$ vanishes. Finally $S_{N-2,N-1}$ intersects $\bar{S}_{N-1,N}$ at \vec{r}_{N-1} contributing -1 to the intersection number. Thus the new $N \times N$ intersection matrix takes the form:

$$I = \begin{pmatrix} 2 & -1 & 0 & 0 & \dots & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & \dots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \dots & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \dots & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & \dots & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & \dots & -1 & 0 & 2 \end{pmatrix}, \quad (19)$$

where the last row and column correspond to the cycle $\bar{S}_{N-1,N}$. This can be easily recognised as the Cartan matrix of D_N algebra. As all the \vec{r}_i 's approach the origin, the area of all of the cycles S_{ij} and \bar{S}_{ij} vanish, and we hit a D_N singularity.⁵ For M -

⁴Note that our derivation of the mass formula for wrapped membranes in M -theory is valid only when the two cycle does not pass close to the $\vec{r} = 0$ point, since in computing the area of the two cycle, we have used the approximate metric that is valid only away from $\vec{r} = 0$. However, we expect that the mass formula will not change even when the correction to the metric is taken into account, since it agrees with the exact BPS bound.

⁵This can also be seen explicitly from the analysis of ref.[17] where it was shown that the M -theory background describing a configuration of N D6-branes in type IIA theory on top of an orientifold 6-plane is the space C^2/Γ_{N-2} , where Γ_{N-2} is the dihedral group. This has a D_N type singularity at the origin.

theory in such a background, the masses of the membranes wrapped around these two cycles vanish, giving rise to enhanced $SO(2N)$ gauge symmetry.

We shall now reinterpret this phenomenon from the viewpoint of type IIA string theory. A repetition of the arguments for the A_{N-1} case shows that the membrane wrapped around the two cycle S_{ij} (\bar{S}_{ij}) in M -theory corresponds to an open string stretched between the D6 branes situated at \vec{r}_i and \vec{r}_j (or its image at $-\vec{r}_j$). The mass formula given in eqs.(6), (18) clearly reproduces the mass formula for open strings stretched between the D6-branes (and their images). Thus from the point of view of type IIA theory, the appearance of extra massless states and hence enhanced $SO(2N)$ gauge symmetry when the D6-branes coincide with the orientifold 6-plane can be reinterpreted as due to the open strings stretched between the D6-branes and their images becoming massless.

4 Tensionless Strings

Another novel phenomenon in non-perturbative string / M -theory that has been discovered during the last two years is the appearance of tensionless strings at special points in the moduli space. This happens for example in the type IIB string theory in a background with A_{N-1} singularity. In this case the tensionless strings come from type IIB three branes wrapped around the two cycles of vanishing area. Another configuration that can give rise to tensionless strings is a set of parallel five-branes in M -theory. This configuration can support open membranes in M -theory, with two ends of the membrane on two five branes. When two or more of these five-branes come close to each other, the membranes stretched between them represent tensionless strings.

Following the procedure outlined in the last two sections, it is easy to show that these two apparently different phenomena are in fact different interpretations of the same phenomenon. For this we start with type IIB string theory in the background of N Kaluza-Klein monopoles. Using the duality between type IIB on S^1 and M -theory on T^2 , we can map this to a configuration of N five branes in M -theory on T^2 , with the five branes being transverse to T^2 [20, 14]. In the type IIB description, we get string like solitonic excitations from three branes wrapped around the two cycles S_{ij} . Since under the duality map between type IIB on S^1 and M -theory on T^2 [26, 27], a three brane of type IIB wrapped on S^1 corresponds to a membrane of M -theory transverse to T^2 , we see that a three brane wrapped on the two cycle S_{ij} in type IIB theory corresponds to a membrane stretched between the i th and the j th five-brane in M -theory. When \vec{r}_i

coincides with \vec{r}_j , this represents a tensionless string. This shows that the appearance of tensionless strings in type IIB string theory from three branes wrapped on collapsed two cycles, and in M -theory from membranes stretched between coincident five branes, are different descriptions of the same phenomenon.

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