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Phenomenology of Pseudo Dirac Neutrinos

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Abstract

We formulate general conditions on 3×3 neutrino mass matrices under which a degenerate pair of neutrinos at a high scale would split at low scale by radiative corrections involving only the standard model fields. This generalizes the original observations of Wolfenstein on pseudo Dirac neutrinos to three generations. A specific model involving partially broken discrete symmetry and solving the solar and atmospheric anomalies is proposed. The symmetry pattern of the model naturally generates two large angles one of which can account for the large angle MSW solution to the solar neutrino problem.

1 Introduction

Dirac neutrinos are associated with an unbroken U(1) symmetry acting on leptons. A small breaking of this symmetry splits a Dirac neutrino into a pair of majorana neutrinos with $(\text{mass})^2$ difference much smaller than the square of the original mass. Such a pair can simultaneously describe small splitting and large mass and is of phenomenological importance in solving solar and/or atmospheric neutrino anomalies [1].

Zeroth order approximation to a pseudo Dirac neutrino is provided by the following texture in case of two generations say, e and μ

$$\begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix} \tag{1}$$

This displays an unbroken $L_e - L_\mu$ symmetry and its breaking (by introducing small non-zero diagonal elements) leads to a pseudo Dirac neutrino. This breaking can be explicitly introduced by allowing for additional fields like Higgs triplet or right handed neutrinos [2] or it can be introduced radiatively by breaking the symmetry $L_e - L_\mu$ in the charged lepton sector [3, 4].

A slightly non-trivial example of the pseudo Dirac neutrino is provided by a mass matrix discussed by Wolfenstein [5]

$$\begin{pmatrix} a & b \\ b & -a \end{pmatrix} \tag{2}$$

Both the textures in eqs.(1) and (2) lead to a pair of neutrinos with equal and opposite eigenvalues. However they differ conceptually and phenomenologically from each other [1]. Unlike in eq.(1), the matrix in eq.(2) cannot be invariant under a $U(1)$ symmetry corresponding to *any* combination of lepton numbers. This has the consequence that the charged current interactions defined in the mass basis of the degenerate pairs violate lepton number [5] and the pseudo Dirac pair gets split automatically by radiative corrections [6]. Thus the theory described by eq. (2) intrinsically defines a pseudo Dirac neutrino while one needs to invoke additional fields in order to break the $L_e - L_\mu$ symmetry in case of eq.(1). At the phenomenological level, the mixing implied by (1) is maximal while it is arbitrary in case (2). The most phenomenological discussions related to pseudo Dirac neutrinos in the literature [2] are in the context of texture in (1). We wish to discuss here instead several interesting aspects related to the Wolfenstein texture, eq.(2) and its generalization to three families.

[1] We are assuming here that these textures are defined in a basis with a diagonal charged lepton mass matrix.

The radiative splitting of neutrinos in case (2) is most simply demonstrated using the relevant renormalization group (RG) equations. Assume that the neutrino mass matrix in (2) is specified at some high scale M_X and the effective theory below this scale is the standard model (SM) or the minimal supersymmetric standard model (MSSM). The neutrino mass matrix at a low scale $\mu \sim M_Z$ is then given by [7]

$$M_\nu(\mu) \approx P M_\nu(M_X) P \ , \tag{3}$$

where in the three-generation case $P = Diag.(1+\delta_e, 1+\delta_\mu, 1+\delta_\tau)$ and

$$\delta_\alpha \equiv -\frac{m_\alpha^2}{v^2 \cos^2\beta (4\pi)^2} \ln \frac{M_X}{\mu} \tag{4}$$

in case of the minimal supersymmetric standard model. $m_\alpha, v \sim 174\,\mathrm{GeV}$ here refer to the charged lepton masses and the weak scale respectively. As before, M_ν is specified in the physical basis of the charged leptons. The texture (2) at high scale gets transformed to the texture

$$\begin{pmatrix} a(1+2\delta_e) & b(1+\delta_e+\delta_\mu) \\ b(1+\delta_e+\delta_\mu) & -a(1+2\delta_\mu) \end{pmatrix} + O(\delta^2) \tag{5}$$

at the low scale. This describes a split pair of neutrinos. In contrast, eq.(1) leads to a degenerate pair even after RG evolution as in eq.(3) is taken into account.

The modified mixing angle and the mass splitting implied by eq.(5) are given by

$$\begin{aligned} \tan 2\theta(\mu) &\approx \tan 2\theta(M_X)(1+O(\delta^2)) \ , \\ \Delta &\approx 4m_0^2 \cos 2\theta \delta_\mu \ . \end{aligned} \tag{6}$$

with $m_0 = \sqrt{a^2+b^2}$ and $\tan 2\theta = \frac{b}{a}$. As follows from eq.(4), typical strength of radiative corrections is $\delta_\mu \sim 10^{-7}$. From phenomenological point of view, the required value of Δ can be as small as $10^{-11}\,\mathrm{eV}^2$. If one starts with a mass matrix having such splitting at M_X then there is a possibility that the radiative corrections may lead to much larger splitting than this. Likewise, maximal mixing angle at M_X can also get destabilized [7]. In the present case, splitting is zero at M_X and it is only induced by radiative corrections.

As follows from eq.(6), the magnitude of this splitting can be in the range $10^{-10} - 10^{-11}\,\mathrm{eV}^2$ for m_0 near the atmospheric neutrino scale. Moreover, the mixing angle at M_X is arbitrary and receives corrections only at $\mathrm{O}(\delta^2)$ in this case. Thus, the texture in (2) is stable against radiative corrections unlike some of the textures discussed in [7]. These properties make the Wolfenstein texture in (2) also phenomenologically realistic. Theoretically, this texture is not a fine tuned possibility but can arise from imposition of the following discrete symmetry on neutrino mass matrix:

$$L_e \to iL_\mu \; ; \qquad L_\mu \to -iL_e \tag{7}$$

This symmetry is broken by hierarchical charged lepton masses which lead to radiative splitting between the degenerate pair.

The 2×2 texture of eq.(2) is successful but not complete from the point of view of simultaneous solution to the solar and atmospheric neutrino anomalies. This would require going beyond two generations. The purpose of this note is to generalize the above considerations to the realistic case of three generations and identify phenomenologically viable models/textures leading to pseudo Dirac neutrinos. We first write down the general conditions on an arbitrary 3×3 neutrino mass matrix under which it leads to a pair of degenerate neutrinos. Such matrices can be classified in two categories, those in which degeneracy is preserved by radiative corrections involving standard model fields and those in which the theory describes a pseudo Dirac state. We formulate general criteria to distinguish between these two categories and show that they can be identified by looking at the structure of the leptonic mixing matrices implied at the *tree level*. Then we discuss an example which satisfies phenomenological requirements to obtain a solution to solar and atmospheric neutrino anomalies. Starting with a generalization of the original Wolfenstein mass matrix, splitting of neutrino states needed for solar neutrino anomaly arises in this example through radiative corrections. The model can lead to either vacuum solution with bi-maximal mixing or MSW solution corresponding to large mixing angle.

2 Pseudo Dirac neutrinos: General analysis

Let us consider a CP conserving theory specified by a general 3×3 real symmetric mass matrix M_ν for the neutrinos:

$$-\mathcal{L}_m = \frac{1}{2}\overline{(\nu'_{\alpha L})^c}(M_\nu)_{\alpha\beta}\nu'_{\beta L} + H.c. \ . \tag{8}$$

The M_ν would contain two equal and opposite eigenvalues if it satisfies the following condition:

$$tr(M_\nu)\sum_i \Delta_i = det M_\nu \ , \tag{9}$$

where Δ_i represents the determinant of the 2×2 block of M_ν obtained by blocking i^{th} (i=1,2,3) row and column. Only M_ν satisfying condition (9) would lead to a Dirac or pseudo Dirac neutrinos. Define a U^ν which diagonalizes such M_ν:

$$U^\nu \ M_\nu U^{\nu T} = Diag.(m, -m, m') \ . \tag{10}$$

The physical mass basis for neutrinos is defined by $\nu_L = U^\nu \nu'_L$. The neutrino masses can be written in terms of Majorana spinors

$$\nu_{1,3} = \nu_{1,3L} + (\nu_{1,3L})^c \qquad \nu_2 = \nu_{2L} - (\nu_{2L})^c \ . \tag{11}$$

Explicitly,

$$-\mathcal{L}_m = \frac{1}{2}\left[m(\bar{\nu}_1\nu_1 + \bar{\nu}_2\nu_2) + m'\bar{\nu}_3\nu_3\right] \ . \tag{12}$$

The degeneracy of two of the Majorana states allows us to define

$$\psi = \frac{1}{\sqrt{2}}(\nu_1 + \nu_2) \tag{13}$$

and rewrite eq.(12) as

$$-\mathcal{L}_m = m\bar{\psi}\psi + \frac{1}{2}m'\bar{\nu}_3\nu_3 \ . \tag{14}$$

Note that the ψ is a four component Dirac field since $\psi^c \neq \psi$. As a result, the system specified by eq.(9) corresponds to a Dirac and a majorana neutrino.

The charged current interactions can be written as follows in the leptonic mass basis:

$$
\begin{aligned}
-\mathcal{L}_W &= \frac{g}{\sqrt{2}}\bar{e}'_{\alpha L}\gamma_\mu \nu'_{\alpha L} W^\mu + H.c.\ , \\
&= \frac{g}{\sqrt{2}}\bar{e}_{\alpha L}\gamma_\mu \left(\frac{1}{\sqrt{2}}(K_{\alpha 1}+K_{\alpha 2})\psi_L + \frac{1}{\sqrt{2}}(K_{\alpha 1}-K_{\alpha 2})(\psi^c)_L + K_{\alpha 3}\nu_{3L}\right) W^\mu + H.c.\ .
\end{aligned}
\tag{15}
$$

Here e_{aL} represents the physical mass basis for the charged leptons and the K represents the leptonic Kobayashi Maskawa (KM) matrix.

Eq.(15) is a straightforward generalization of the 2×2 case considered in [5]. It shows that although mass term for ψ is invariant under a U(1) symmetry the charged current violates it and the Dirac state will split by radiative corrections [6]. Thus any 3×3 matrix satisfying condition (9) at tree level would generically lead to a pseudo Dirac state.

While the lepton number violation is generically present, it is easy to identify all the 3×3 structures for M_ν which admit an unbroken $U(1)$ symmetry corresponding to a truly Dirac neutrino. Necessary condition for this to happen is easy to write down using eq.(15):

$$
K_{\alpha 1} = \eta_\alpha K_{\alpha 2} \tag{16}
$$

Here $\eta_\alpha = \pm 1$. The above equation ensures that either only ψ or ψ^c couples to a given charged lepton e_α. As a result, phase rotation of ψ can be symmetry of eq.(15).

The above condition is also sufficient to ensure truly Dirac state in case of two generations. But the couplings of the third generation to the first two requires additional constraint to obtain a $U(1)$ symmetry. To see this, note that orthogonality of K does not allow all the η_a in eq.(16) to have the same sign. Without loss of generality we can choose $\eta_1 = -\eta_2 = -\eta_3 = 1$. Other solutions of eq.(16) are obtained by interchange $1 \leftrightarrow 2, 1 \leftrightarrow 3$ or an overall multiplication of η_a by -1 in all these cases. It is easy to write a parameterization of K with this choice of η using orthogonality.

$$
K = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-c}{\sqrt{2}} & \frac{c}{\sqrt{2}} & s \\ \frac{s}{\sqrt{2}} & \frac{-s}{\sqrt{2}} & c \end{pmatrix} \tag{17}
$$

Apart from interchange of rows and overall multiplications of any column by -1, this is the most general form for any real K satisfying eq.(16). Using above eq. (17) in (15) we immediately see that if $s = 0$, the third neutrino does not mix with the first two and eq.(15) is invariant under the symmetry,

$$\psi_L \to e^{i\alpha}\psi_L; \quad e_L \to e^{i\alpha} e_L \;; \quad \mu_L \to e^{-i\alpha}\mu_L$$

In this case, the third neutrino is decoupled and we get an unbroken $L_e - L_\mu$ symmetry and a truly Dirac neutrino.

Even when $s \neq 0$, W interactions can still be made invariant under the following symmetry

$$\psi_L \to e^{i\alpha}\psi_L; \quad e_L \to e^{i\alpha} e_L \;; \quad (\mu_L, \tau_L, \nu_{3L}) \to e^{-i\alpha}(\mu_L, \tau_L, \nu_{3L})$$

But this symmetry would be violated by the mass of the third neutrino and hence we would expect splitting of the neutrino pairs radiatively in this case even though the charged current interactions possess an unbroken symmetry.

It follows from the above discussion that there are two ways in which a degenerate pair at high scale would be split by the radiative corrections involving W: (i) When mixing matrix does not satisfy the necessary condition in (16) and (ii) when mixing matrix satisfies this condition but the third neutrino has a mass and is not decoupled from the first two. The latter situation is more interesting since in this case the implied form of K, eq.(17) corresponds to bi-maximal mixing which is argued [8, 9] to be useful in simultaneous solution of the solar and atmospheric neutrino anomalies. The mass of the third generation plays a non-trivial role in splitting the degenerate pair in this case. This was found to be true in specific example considered in [10]. The present discussion highlights the importance of a non-zero third generation mass from more general considerations.

All the above conclusions were based on the $U(1)$ invariance of the W interactions displayed in eq. (15). The same conclusions can be drawn from the study of the RG evolution of the texture as in eq.(3). The matrix K in eq.(17) implies the following texture for the neutrino mass matrix in the charged lepton mass basis:

$$\begin{pmatrix} 0 & -mc & ms \\ -mc & m's^2 & m'cs \\ ms & m'cs & m'c^2 \end{pmatrix} \tag{18}$$

Assuming this texture to be true at M_X, texture at a lower scale can be worked out using eq.(3)

$$\begin{pmatrix} 0 & -mc(1+\delta_e+\delta_\mu) & ms(1+\delta_e+\delta_\tau) \\ -mc(1+\delta_e+\delta_\mu) & m's^2(1+2\delta_\mu) & -m'cs(1+\delta_\tau+\delta_\mu) \\ ms(1+\delta_e+\delta_\tau) & -m'cs(1+\delta_\tau+\delta_\mu) & m'c^2(1+2\delta_\tau) \end{pmatrix} \tag{19}$$

This texture is seen to lead to a degenerate pair at low scale if $s = 0$ or $m' = 0$ in accordance with the conditions discussed above. Otherwise, it describes a pseudo Dirac state. In the former case, the neutrino mass matrix in the charged lepton mass basis respects either $L_e - L_\mu$ ($s = 0$) or $L_e - L_\mu - L_\tau$ ($m' = 0$) symmetry. In the latter case, splitting occurs but only at O(δ^2) as can be seen using eq.(19). This makes the case (ii) discussed above also more natural from the point of view of stability as argued in [10].

3 A Specific Model

A very economical scheme for understanding the solar and atmospheric neutrino anomaly is provided by the following scenario [3, 9]. The neutrino spectrum at tree level consists of one massless neutrino and two degenerate neutrinos with mass in the atmospheric neutrino range. The radiative corrections split this pair and provides the scale needed to understand the solar neutrino anomaly. Different possibilities realizing this scenario have been proposed [3, 9]. We give here a specific and very economical example in the context of pseudo Dirac texture where one does not need to invoke any new physics and the W interactions provide a source for the solar scale.

As we indicated in introduction, the Wolfenstein type structure as in eq.(2) can result from a discrete symmetry. A similar symmetry can be used to obtain a 3×3 generalization of eq. (2). Consider,

$$(L'_e, L'_\mu) \to i(L'_\mu, L'_e) \; ; \qquad (L'_\tau, e'_R, \mu'_R, \tau'_R) \to -i(L'_\tau, \mu'_R, e'_R, \tau'_R) \tag{20}$$

Here, L'_α and (e'_R, μ'_R, τ'_R) denote the leptonic doublets and singlets respectively. All other fields are assumed neutral with respect to the above symmetry. The neutrino mass matrix invariant under this symmetry has the following form:

$$M_\nu(M_X) = \begin{pmatrix} a & 0 & b \\ 0 & -a & b \\ b & b & 0 \end{pmatrix} \tag{21}$$

This mass matrix satisfies condition, eq.(9) and hence leads to a degenerate pair of neutrinos with mass $m = \sqrt{a^2 + 2b^2}$ at the tree level. It is not in the basis with diagonal charged leptons. The symmetry in eq.(20) allows the following general form for the charged lepton masses:

$$M_l = \begin{pmatrix} m_1 & m_2 & m_3 \\ -m_2 & -m_1 & -m_3 \\ m_4 & m_4 & m_5 \end{pmatrix} \tag{22}$$

Here m_i are parameters arising in the standard way through Yukawa couplings of leptons to the Higgs field. The M_l can be diagonalized in the standard manner. Eq.(22) leads to

$$M_l M_l^\dagger = \begin{pmatrix} A & B & C \\ B & A & -C \\ C & -C & D \end{pmatrix} \tag{23}$$

The parameters in the above matrix can be read off from eq.(22). One finds,

$$U^l M_l M_l^\dagger U^{l\dagger} = Diag.(m_e^2, m_\mu^2, m_\tau^2) \tag{24}$$

With

$$U^l = R_{23}(\phi) R_{12}(\pi/4) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-c_\phi}{\sqrt{2}} & \frac{c_\phi}{\sqrt{2}} & s_\phi \\ \frac{s_\phi}{\sqrt{2}} & \frac{-s_\phi}{\sqrt{2}} & c_\phi \end{pmatrix} \tag{25}$$

where $c_\phi = \cos\phi$; $s_\phi = \sin\phi$ and

$$\tan 2\phi = \frac{2\sqrt{2}C}{D - A + B}$$

While fairly large ranges can be allowed for the parameters m_i, we will make the following choice for illustrative purpose:

$$m_1 \sim m_2 \sim \frac{1}{2} m_e \; ; \qquad m_3 \sim O(m_\mu) \; ; \qquad m_5 \sim O(m_\tau) \tag{26}$$

It can be seen that this choice reproduces the charged lepton masses correctly. Moreover, with this choice,

$$\tan 2\phi \sim O(\frac{m_\mu}{m_\tau}) \tag{27}$$

The neutrino mass matrix in eq.(21) is diagonalized by

$$U_\nu = \begin{pmatrix} \cos^2\frac{\theta}{2} & \sin^2\frac{\theta}{2} & \frac{1}{\sqrt{2}}\sin\theta \\ \sin^2\frac{\theta}{2} & \cos^2\frac{\theta}{2} & -\frac{1}{\sqrt{2}}\sin\theta \\ -\frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{2}}\sin\theta & \cos\theta \end{pmatrix} \tag{28}$$

with $\tan\theta \equiv \frac{\sqrt{2}b}{a}$.

In spite of the complicated form for U_ν, the KM matrix $K \equiv U^l U^{\nu T}$ has a bi-maximal form given in eq.(17). This form coupled with the masslessness of the third generation ensures that the degenerate pair does not split radiatively. This is not surprising since the charged lepton mass matrix, eq.(22) is also invariant under the symmetry of eq.(20) which was responsible for the degeneracy of neutrinos. We need to break this symmetry in order to obtain splitting of the degenerate pairs. One possible breaking is as follows:

$$\tilde{m}(\bar{e'}_L e'_R + \bar{\mu'}_L \mu'_R) \tag{29}$$

Such a term can arise from Yukawa couplings with the standard Higgs in which case it breaks the symmetry explicitly. This type of hard breaking advocated in several papers [4] can make the theory non-renormalizable. This is avoided by Yukawa couplings of an additional Higgs field which is odd under the discrete symmetry. The discreet symmetry would then be spontaneously broken. Alternatively, we can work with the hard breaking in (29) but assume the model to be embedded in the supersymmetric theory which would not make the model non-renormalizable. We shall choose the latter alternative in what follows.

The inclusion of the term in eq.(29), would be expected to split the degeneracy between the neutrinos. This can be seen after some algebra. Eq.(23) is now replaced by

$$M_l M_l^\dagger = \begin{pmatrix} A' + \epsilon_1 & B & C + \epsilon_2 \\ B & A' - \epsilon_1 & -C + \epsilon_2 \\ C + \epsilon_2 & -C + \epsilon_2 & D \end{pmatrix} \tag{30}$$

with $A' = A + \tilde{m}^2$, $\epsilon_1 = 2m_1\tilde{m}$ and $\epsilon_2 = \tilde{m}m_4$. The above matrix can be diagonalized by the following U^l if one assumes

$$\tan\theta_{13} \approx \frac{\epsilon_1 \sin\phi' + \sqrt{2}\epsilon_2 \cos\phi'}{m_\tau^2} \ll 1$$

$$U^l = R_{12}(\theta_{12}) R_{23}(\phi') R_{12}(\pi/4) \tag{31}$$

where

$$\begin{aligned} \tan 2\phi' &\approx \frac{2\sqrt{2}C'}{D - A' + B'}; \\ \tan 2\theta_{12} &\approx \frac{2(\epsilon_1 \cos\phi' - \sqrt{2}\epsilon_2 \sin\phi')}{B(1+\cos^2\phi') + A'\sin^2\phi' + \sqrt{2}C'\sin 2\phi' - D\sin^2\phi'} \end{aligned} \tag{32}$$

The KM matrix following from eqs.(28,31) is given by

$$K(M_X) = R_{12}(\theta_{12}) R_{23}(\theta_{23}) R_{12}(\pi/4) \tag{33}$$

$\theta_{23} = \phi' + \theta$ in the above equation. The mixing angle θ_{12} introduced by the symmetry breaking parameter $\tilde{m}$ has two important effects. It causes a departure from the exact bi-maximal mixing obtained in the symmetric limit and it leads to splitting among the degenerate neutrinos radiatively. Both these features go in the right direction in solving the solar neutrino problem.

In order to evaluate this splitting using eq.(3), one needs to express the neutrino mass matrix at M_X in the charged lepton mass basis. This can be done using eqs.(33),

$$M_\nu(M_X) = m \begin{pmatrix} -c_{23}\sin 2\theta_{12} & -c_{23}\cos 2\theta_{12} & s_{23}c_{12} \\ -c_{23}\cos 2\theta_{12} & c_{23}\sin 2\theta_{12} & -s_{23}s_{12} \\ s_{23}c_{12} & -s_{23}s_{12} & 0 \end{pmatrix} \tag{34}$$

where $c_{23} \equiv \cos(\phi' + \theta)$; $s_{23} \equiv \sin(\phi' + \theta)$. In the absence of the symmetry breaking term $\tilde{m}$, $\theta_{12} = 0$ and the above matrix has a $L_e - L_\mu - L_\tau$ symmetry. Its evolution using eq.(3) does not lead to any splitting. With non-zero θ_{12}, one finds using eq.(3)

$$\Delta_S \equiv m_{\nu_2}^2 - m_{\nu_1}^2 \approx -4\delta_\mu m^2 c_{23} \sin 2\theta_{12} \tag{35}$$

Where δ_μ is radiative correction defined in eq.(4). We have neglected the electron Yukawa couplings in the above equation. Note that due to the specific texture of eq.(34), the splitting is determined by the μ Yukawa couplings rather than the more dominant tau coupling. With $\Delta_A \equiv m^2$, one obtains,

$$\frac{\Delta_S}{\Delta_A} \sim -\frac{2.7 \;\; 10^{-7}\,\mathrm{eV}^2 c_{23} \sin 2\theta_{12}}{\cos^2\beta} \tag{36}$$

The evolution from M_X to the low scale also affects the leptonic mixing. This can be determined after evolving eq.(34) to the low scale. One finds that if the leading terms corresponding to only τ Yukawa couplings are kept then the mixing matrix at low scale μ remains formally the same as eq.(33):

$$K(\mu) = R_{12}(\theta_{12}) R_{23}(\theta_{23}(\mu)) R_{12}(\pi/4) \tag{37}$$

where

$$\tan\theta_{23}(\mu) = \tan\theta_{23}(1+\delta_\tau)\ .$$

It follows that the RG evolution does not change the mixing matrix appreciably compared to its form eq.(33) at a high scale M_X.

Phenomenological implications of eqs.(36,37) depend upon the value of θ_{12} which is determined by the symmetry breaking parameter $\tilde{m}$. We take it as a free parameter and consider two interesting extremes corresponding to very small and large θ_{12} respectively. For small θ_{12}, the leptonic mixing matrix eq.(37) has a nearly bi-maximal form. The solar mass scale in eq.(36) could lie in the range corresponding to vacuum oscillation solution. This happens for moderately large value of $\tan\beta$ which is determined by θ_{12}. More specifically, one needs $\frac{\sin 2\theta_{12}}{\cos^2\beta} \sim 10^{-1}$ in order to obtain the solar scale around $\Delta_S \approx 10^{-11}\,\mathrm{eV}^2$.

Recent observations [11] of neutrino energy spectrum and the day night asymmetry are found to be less favourable for the vacuum and small angle MSW (SAMSW) solutions respectively. In contrast, the LAMSW solutions including the one with the low Δ_S are found to be allowed. One can obtain this LOW MSW solution for somewhat larger value of θ_{12} in our case. The mixing matrix (37) departs from purely bi maximal mixing in this case:

$$K(\mu) = \begin{pmatrix} \frac{c_{12}-s_{12}c_{23}^\mu}{\sqrt{2}} & \frac{c_{12}+s_{12}c_{23}^\mu}{\sqrt{2}} & s_{12}s_{23}^\mu \\ \frac{-s_{12}-c_{12}c_{23}^\mu}{\sqrt{2}} & \frac{-s_{12}+c_{12}c_{23}^\mu}{\sqrt{2}} & c_{12}s_{23}^\mu \\ \frac{s_{23}^\mu}{\sqrt{2}} & -\frac{s_{23}^\mu}{\sqrt{2}} & c_{23}^\mu \end{pmatrix} \tag{38}$$

where $c^\mu_{23} = \cos\theta_{23}(\mu); s^\mu_{23} = \sin\theta_{23}(\mu)$.

The above matrix has the correct form to simultaneously solve the solar and atmospheric neutrino anomalies without violating the constraint from CHOOZ. The latter constraint does not allow θ_{12} to be very large as it needs

$$K_{13}(\mu) = s_{12}s^\mu_{23} \leq 0.2$$

The data on atmospheric neutrinos require

$$\sin^2 2\theta_A = 4K^2_{\mu 3}(1 - K^2_{\mu 3}) = 4s^{\mu\,2}_{23}c^2_{12}(1 - c^2_{12}s^{\mu\,2}_{23}) \approx 0.84 - 1$$

The MSW LOW solution is obtained for

$$\begin{aligned} \Delta_S &\approx (7 - 20) \times 10^{-8}\,\text{eV}^2 \\ \sin^2 2\theta_S &\equiv 4K^2_{e1}K^2_{e2} = (c^2_{12} - s^2_{12}c^{\mu\,2}_{23})^2 \approx 0.68 - 0.98. \end{aligned} \tag{39}$$

Ranges in values of $\theta_{12}, \theta_{23}(\mu)$ and $\tan\beta$ exist which satisfy the last three equations simultaneously and lead to the solar scale in the range required for the MSW LOW solutions. e.g.

$$s_{12} \sim 0.28; \quad s^\mu_{23} = c^\mu_{23} \sim \frac{1}{\sqrt{2}}, \quad \tan\beta \sim 20$$

satisfy the CHOOZ constraint and imply

$$\sin^2 2\theta_S \sim 0.77; \quad \sin^2 2\theta_A \sim 0.99; \quad \Delta_S \sim 1.6 \times 10^{-7}\,\text{eV}^2$$

when $\Delta_A \sim 4 \times 10^{-3}\,\text{eV}^2$. Since the mass splitting in the model is governed by the μ Yukawa coupling, it is relatively small and one needs to have large $\tan\beta$ in order to obtain the scale relevant for the LOW solution.

4 Summary

Solution to the neutrino anomalies, specifically the solar neutrino deficits, may require scales as small as $\Delta_S \sim 10^{-7} - 10^{-11}\,\text{eV}^2$. It is interesting to suppose that such a scale is generated radiatively. The most economical possibility in this context would be to assume that the standard model interactions themselves are responsible for generating such a small mass difference. This requires specific textures for the neutrino mass matrix at high scale. We have discussed in this paper general conditions which ensure degeneracy of the masses even after radiative evolution at a low scale. It requires specific form, eq.(17) and massless or decoupled third neutrino.

We also argued that pseudo Dirac structure originally discussed by Wolfenstein can arise from a broken discrete symmetry and presented a specific example based on discrete symmetry in the context of three generations. This example provides a nice realization of the phenomenologically successful large angle solutions to the neutrino anomalies.

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References

[1] Different possibilities which simultaneously solve the solar and atmospheric neutrino anomalies are reviewed in A. Yu. Smirnov, Talk given at the International WEIN Symposium: A Conf. on Physics Beyond Standard Model (WEIN98), Santa Fe, June 1998 (hep-ph/9901208); S. M. Barr and I. Dorsner, hep-ph/0003058; Anjan S. Joshipura, Pramana, **54** (2000) 119.

[2] G. Dutta and A. S. Joshipura, *Phys. Rev.* **D51** (1995) 3838 (hep-ph/9405291); Y. Nir, Journal of High Energy Physics **0006** (2000) 039 (hep-ph/0002168); D. Chang and O. C. Kong, hep-ph/9912268; A. Geiser, hep-ph/9901433

[3] S. Doi et al, *Phys. Rev.* **D30** (1984) 626; Anjan S. Joshipura and S. D. Rindani, Euro. Phys. Journal, **C14** (2000) 85 (hep-ph/9811252); W. Grimus and H. Neufeld, hep-ph/9911465; S. Lavoura, hep-ph/0005321, T. Kitabayashi and M. Yasue, hep-ph/0006014

[4] E. Ma, hep-ph/9812344; *Phys. Rev. Lett.* **83** (1999) 2514 (hep-ph/9909249); R. Adhikari, E. Ma and G. Rajasekaran, hep-ph/0004197; E. J. Chun and S. K. Kang, hep-ph/9912524.

[5] L. Wolfenstein, *Nucl. Phys.* **B186** (1981) 147

[6] S. T. Petcov, *Phys. Lett.* **110B** (1982) 245; S. T. Petcov and C. N. Leung, *Phys. Lett.* **125B** (1983) 461

[7] K. S. Babu, C. N. Leung and J. Pantaleone, *Phys. Lett.* **B139** (1993) 191; P. H. Chankowski and Z. Pluciennik, *Phys. Lett.* **B316** (1993) 312; M. Tanimoto, *Phys. Lett.* **B360** (1995) 41; J. Ellis et al, *Eur. Phys. J.* **C9** (1999) 310; J. Ellis and S. Lola, *Phys. Lett.* **B458** (1999) 389; A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, *Nucl. Phys.* **B556** (1999) 3; JHEP **9909**(99) 015; *Nucl. Phys.* **B569**(2000) 82; K. R. S. Balaji et al, *Phys. Rev. Lett.* **84** (2000) 5034; *Phys. Lett.* **B481** (2000) 33; S. Lola, hep-ph/0005093.

[8] V. Barger, S. Pakvasa, T. J. Weiler and K. Whisnant, *Phys. Lett.* **B437** (1998) 107 (hep-ph/9806387); F. Vissani, hep-ph/9708483.

[9] R. Barbieri et al, *Phys. Lett.* **B445** (1999) 239; A. S. Joshipura, *Phys. Rev.* **D60** (1999) 053002; G. Altarelli and F. Fruglio, hep-ph/9905536; A. Ghosal, hep-ph/0004171, hep-ph/9905470; Q. Shafi and Z. Tavartkiladze, hep-ph/0002150; R. N. Mohapatra et al, Phys. Lett. **474** (2000) 355; R. Mohapatra and S. Nussinov, *Phys. Rev.* **D60** (1999) 013002 and *Phys. Lett.* **B441** (1998) 299; S. Davidson and S. F. King, *Phys. Lett.* **B445** (1998) 191; C. H. Albright and S. M. Barr, *Phys. Lett.* **B461** (1999) 218; A. S. Joshipura and S. D. Rindani, *Phys. Lett.* **B464** (1999) 239; M. Masahisa et al, hep-ph/0005147.

[10] R. Barbieri, G. Ross and A. Strumia, hep-ph/9906470

[11] Talk by Y. Suzuki at XIX Int. Conf. on Neutrino Physics and Astrophysics, http://ALUMNI.LAURENTIAN.CA/www/physics/nu2000.