UCRHEP-T326
TIFR/TH/01-48
November 2001

Anomalous Neutrino Interaction, Muon g-2, and Atomic Parity Nonconservation

Ernest Ma[a] **and D. P. Roy**[b]

[a] *Physics Department, University of California, Riverside, California 92521, USA*

[b] *Tata Institute of Fundamental Research, Mumbai (Bombay) 400005, India*

Abstract

We propose a simple unified description of two recent precision measurements which suggest new physics beyond the Standard Model of particle interactions, i.e. the deviation of $\sin^2\theta_W$ in deep inelastic neutrino-nucleon scattering and that of the anomalous magnetic moment of the muon. Our proposal is also consistent with a third precision measurement, i.e. that of parity nonconservation in atomic Cesium, which agrees with the Standard Model.

The minimal Standard Model (SM) of particle interactions is consistent with all present experimental data with only a few possible exceptions. One such is a recent measurement [1] of the electroweak parameter $\sin^2\theta_W$ from ν_μ and $\bar{\nu}_\mu$ interactions with nucleons, which claims a three-standard-deviation departure from the SM prediction. Another is the measurement [2] of the anomalous magnetic moment of the muon, which originally claimed a value higher than the SM prediction by 2.6 standard deviations [3], but is now revised down to only 1.6σ after a theoretical sign error has been corrected [4]. A third important constraint comes from the measurement [5] of parity nonconservation in atomic Cesium, which was thought to be in disagreement with the SM, but subsequent improved theoretical calculations [6] have shown it to be in good agreement. In addition, the phenomonena of neutrino oscillations are now well-established [7, 8] which suggest strongly that neutrinos have mass and mix with one another.

In this paper we propose a simple unified description of all the above effects by extending the SM to include the gauge symmetry $L_\mu - L_\tau$ [9]. The relevance of this symmetry to the muon $g-2$ value and neutrino mass has been discussed by us in a previous paper [10, 11]. Here we focus on how it can also explain the NuTeV result [1] and its other possible experimental consequences.

Our model assumes the anomaly-free gauge symmetry $U(1)_X$ with gauge boson X which couples to $(\nu_\mu, \mu)_L$, μ_R with charge $+1$ and to $(\nu_\tau, \tau)_L$, τ_R with charge -1, but not to any other fermion. This means that it has the contribution

$$\Delta a_\mu = \frac{g_X^2 m_\mu^2}{12\pi^2 M_X^2} \tag{1}$$

to the muon anomalous magnetic moment. It also contributes to ν_μ and $\bar{\nu}_\mu$ interactions, but since X does not couple to quarks, the NuTeV result [1] is only affected if X mixes with the Z boson of the SM. This also applies to atomic parity nonconservation.

In our previous paper [10], we assume for simplicity that $X-Z$ mixing is zero by the imposition of an interchange symmetry in the Higgs sector, but we also mention that this symmetry cannot be maintained for the entire theory, so that a small deviation is to be expected. This small deviation (corresponding to a mixing angle of order 10^{-3}) turns out to be just what is needed to explain the NuTeV result, as shown below.

The Higgs sector of our model consists of three doublets: $\Phi = (\phi^+, \phi^0)$ with charge 0 and $\eta_{1,2} = (\eta^+_{1,2}, \eta^0_{1,2})$ with charge ± 1 under $U(1)_X$. The mass matrix spanning X and Z is then given by

$$\mathcal{M}^2_{XZ} = \begin{bmatrix} 2g_X^2(v_1^2 + v_2^2) & g_X g_Z (v_1^2 - v_2^2) \\ g_X g_Z (v_1^2 - v_2^2) & (g_Z^2/2)(v_0^2 + v_1^2 + v_2^2) \end{bmatrix}, \tag{2}$$

where $v_0 \equiv \langle \phi^0 \rangle$ and $v_{1,2}^2 \equiv \langle \eta^0_{1,2} \rangle$ with $v_0^2 + v_1^2 + v_2^2 = (2\sqrt{2} G_F)^{-1}$. Assuming that $v_1 \simeq v_2$ so that the $X - Z$ mixing is small, we then have

$$M_Z^2 \simeq \frac{1}{2} g_Z^2 (v_0^2 + 2v_1^2), \quad M_X^2 \simeq 4 g_X^2 v_1^2, \tag{3}$$

with the $X - Z$ mixing angle given by

$$\sin\theta \simeq \frac{g_X g_X (v_1^2 - v_2^2)}{M_X^2 - M_Z^2}. \tag{4}$$

The effective ν_μ and $\bar{\nu}_\mu$ interactions with quarks has the same structure as the SM, but the effective strength is changed from g_Z^2/M_Z^2 to

$$\begin{aligned} & g_Z^2 \left(\frac{\cos^2\theta}{M_Z^2} + \frac{\sin^2\theta}{M_X^2} \right) - 2 g_X g_Z \sin\theta\cos\theta \left(\frac{1}{M_Z^2} - \frac{1}{M_X^2} \right) \\ \simeq {} & \frac{g_Z^2}{M_Z^2} \left[1 + \frac{2g_X}{g_Z} \left(\frac{M_Z^2}{M_X^2} - 1 \right) \sin\theta \right] \equiv \frac{g_Z^2}{M_Z^2} \rho_\mu. \end{aligned} \tag{5}$$

Note that the factor of 2 in the $\sin\theta$ term comes from the fact that X couples to ν_μ with strength 1 whereas Z couples to ν_μ with strength 1/2 ($= I_3$).

In the NuTeV analysis, if $\rho_\mu = 1$ is assumed, then $\sin^2\theta_W = 0.2277 \pm 0.0013 \pm 0.0009$, which deviates from the SM prediction of 0.2227 ± 0.00037 by approximately 3σ. On the other hand, if a simultaneous fit to both ρ_μ and $\sin^2\theta_W$ is made, they obtain

$$\rho_\mu = 0.9983 \pm 0.0040, \quad \sin^2\theta_W = 0.2265 \pm 0.0031, \tag{6}$$

with a correlation coefficient of 0.85 between the two parameters. They then suggest that one but not both of them may be consistent with SM expectations. Here we choose to consider the deviation of the NuTeV result as being due to ρ_μ.

The NuTeV analysis also makes a two-parameter fit in terms of the isoscalar combinations of the effective neutral-current quark couplings, resulting in

$$(g_L^{eff})^2 = 0.3005 \pm 0.0014, \quad (g_R^{eff})^2 = 0.0310 \pm 0.0011, \tag{7}$$

with a negligibly small correlation coefficient, whereas the SM predictions are

$$(g_L^{eff})^2_{SM} = 0.3042, \quad (g_R^{eff})^2_{SM} = 0.0301. \tag{8}$$

Now if we take for example $\rho_\mu = 0.9962$, then the above two values become $(g_L^{eff})^2 = 0.3019$ and $(g_R^{eff})^2 = 0.0299$, placing them both within 1σ of the experimental measurements.

In atomic parity nonconservation, because X does not couple to electrons, we have

$$\rho_e = \cos^2\theta + \sin^2\theta \left(\frac{M_Z^2}{M_X^2}\right) \simeq 1 \tag{9}$$

to a very good approximation. Thus there should be no deviation from the SM, in agreement with experiment.

From Eq. (5) we obtain

$$\sin\theta = (\rho_\mu - 1)\left(\frac{g_Z}{2g_X}\right)\left(\frac{M_X^2}{M_Z^2 - M_X^2}\right), \tag{10}$$

which is of order 10^{-3} for $\rho_\mu = 0.9962$. This will affect precision data at the Z resonance in the following way. First, the observed resonance is of course the physical Z boson which has a small X component. However, since X does not couple to electrons, the production of Z is only suppressed by $\cos^2\theta$ which is indistinguishable from 1. The decay of Z to most fermions is also unaffected because the suppression factor is again just $\cos^2\theta$. The exceptions are $Z \to \mu^+\mu^-,\ \bar{\nu}_\mu\nu_\mu,\ \tau^+\tau^-,\ \bar{\nu}_\tau\nu_\tau$. Their effective couplings are

$$\mu \ : \ g_V = -\frac{1}{2} + 2\sin^2\theta_W - 2\left(\frac{g_X}{g_Z}\right)\sin\theta, \quad g_A = -\frac{1}{2}, \tag{11}$$

$$\nu_\mu \ : \ g_V = \frac{1}{2} - 2\left(\frac{g_X}{g_Z}\right)\sin\theta, \quad g_A = \frac{1}{2} - 2\left(\frac{g_X}{g_Z}\right)\sin\theta, \tag{12}$$

$$\tau \ : \ g_V = -\frac{1}{2} + 2\sin^2\theta_W + 2\left(\frac{g_X}{g_Z}\right)\sin\theta, \quad g_A = -\frac{1}{2}, \tag{13}$$

$$\nu_\tau \ : \ g_V = \frac{1}{2} + 2\left(\frac{g_X}{g_Z}\right)\sin\theta, \quad g_A = \frac{1}{2} + 2\left(\frac{g_X}{g_Z}\right)\sin\theta. \tag{14}$$

Precision measurements of Z couplings at LEP-I give [12]

$$g_V^\mu = -0.0359 \pm 0.0033, \quad g_V^\tau = -0.0366 \pm 0.0014, \tag{15}$$

where the smaller error on g_V^τ is due to the use of τ polarization along with the forward-backward asymmetry. Thus

$$g_V^\tau - g_V^\mu = 4(g_X/g_Z)\sin\theta = -0.0007 \pm 0.0036, \tag{16}$$

adding the two errors in quadrature. Consider now Eq. (10) with the more conservative choice

$$\rho_\mu = 0.9976 \tag{17}$$

which is within 1.6σ of the NuTeV measurement of $(g_L^{eff})^2$. Comparing it to Eq. (16), we then obtain the following 2σ bounds on M_X:

$$M_X < 72 \text{ GeV or } M_X > 178 \text{ GeV}. \tag{18}$$

A lower bound on M_X as a function of g_X is also available from LEP-I data on Z decay into the 4-muon final state via $Z \to \mu^+\mu^- X$ [10]. For example, if $g_X = 0.2$, then $M_X > 58$ GeV. Furthermore, Eq. (3) requires

$$g_X > \frac{g_Z M_X}{2M_Z}. \tag{19}$$

In Figure 1 we show the above lower limit on g_X as well as the 2σ upper limits on g_X as functions of M_X from $Z \to \mu^+\mu^- X$ decay and the difference of the $Z \to e^+e^-$ and $Z \to \mu^+\mu^-$ partial widths as the result of the X radiative contribution. Details are provided in Ref. [9]. The Z decay limit essentially rules out $M_X < 60$ GeV. The analogous process $e^+e^- \to \mu^+\mu^- X$ at LEP-II does not improve this bound, as already shown [10]. Thus we conclude that M_X between 60 and 72 GeV is still allowed, but perhaps $M_X > 178$ GeV is more likely.

Going back to Eq. (1) for the muon $g-2$ discrepancy, we note that there is a theoretical <u>lower</u> bound [10] of 1.56×10^{-9} in this model, whereas the corrected [4] range of the experimental discrepancy is $2.65 \pm 1.65 \times 10^{-9}$. This is entirely consistent with the low M_X solution, while in the case of the high M_X solution, the maximum deviation we get is 2.7×10^{-9}. In either case, the X boson signal will be too small to be observable at the Fermilab Tevatron, but will be clearly visible at the CERN LHC [10] via the associated production processes $u\bar{u}(d\bar{d}) \to \mu\mu X$ and $u\bar{d}(d\bar{u}) \to \mu\nu X$. At a future muon collider, X would be copiously produced, especially if it turns out to be light.

To obtain naturally small Majorana neutrino masses, we may add one heavy neutral fermion singlet N_R with $U(1)_X$ charge 0 as in our previous paper, but then an extra charged scalar boson ζ^+ with charge $+1$ is needed there to get a second neutrino mass term, i.e. $\nu_e\nu_\tau$, radiatively. A possible alternative is to add two N_R's. One is assumed to couple only to a linear combination of $(\nu_\mu\eta_2^0 - \mu_L\eta_2^+)$ and $(\nu_\tau\eta_1^0 - \tau_L\eta_1^+)$, and the other to $(\nu_e\phi^0 - e_L\phi^+)$ as well. Using the canonical seesaw mechanism [13], this structure allows for the appearance of two massive neutrinos: one is predominantly a mixture of ν_μ and ν_τ, the other is a linear combination of ν_e and the orthogonal $\nu_\mu - \nu_\tau$ mixture. This may then lead to a consistent pattern of neutrino masses and mixing for explaining the present atmospheric [7] and solar [8] neutrino data.

The interchange symmetry $\eta_1 \leftrightarrow \eta_2$ in the Higgs sector allows us to assume $v_1 = v_2$, but this cannot be maintained for the entire theory. If we try to extend this to the gauge sector, then $\mu \leftrightarrow \tau$ is implied. Hence $m_\mu \neq m_\tau$ in the Yukawa sector would break this symmetry. However, the size of this breaking is only of order $(m_\tau^2 - m_\mu^2)/v_0^2$ which is smaller than what we require for $\sin\theta$. In other words, $X - Z$ mixing of order 10^{-3} is a very reasonable value.

In conclusion we have shown in this paper how the gauge symmetry $L_\mu - L_\tau$ (as realized specifically by us in a previous paper [10]) explains naturally the recent NuTeV result [1] on the possible deviation from the Standard Model in ν_μ and $\bar{\nu}_\mu$ scattering with nucleons. Our proposal also explains the possible discrepancy in the recent measurement [2] of the anomalous magnetic moment of the muon. It further explains why there is no deviation from the Standard Model in atomic parity nonconservation [5]. Our model is constrained by the precision measurements of $Z \to \mu^+\mu^-$ and $Z \to \tau^+\tau^-$, from which we predict that the new gauge boson X is likely to have a mass between 60 and 72 GeV, or be heavier than 178 GeV. As such, our model is verifiable experimentally in the future at the LHC.

We thank S. N. Ganguli and A. Gurtu for discussions on the LEP data. This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

References

[1] G. P. Zeller *et al.*, NuTeV Collaboration, hep-ex/0110059.

[2] H. N. Brown *et al.*, Phys. Rev. Lett. **86**, 2227 (2001).

[3] This is subject to theoretical uncertainties in the calculation of the hadronic contributions. A. Czarnecki and W. J. Marciano, Phys. Rev. **D64**, 013014 (2001); J. Erler and M. Luo, Phys. Rev. Lett. **87**, 071804 (2001); F. J. Yndurain, hep-ph/0102312; S.

Narison, Phys. Lett. **B513**, 53 (2001); F. Jegerlehner, hep-ph/0104304; K. Melnikov, hep-ph/0105267.

[4] M. Knecht and A. Nyffeler, hep-ph/0111058; M. Knecht, A. Nyffeler, M. Perrottet, and E. de Rafael, hep-ph/0111059; M. Hayakawa and T. Kinoshita, hep-ph/0112102; I. Blokland, A. Czarnecki, and K. Melnikov, hep-ph/0112117.

[5] S. C. Bennett and C. E. Wieman, Phys. Rev. Lett. **82**, 2484 (1999).

[6] A. Derevianko, Phys. Rev. Lett. **85**, 1618 (2000); M. G. Kozlov, S. G. Porsev, and I. I. Tupitsyn, Phys. Rev. Lett. **86**, 3260 (2001); V. A. Dzuba, C. Harabati, W. R. Johnson, and M. S. Safronova, Phys. Rev. **A63**, 044103 (2001).

[7] S. Fukuda *et al.*, Super-Kamiokande Collaboration, Phys. Rev. Lett. **85**, 3999 (2000) and references therein.

[8] S. Fukuda *et al.*, Super-Kamiokande Collaboration, Phys. Rev. Lett. **86**, 5656 (2001) and references therein. See also Q. R. Ahmad *et al.*, SNO Collaboration, Phys. Rev. Lett. **87**, 071301 (2001).

[9] X. G. He, G. C. Joshi, H. Lew, and R. R. Volkas, Phys. Rev. **D43**, 22 (1991); **44**, 2118 (1991).

[10] E. Ma, D. P. Roy, and S. Roy, hep-ph/0110146 (Phys. Lett. **B**, in press).

[11] See also S. Baek, N. G. Deshpande, X.-G. He, and P. Ko, Phys. Rev. **D64**, 055006 (2001); S. N. Gninenko and N. V. Krasnikov, Phys. Lett. **B513**, 119 (2001).

[12] Particle Data Group, Eur. Phys. J. **C15**, 1 (2000).

[13] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida,

in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979), p. 95; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

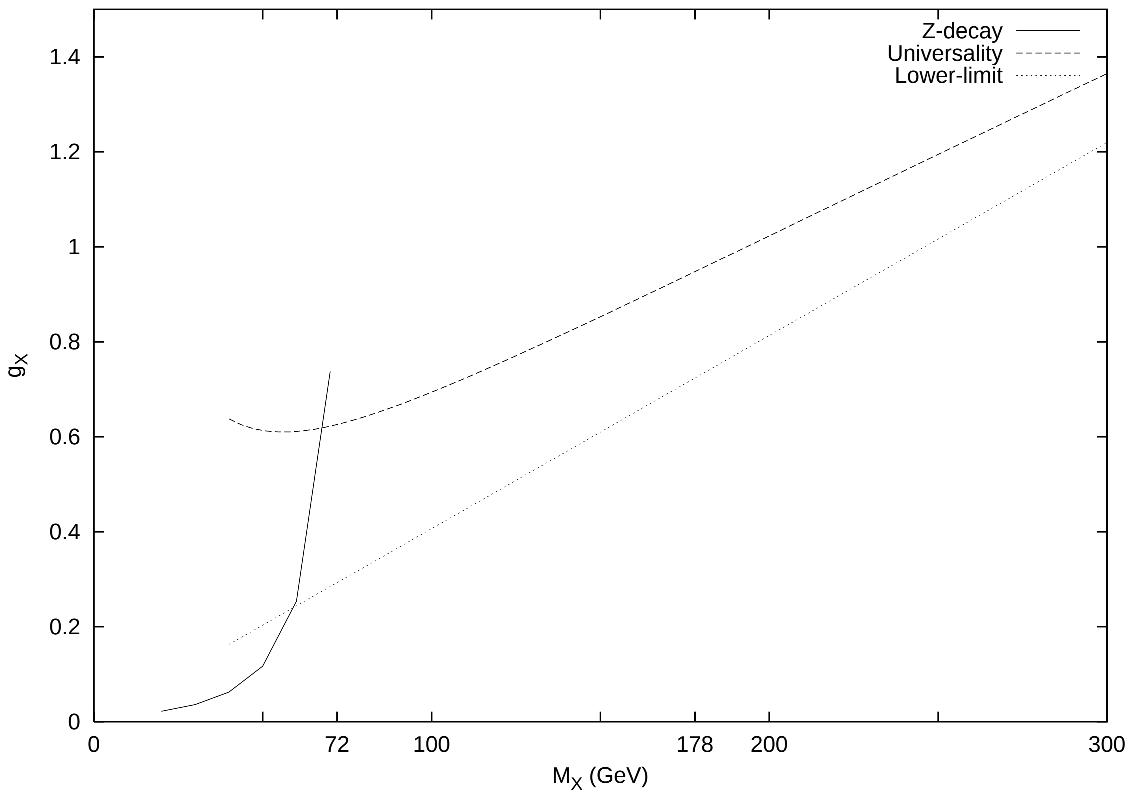

Figure 1: The predicted lower limit of the X boson coupling shown along with the LEP-I upper limits from $Z \to \mu^+\mu^- X$ decay and the universality relation between the $Z \to e^+e^-$ and $\mu^+\mu^-$ partial widths. The X mass ranges of interest to the NuTeV anomaly are $M_X = 60 - 72$ GeV or $M_X > 178$ GeV.