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# HIDDEN VARIABLE THEORIES WITHOUT NON-LOCAL SIGNALLING AND THEIR EXPERIMENTAL TESTS

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## ABSTRACT

We formulate hidden variable theories obeying the local causality condition of absence of faster than light signals, a condition weaker than the Einstein-Bell locality condition. We propose experimental tests of such theories and show that quantum mechanics can be embedded in them.

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Considering two systems $S_1$ and $S_2$ which may have interacted in the past but are now spatially separated, Einstein asserted [1] "on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system $S_2$ is independent of what is done with the system $S_1$". Building on this principle Einstein, Podolsky and Rosen argued [2] that quantum mechanics cannot be a complete description of physical reality. Much later, the celebrated work of Bell [3] provided a mathematically precise formulation of Einstein's principle of local realism or local causality and showed that quantum mechanics was inconsistent with this principle. In spite of the philosophical attractiveness of this principle leading to Bell's inequalities [3], there is now mounting experimental evidence [4] which agree with quantum mechanics and disagree with Bell's inequalities. Therefore one may be allowed to examine the possibility that the physical world obeys a principle of local causality weaker than Einstein-Bell locality.

In this paper we redefine local causality to mean the absence of faster than light signals, and proceed to formulate it precisely in a hidden variable framework. It is worthwhile to note that Bell himself called such a possible redefinition of local causality the "human version" and granted that it was sensible [5]. Among our motivations to adopt it is the fact that relativistic quantum field theory violates Einstein-Bell locality but does not allow faster than light signalling [5,6].

Consider the Bohm-Aharonov version [7] of the EPR situation. A system of two particles (spin-$\frac{1}{2}$ particles, photons, etc.) is prepared so as to move in different directions (say left and right) towards two measuring devices with orientations or other parameters (of Stern-Gerlach magnets, photon analyzers, etc.) collectively denoted by a and b respectively. The devices measure variables A and B which, by their definition, can only take values $|A| \leq 1$, $|B| \leq 1$ (e.g., for a Stern-Gerlach apparatus the particle hitting one of the two detectors on the left may correspond to A = +1 and hitting the other may correspond to A = -1; for a photon-analyzer transmission may correspond to A = +1 and non-transmission to A = -1). Quantum mechanically, for a pair of spin-$\frac{1}{2}$ particles A may correspond to measuring $\vec{\sigma}_1 \cdot \vec{a}$ and B to $\vec{\sigma}_2 \cdot \vec{b}$ where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are Pauli spin matrices. In a hidden variable framework, let us assume that A and B are specified if the hidden variables of the system, of the left apparatus and of the right apparatus, and the orientations or other parameters a and b of the two apparatus are known, i.e., $A = A(\lambda,a,b)$, $B = B(\lambda,a,b)$ where each argument may consist of many parameters discrete or continuous. The expectation values of A,B and AB are then

$$\langle A \rangle = \int d\lambda \, \rho(\lambda,a,b) \, A(\lambda,a,b) \, , \qquad (1)$$

$$\langle B\rangle = \int d\lambda\ \rho(\lambda,a,b)\ B(\lambda,a,b), \tag{2}$$

$$\langle AB\rangle = \int d\lambda\ \rho(\lambda,a,b)\ A(\lambda,a,b)\ B(\lambda,a,b), \tag{3}$$

where $\lambda$ denotes collectively all the hidden variables, $\rho(\lambda,a,b)$ being the normalized non-negative probability distribution function for the hidden variables. Thus,

$$\rho(\lambda,a,b) \geqslant 0\ ,\ \int d\lambda\ \rho(\lambda,a,b) = 1\ . \tag{4}$$

Further, by definition

$$|A(\lambda,a,b)| \leq 1\ ,\ |B(\lambda,a,b)| \leq 1\ . \tag{5}$$

At this point, Einstein-Bell locality would require $A(\lambda,a,b)$ to be independent of the hidden variables and orientations b of the spatially separated apparatus measuring B, and similarly require $B(\lambda,a,b)$ to be independent of a and hidden variables of the apparatus measuring A, and $\rho(\lambda,a,b)$ to be independent of a and b.

Denoting the average of A over hidden variables of the apparatus measuring it by $\bar{A}$, and a similar average of B by $\bar{B}$, Bell would obtain

$$\langle AB\rangle_{Bell} = \int d\lambda_0\ \rho_0(\lambda_0)\ \bar{A}(\lambda_0,a)\ \bar{B}(\lambda_0,b), \tag{6}$$

where $\lambda_0$ now denote the hidden variables of the system only, and $|\bar{A}| < 1$, $|\bar{B}| < 1$. From this follow the Bell inequalities [3] and the generalized Bell inequalities [8] which constitute experimental tests of Einstein-Bell locality.

Here we do not require Einstein-Bell locality. Recall Bohr's position [9]: "Of course, there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of an influence on the very conditions which define the possible types of predictions regarding the future behaviour of the system". Accordingly, we allow $A(\lambda,a,b)$, $B(\lambda,a,b)$ and $\rho(\lambda,a,b)$ to depend on variables of the distant apparatus. We make, however, the following hypothesis of local causality (≡ absence of faster than light signals): "the expectation value $\langle A\rangle$ of a quantity A cannot depend on whether or not a quantity B referring to a space-like separated region is measured; in particular $\langle A\rangle$ cannot depend on variables of the apparatus measuring B". Clearly, the violation of this hypothesis would imply faster than light signals. We thus assume that

$$\langle A\rangle = A(a)\ ,\ \langle B\rangle = B(b). \tag{7}$$

This result would of course follow from Einstein-Bell locality but constitutes a much weaker hypothesis.

## Experimental tests

We have defined the class of hidden variable theories without non-local signalling. We show now that predictions of this class of theories can easily be tested experimentally if the expectation values $\langle A\rangle$, $\langle B\rangle$, and the polarization correlations $\langle AB\rangle \equiv P(a,b)$ are measured. From

$$P(a,b) \pm A(a) = \int d\lambda\, \rho(\lambda,a,b)\, A(\lambda,a,b)\left[B(\lambda,a,b) \pm 1\right], \tag{8}$$

using $\rho \geq 0$, $|A(\lambda,a,b)| \leq 1$, $B(\lambda,a,b) + 1 \geq 0$ and $B(\lambda,a,b)-1 \leq 0$, we obtain

$$\begin{aligned} |P(a,b) \pm A(a)| &\leq \int d\lambda\, \rho(\lambda,a,b)\left[1 \pm B(\lambda,a,b)\right] \\ &= 1 \pm B(b) \end{aligned} \tag{9}$$

In the last step we used the normalization condition for $\rho(\lambda,a,b)$ and the hypothesis $\langle B\rangle = B(b)$. Since the right-hand sides of the above pair of inequalities are independent of a, we obtain

$$\max_{(a)} |P(a,b) \pm A(a)| \leq 1 \pm B(b) \tag{10}$$

Analogously, we obtain

$$\max_{(b)} |P(a,b) \pm B(b)| \leq 1 \pm A(a) \tag{11}$$

We propose that the above two pairs of inequalities (10) and (11) be directly tested by experiments which are similar to those already used to test Bell's inequalities [4]. We shall then be testing hidden variable theories without non-local signalling.

## Embedding quantum mechanics

Consider the Bohm-Aharonov example [7] of a pair of spin-½ particles with quantum wave function

$$\psi = (\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2} \tag{12}$$

which yields the quantum mechanical answers for <A>, <B> and <AB>:

$$
\begin{aligned}
A(a) &= \langle \vec{\sigma}_1 \cdot \vec{a} \rangle = 0, \\
B(b) &= \langle \vec{\sigma}_2 \cdot \vec{b} \rangle = 0, \\
P(a,b) &= -\vec{a} \cdot \vec{b} \quad .
\end{aligned}
\tag{13}
$$

Substituting these quantum mechanical values in the inequalities (10) and (11) above we see that the inequalities are obeyed (in fact saturated) by quantum mechanics.

We may thus hope that quantum mechanics can be embedded in the class of hidden variable theories considered here. We proceed to show that this is indeed the case by explicitly exhibiting a model of hidden variables. We choose the hidden variables $\lambda$ to be represented by a unit vector $\vec{\lambda}$ from the origin which specifies the points on the surface of a unit sphere. We further require the probability measure for the hidden variables to be uniform over the surface of the unit sphere, i.e.,

$$\rho(\lambda)\, d\lambda = d\Omega_{\vec{\lambda}} \,/\, (4\pi) \quad .$$

With such a choice of hidden variables, Bell [3] already demonstrated that the quantum mechanical results can be reproduced if A is allowed to depend on $\vec{b}$ also, e.g., $A = \mathrm{sgn}\vec{\lambda}\cdot\vec{a}'$, $B = -\mathrm{sgn}\vec{\lambda}\cdot\vec{b}$, where $\vec{a}'$ is a unit vector obtained from $\vec{a}$ by rotation towards $\vec{b}$ until $(1-\vec{a}\cdot\vec{b}) = (2/\pi)\cos^{-1}\vec{a}'\cdot\vec{b}$. In order to show that the above unsymmetrical dependence of A and B on their arguments is not an essential feature we give here another example.

Draw the unit vectors $\hat{a}$ and $\hat{b}$ from the origin of the unit sphere and also the unit vector $\vec{c} = (\hat{a}+\hat{b})/|\hat{a}+\hat{b}|$. Let the two cones with $\vec{c}$ and $-\vec{c}$ as axes, and origin as apex be drawn such that the solid angle subtended by each of them at the origin is $\pi(1+\hat{a}\cdot\hat{b})$. This can be done by choosing the half-opening angle $\theta_0$ of the cones to be given by $2(1-\cos\theta_0) = 1+\hat{a}\cdot\hat{b}$. We choose

$$A(\hat{\lambda},\hat{a},\hat{b}) = +1\,, \quad B(\hat{\lambda},\hat{a},\hat{b}) = -1$$

provided $\vec{\lambda}$ lies inside the upper cone with axis $\vec{c}$, and

$$A(\hat{\lambda},\hat{a},\hat{b}) = -1, \quad B(\hat{\lambda},\hat{a},\hat{b}) = +1$$

when $\vec{\lambda}$ lies inside the lower cone with axis $-\vec{c}$.

The remaining region on the unit sphere outside the above two cones is divided into two equal regions such that each of them subtends a solid angle $\pi(1-\hat{a}\cdot\hat{b})$ at the origin. We choose

$$A(\hat{\lambda},\hat{a},\hat{b}) = B(\hat{\lambda},\hat{a},\hat{b}) = \mathrm{Sign}\,(\hat{\lambda}\cdot(\hat{a}\times\hat{b}))$$

when $\hat{\lambda}$ lies outside the two cones. It can be easily verified that in this explicit model we do reproduce the quantum mechanical results given by (13).

General proof

We give now a general proof that quantum mechanical predictions will obey our requirements (1) to (5) and (7) for a hidden variable theory without non-local signalling.

Let the two observables referring to measurements in spacelike separated regions be represented by the operators A and B which satisfy the operator inequalities $-1 \leq A \leq 1$, $-1 \leq B \leq 1$. In relativistic quantum theory we must have $[A,B] = 0$. Let the quantum mechanical state, coherent or incoherent, be represented by the density matrix $\rho$. In a basis $\{|n\rangle\}$ in which both A and B are diagonal, we obtain

$$\langle A\rangle = \mathrm{Tr}\,\rho A = \sum_n \rho_{nn} A_{nn}$$
$$\langle B\rangle = \mathrm{Tr}\,\rho B = \sum_n \rho_{nn} B_{nn}$$
$$\langle AB\rangle = \mathrm{Tr}\,\rho AB = \sum_n \rho_{nn} A_{nn} B_{nn} ,$$

with

$$\rho_{nn} \geq 0 , \quad \sum_n \rho_{nn} = 1 ,$$
$$|A_{nn}| \leq 1 , \quad |B_{nn}| \leq 1 ,$$

which are clearly of the form (1) to (5). In general $\rho_{nn}$, $A_{nn}$ and $B_{nn}$ may depend on a and b since the basis $|n\rangle$ may depend on these orientations. Finally, the locality condition (7) follows from the arguments of Ghirardi, Rimini and Weber [6] asserting that $\langle A\rangle$ cannot depend on whether or not B is previously measured. Briefly, on measurement of B, $\rho \rightarrow \Sigma_\beta P_\beta \rho P_\beta$, where $P_\beta$ are projectors to different eigenvalues $\beta$ of B; but we see easily that $\mathrm{Tr}(\Sigma_\beta P_\beta \rho P_\beta)A = \mathrm{Tr}\rho A$, since $[A,P_\beta] = 0$ and $\Sigma_\beta P_\beta^2 = \Sigma_\beta P_\beta = 1$.

Generalizations

We formulated theories with no faster than light signalling in the hidden variable framework in which Bell had formulated Einstein-Bell locality. However, Bell's hidden variable framework was substantially generalized later [10] and still led to Bell's inequalities. We feel that the inequalities (10) and (11) here derived from no non-local signalling in the hidden variable framework could actually be derived also in a larger framework. Their experimental failure would imply a failure not only of quantum mechanics but also of a larger class of theories which have no non-local signalling. The experimental tests should take appropriate account of analyzer and detector efficiencies [11] as in tests of Bell's inequalities. Notice the striking feature of the inequalities (10) and (11) proposed here: they predict constraints on polarization correlations P(a,b) in terms of A(a) which can be measured by the left apparatus alone and B(b) which can be measured by the right apparatus alone.

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