J. Astrophys. Astr. (1981) **2**, 289–307

A Doppler Theory of Quasars

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Received 1981 April 28; accepted 1981 July 16

Abstract. We examine a Doppler theory of quasars in which it is assumed that a fraction of the total population of quasars are fired from centres of explosion with moderate cosmological redshifts. It is argued that the substantial part of the redshift of a typical high redshift quasar could be of Doppler origin. If Hoyle's recent hypothesis that quasars emit the bulk of their radiation in a narrow backward cone is given a quantitative form, it is shown that the kinematic and emission parameters of this model can explain the observed features of the four aligned triplets of quasars discovered by Arp and Hazard (1980) and by Saslaw (personal communication). The model predicts a small but nonzero fraction of quasars with blueshifts. Further observational tests of the model are discussed.

***Key words*:** quasar alignments—non cosmological redshifts of quasars

1. Introduction

Doppler theories of quasars have been in the astronomical literature since the early days of the discovery of quasars. The first of these was proposed by Terrell (1964) who argued that quasars are small stellar mass objects ejected from our own Galaxy. On a somewhat larger scale, Hoyle and Burbidge (1966) suggested that quasars were ejected in violent explosions in nearby galaxies, citing NGC 5128 as a likely source. For many years Arp has been presenting evidence in support of the point of view that high redshift quasars are physically associated with low redshift galaxies (Arp 1966). Recently one of us (Narlikar and Das 1980) was concerned with explaining the existing data on quasar–galaxy associations in. terms of a cosmology of variable particle masses in which the quasars are considered as eject a from nearby galaxies.

We do not wish to enter into the debate on quasar redshifts; we simply wish to examine whether a Doppler model can be excluded on the, basis of current observations.

With this view we investigate here in some detail a variation on the theme proposed by Hoyle and Burbidge (1966)— a variation based on. Hoyle's recent hypothesis that quasars emit most of their radiation in a narrow *backward* cone as they travel through the intergalactic medium (Hoyle 1980).

In Section 2 we outline the essential features of Hoyle's hypothesis. Although Hoyle assumed that the entire redshift of the quasar is due to the Doppler effect we will suppose that a small component of the redshift may be due to the expansion of the universe. As we discuss in Sections 3 and 4, it then becomes possible to introduce additional inputs into the theory which can be tested by observations already available. In Section 5 we discuss limitations of this theory and outline ways in which it can be tested further.

2. Hoyle's hypothesis

The main objection to the Burbidge-Hoyle theory was that it predicted a preponderance of blueshifted, quasars over redshifted ones. According to a calculation made by Strittmatter (1967) if quasars are emitted isotropically from exploding nuclei of galaxies within a cosmologically nearby region (*i.e.* up to 10–100 Mpc), then in a flux limited survey the number of blueshifted quasars (N_b) is related to the number of redshifted quasars (N_r) by the formula

$$N_b/Nr = (1+Zm)^4, \qquad (1)$$

where z_m is the maximum redshift observed in the survey. This calculation makes the following assumptions.

(1) Each quasar emits radiation isotropically in its rest frame.
(2) Quasars are emitted with the same speed from their respective sources.
(3) All quasars have the same luminosity.
(4) The cosmological redshift of a source of explosion is zero.
(5) The sample is complete.

For a characteristic value of z_m = 2, equation (1) gives 81 times as many blueshifted quasars as the redshifted ones. The fact that *no* blueshifted quasars have been observed to date, effectively disposes of the theory.

Burbidge and Burbidge (1967) have discussed possible reasons for the non-observance of blueshifts. Among the various causes, they mention that blueshifts could be avoided if each quasar emits in backward direction with respect to its motion, so that a quasar moving towards us would be invisible and no blueshift would be seen from it.

Hoyle (1980) has calculated the precise angle of the backward cone within which a quasar must emit its radiation in order not to exhibit a blueshift to *any* observer. The angle, as Hoyle pointed out, does not depend on where the observer is located provided he is at rest relative to the intergalactic medium. The angle depends *only* on the speed V of the quasar relative to the intergalactic medium and is given by $2\theta_H$ where

$$\theta_{\mathrm{H}} = \cos^{-1}\left[\frac{1-(1-V^2)^{1/2}}{V}\right] \quad \text{(speed of light } c = 1\text{).} \tag{2}$$

We will refer to θ_{H} as the Hoyle-angle and the backward cone of semi-vertical angle θ_{H} with its axis along the line of motion as the Hoyle-cone.

The Hoyle-angle tends to zero as $V \rightarrow$ the speed of light. However, even for substantial relativistic speeds θ_{H} is fairly large. Thus for $V = 0{\cdot}8$, $\theta_{\mathrm{H}} = 60°$ and for $V = 0{\cdot}95$, $\theta_{\mathrm{H}} \simeq 43°$. Hence even for such high speeds the ejection does not have to be confined to a very narrow beam. Hoyle did not discuss the type of emission mechanism which would be confined in such a fashion, beyond pointing out that a quasar moving rapidly through the intergalactic medium may tend to pile up gas in the forward direction and leave a relatively rarefied region in the backward direction. Thus the opacity of gas in the forward direction will be high and in the backward direction low. While this argument supplies a qualitative basis for Hoyle's hypothesis, only a detailed investigation of relativistic plasma physics may tell us how, if at all, radiation is allowed to escape only in a specified backward cone. Before such an investigation could be undertaken it is desirable first to see whether observations provide any support for Hoyle's hypothesis.

The observational motivation which led Hoyle to resurrect the Doppler theory was provided by the discovery of two perfectly aligned triplets of quasars by Arp and Hazard (1980). These quasars designated (B, A, C) and (X, Y, Z) by Arp and Hazard lie in adjacent areas of the sky at $11^{\mathrm{h}}\,30^{\mathrm{m}} + 10°{\cdot}6$. In each triplet a bright central quasar is flanked by fainter ones of larger redshifts. The striking feature about each triplet is that the three quasars lie on a straight-line within the observational accuracy (claimed to be $\sim$1 arcsec). In addition to these triplets Saslaw (personal communication) found two more in another field studied by Arp and Hazard (1980) at $11^{\mathrm{h}}\,46^{\mathrm{m}} + 11°{\cdot}1$. These triplets also show similar characteristics as (B, A, C) and (X, Y, Z). We will refer to these four triplets by I, II, III and IV respectively.

Are these triplets cases of chance projection on the sky in our frame of reference? If the cosmological hypothesis is to survive this is the conclusion one must come to; for in each triplet the vastly different redshifts of the member quasars imply (according to this hypothesis) that they are at very, different distances from us. If the probability of chance projection is moderately large (say $\gtrsim$ 10 per cent) then such observations pose no threat to the cosmological hypothesis. If the probability is small (say $\lesssim$ 1 per cent) then we must conclude that the members of a triplet form a part of the same physical system. The problem of estimating this probability is not simple and it requires the specification of the selection procedure as well as an accurate knowledge of the quasar surface density at a given magnitude. We do not wish to enter into the arguments for and against the cosmological hypothesis based on probability estimates. We simply refer the reader to Hoyle (1980) and Arp and Hazard (1980) who have argued that the chance probability is low and to Edmunds and George (1981) who have argued that the probability is moderately large.

Hoyle took the view that the probability of chance alignment is low, and argued further that triplets like these present *prima facie* examples of ejection of the three quasars from a central explosion, in a straight-line Thus in Hoyle's theory the redshifts of A, B and C are entirely of Doppler origin and in the centre of mass frame A, B and C are moving in a straight-line. Would this alignment be preserved in the rest frame of any other observer? The answer is in the negative. As shown by one of us (Narlikar 1981) the angle between the lines joining the end quasars to the middle one is given by

$$\chi = \frac{n}{(1-f^2)^{1/2}} \frac{V \sin\theta}{(1-V^2)^{1/2}} \frac{\theta_{AB}+\theta_{AC}}{2}. \tag{3}$$

The notation in formula (3) is explained in Fig. 1. The above formula is valid for small χ. Although χ could in principle be as high as 90° in the most extreme case, the smallness of θ_{AB} and θ_{BC} make χ small. For the example worked out by Hoyle (personal communication) the angle χ is on the borderline of the alignment accuracy claimed by Arp and Hazard. Thus in. future more accurate positional measurements may provide a good observational test of Hoyle's theory.

In this paper we take a middle-of-the-road point of view. There is considerable observational evidence for a moderate cosmological redshift (z < 0·5) for at least some quasars *e.g.* 3C 273 (Stockton 1978, 1980; Wyckoff *et al.* 1980). We assume that there might be another population of quasars, ejected with large speeds from centres of explosion which have moderate cosmological redshifts. We consider whether such quasars may have a large Doppler component in their redshifts which may substantially exceed the redshifts of the centres of explosion. For a non-cosmological theory we expect the cosmological component of the redshift of any such ballistic quasar not to exceed $z \sim 1$ which is the limit to which galaxies are being observed today.

In a typical quasar triplet we will assume that the redshift of the middle quasar is wholly cosmological. This could happen either by the firing of the end quasars with large speeds and the middle one with a small speed in a single explosion, or by the middle quasar itself acting as a source of explosion. In either case the three quasars will appear well aligned to us provided they were so in the rest frame of the middle quasar. Such alignment-preserving ejections are not unknown in high energy astrophysics: the double radio sources appear to show the same pattern.

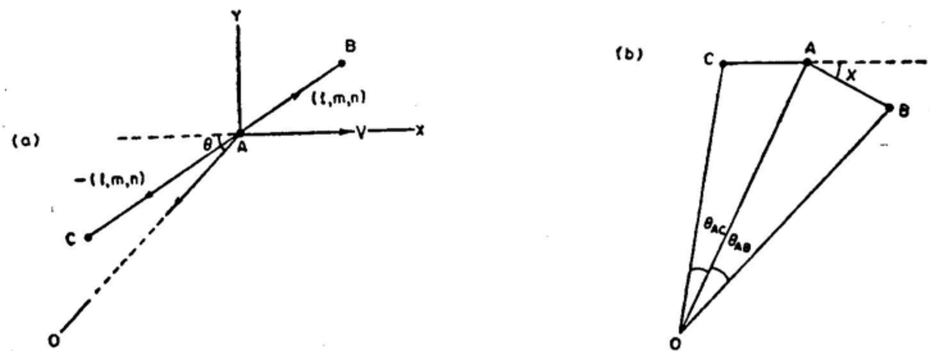

Figure 1. (a) The quasar A moves with speed V, relative to the observer O, in the direction shown by the arrow. θ is the angle measured by A between its backward direction of motion and, the direction AO along which it must emit radiation reaching O. Take rectangular axes AXYZ in the rest frame of A with AX along A's direction of motion, AY in the plane OAX and AZ (not shown) perpendicular to the plane OAX. ± (l, m, n) are the direction cosines of AB and AC. $F \equiv l \cos\theta + m \sin\theta$ is the projection of the unit vector (l, m, n) on the line OA.

(b) As seen from O the line CAB of (a) appears bent with an angle χ between the segments CA and AB. (Figure not drawn to scale).

This assumption enables us to calculate the kinematic parameters of each of the four triplets uniquely. We will outline the calculation in Section 3. The uniqueness of the solution enables us to test an emission function which gives a quantitative form to Hoyle's hypothesis that quasars emit their radiation preferentially backwards.

To avoid confusion with the notation of Arp and Hazard we will denote a triplet by (L, M, N), of which M is the middle quasar with wholly cosmological redshift.

3. The kinematics of triplets

We will assume that in a typical triplet (L, M, N) the end quasars L and N were fired at relativistic speeds in *opposite* directions from the central quasar M. We will denote the redshifts of L, M and N by z_L z_M and z_N respectively; z_M is cosmological in origin. Our calculations can also be applied to quasars ejected from galaxies although so far no linear alignment as good as those discussed in Section 2 is known in which the middle member is a galaxy. If, however, more than two quasars are ejected in a single explosion from M, as Arp has suggested, then linear alignments would not be expected.

We will assume that the cosmological component in the redshift of L or N is also z_M and define the non-cosmological components in their redshifts by Z_L and Z_N respectively where

$$1+Z_L=\frac{1+z_L}{1+z_M}, \qquad 1+Z_N=\frac{1+z_N}{1+z_M}. \tag{4}$$

To determine the kinematic parameters of the triplets we will assume that the large scale properties of spacetime are determined by the Robertston-Walker metric

$$ds^2=dt^2-S^2(t)\left[\frac{dr^2}{1-kr^2}+r^2\left(d\theta^2+\sin^2\theta\, d\phi^2\right)\right], \tag{5}$$

where k = 1, 0 or—1 and $S(t)$ is the scale factor of expansion.

Since the angular separation of L or N from M is small in a typical triplet ($< 10^{-2}$) we are able to make several simplifying assumptions in our calculations. To begin with, we can assume that L and N are moving in the locally inertial coordinate system in which M is at rest. Fig. 2 illustrates the geometry of the problem.

The circled region at M is where the locally inertial coordinates operate and where special relativity can be used. The passage of light from M to the observer O (*i.e.* ourselves) is across the curved spacetime of equation (5). Through M draw the line JMK perpendicular to MO. The quasars L and N are fired along a line making angle α with OM. The light ray from L to O crosses JK at a point E. In the rest frame of M, denoted by $\mathfrak{I}$, the line LE is parallel to MO because of the smallness of the angle MOL. Let V_L and V_N be the velocities with which L and N were fired from M, at the cosmic time $t = \bar{t}$. Suppose t_L and t_N are the times in the locally inertial frame when light signals left L and N respectively in order to arrive simultaneously at O at the cosmic time t_0.

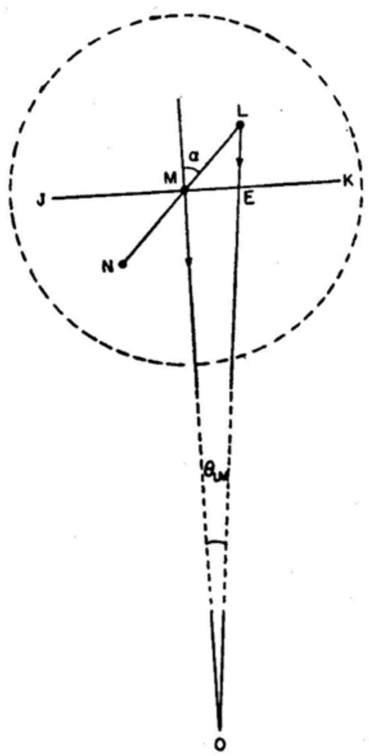

Figure 2. The circled region is local to the system LMN wherein the spacetime may be taken as flat. Cosmological effects arise when observations are made from O lying well outside the circle. (Figure not drawn to scale).

Let the Robertson-Walker space coordinates of M be $r = r_M$, $\theta = \theta_M$, $\phi = \phi_M$, the origin of the coordinates being at O. Suppose light leaving M at $t = t_M$ arrives at O at $t = t_0$ then

$$\int_0^{r_M} \frac{dr}{(1-kr^2)^{1/2}} = \int_{t_M}^{t_0} \frac{dt}{S(t)}. \tag{6}$$

The light tracks from L and N towards O are small variations on the light track from M to O. For motion or light propagation in the circled region it is a sufficiently good approximation to freeze $S(t)$ at a typical value, say $S(t_M)$.

In $\mathfrak{T}$, the distances of interest are

$$\mathrm{ML} = V_L(t_L - \bar{t}), \qquad \mathrm{ME} = V_L(t_L - \bar{t}) \sin \alpha,$$

$$\mathrm{LE} = V_L(t_L - \bar{t}) \cos \alpha. \tag{7}$$

In terms of the Robertson-Walker metric we have

$$\mathrm{LE} \simeq S(t_{\mathrm{M}}) \int_{r_{\mathrm{E}}}^{r_{\mathrm{L}}} \frac{dr}{(1-kr^2)^{1/2}} \simeq S(t_{\mathrm{M}}) \int_{r_{\mathrm{M}}}^{r_{\mathrm{L}}} \frac{dr}{(1-kr^2)^{1/2}}, \tag{8}$$

$$\mathrm{ME} \simeq r_{\mathrm{M}}\, S(t_{\mathrm{M}})\, \theta_{\mathrm{LM}} \tag{9}$$

Here θ_{LM} and θ_{NM} are the angles subtended by the segments LM and NM at O.

Light propagation from L to O can be split into two parts: first from L to E in the local region of M and then from E to O in the expanding universe

$$\int_0^{r_{\mathrm{L}}} \frac{dr}{(1-kr^2)^{1/2}} = \int_{t_{\mathrm{L}}}^{t_0} \frac{dt}{S(t)},$$

i.e.

$$\int_0^{r_{\mathrm{M}}} \frac{dr}{(1-kr^2)^{1/2}} + \frac{V_{\mathrm{L}}\,(t_{\mathrm{L}} - \bar{t}) \cos \alpha}{S(t_{\mathrm{M}})} = \int_{t_{\mathrm{M}}}^{t_0} \frac{dt}{S(t)} + \frac{t_{\mathrm{M}} - t_{\mathrm{L}}}{S(t_{\mathrm{M}})}\,. \tag{10}$$

In view of equation (6) we get

$$t_{\mathrm{M}} - t_{\mathrm{L}} = V_{\mathrm{L}}\,(t_{\mathrm{L}} - \bar{t}) \cos \alpha, \tag{11}$$

which can be rewritten in the form

$$(t_{\mathrm{L}} - \bar{t})\,(1 + V_{\mathrm{L}} \cos \alpha) = t_{\mathrm{M}} - \bar{t} \equiv T. \tag{12}$$

By symmetry we get a similar relation for quasar N by changing α to $\alpha + \pi$

$$(t_{\mathrm{N}} - \bar{t})\,(1 - V_{\mathrm{N}} \cos \alpha) = T. \tag{13}$$

A variation of the propagation relation for light from L to O gives us the redshift of L seen by O

$$1 + z_{\mathrm{L}} = \frac{1 + V_{\mathrm{L}} \cos \alpha}{(1 - V_{\mathrm{L}}^2)^{1/2}}\,(1 + z_{\mathrm{M}}) \tag{14}$$

and a similar equation is obtained for N.

Next, from equations (7), (9) and (12) we get

$$\theta_{LM} = \frac{V_L T \sin \alpha}{(1 + V_L \cos \alpha) r_M S(t_M)} \tag{15}$$

and a similar relation for θ_{NM}.

From the above we have three equations relating the observable quantities θ_{LM}, θ_{NM}, Z_L and Z_N to the three unknowns V_L, V_N and α

$$\frac{1 + V_L \cos \alpha}{(1 - V_L^2)^{1/2}} = 1 + Z_L, \tag{16}$$

$$\frac{1 - V_N \cos \alpha}{(1 - V_N^2)^{1/2}} = 1 + Z_N, \tag{17}$$

$$\frac{V_L (1 - V_N \cos \alpha)}{V_N (1 + V_L \cos \alpha)} = \frac{\theta_{LM}}{\theta_{NM}} \equiv k. \tag{18}$$

These equations can be solved with the help of a little algebra. It is convenient to define the known quantities

$$\lambda_L = (1 + k)^{-1} (1 + Z_L)^{-1}, \quad \lambda_N = (1 + k^{-1})^{-1} (1 + Z_N)^{-1}. \tag{19}$$

We then. have

$$V_L = \left[1 - \left(\frac{2\lambda_L}{1+\lambda_L^2 - \lambda_N^2}\right)^2\right]^{1/2}, \tag{20}$$

$$V_N = \left[1 - \left(\frac{2\lambda_N}{1+\lambda_N^2 - \lambda_L^2}\right)^2\right]^{1/2}, \tag{21}$$

and

$$\cos \alpha = \frac{(1 - k)/(1 + k) + \lambda_N^2 - \lambda_L^2}{\{[1-(\lambda_N+\lambda_L)^2]\,[1-(\lambda_N-\lambda_L)^2]\}^{1/2}}. \tag{22}$$

The angle made by the light ray leaving quasar L with the backward direction of motion of L as measured in the rest frame of L, is $\tilde{\alpha}_L$ where

$$\cos \tilde{\alpha}_L = \frac{\cos \alpha + V_L}{1 + V_L \cos \alpha}. \tag{23}$$

Similarly,

$$\cos \tilde{\alpha}_N = \frac{V_N - \cos \alpha}{1 - V_N \cos \alpha}. \tag{24}$$

The corresponding Hoyle-angles are $\tilde{\alpha}_{LH}$ and $\tilde{\alpha}_{NH}$ where

$$\cos \tilde{\alpha}_{LH} = \left[\frac{1 - (\lambda_L + \lambda_N)}{1 + (\lambda_L + \lambda_N)} \times \frac{1 - (\lambda_L - \lambda_N)}{1 + (\lambda_L - \lambda_N)}\right]^{1/2} \tag{25}$$

$$\cos \tilde{\alpha}_{NH} = \left[\frac{1 - (\lambda_N + \lambda_L)}{1 + (\lambda_N + \lambda_L)} \times \frac{1 - (\lambda_N - \lambda_L)}{1 + (\lambda_N - \lambda_L)}\right]^{1/2} \tag{26}$$

Table 1 gives the values of all the kinematic parameters for the four triplets mentioned in Section 2. (Of these, I and II are the triplets of Arp and Hazard and III and IV are Saslaw's triplets.)

Note that all end quasars are emitting within their Hoyle-cones, even though we have now added the complication of a cosmological redshift for the central quasar. This is hardly surprising, however, since our general calculation above gives

$$\cos \tilde{\alpha}_L - \cos \tilde{\alpha}_{LH} = \frac{1 - V_L^2}{V_L (1 + V_L \cos \alpha)} Z_L > 0, \tag{27}$$

$$\cos \tilde{\alpha}_N - \cos \tilde{\alpha}_{NH} = \frac{1 - V_N^2}{V_N (1 - V_N \cos \alpha)} Z_N > 0. \tag{28}$$

So long as the end quasars in a triplet have higher redshifts than the middle quasar, the property of emission within the Hoyle-cone will always hold. This property applies not only in the local special relativistic case considered by Hoyle but also in the 'mixed' case where the quasar has been fired from a source which itself has a significant cosmological redshift with respect to the observer.

It is necessary to make this distinction between Hoyle's purely local theory and the mixed theory being discussed here. In Hoyle's theory the intergalactic medium in

Table 1. The kinematic parameters of quasar triplets I, II, III and IV.

Triplet	Observed quantities					Computed quantities						
No.	z_M	z_L	z_N	$\theta_{LM}\times 10^3$	$\theta_{MN}\times 10^3$	α^0	V_L	$\tilde{\alpha}^0_{LH}$	$\tilde{\alpha}^0_L$	V_N	$\tilde{\alpha}^0_{NH}$	$\tilde{\alpha}^0_N$
I	0·51	2·15	1·72	1·66	3·08	73·8	0·80	59·8	27·9	0·91	50·1	32·4
II	0·54	2·12	1·61	0·83	2·08	66·6	0·76	62·3	27·0	0·93	47·5	32·8
III	1·01	2·22	1·93	3·64	3·58	88·8	0·77	61·8	38·6	0·74	63·9	43·3
IV	1·01	1·67	2·12	5·03	1·01	125·2	0·94	45·8	38·0	0·54	73·1	31·8

which the quasar moves is also the medium in which the observer is at rest. In the present theory, however, the intergalactic medium against which the quasars move has its standard of rest redshifted (cosmologically) relative to the observer. The Hoyle-cone for a quasar is therefore determined by the speed of the quasars relative to its local intergalactic medium and not relative to the intergalactic medium of the observer.

4. Constraints on quasar emission

We now try to test the hypothesis that quasars emit preferentially within their backward Hoyle-cone. Until a quasar emission mechanism has been worked out in detail (in the lines indicated at the end of Section 2, we do not have a precise theory to test. Nevertheless an empirical approach can still be useful, and we outline it here.

In this approach we postulate an *ad hoc* anisotropic emission pattern in the rest frame of a typical quasar and then test it against the data on the four triplets discussed above. To this end we first outline the emission pattern and then discuss how the relative brightness of the end quasars would appeal to a remote observer.

If a quasar were emitting isotropically, then its total luminosity $\mathscr{L}$ would be evenly distributed over the solid ankle 4π so that the luminosity $\mathscr{L}$ across a solid angle $d\,\Omega$ would be simply $\mathscr{L}\, d\,\Omega/4\pi$. In the case of anisotropic emission we have to modify this expression to

$$\mathscr{L}\, Q\,(\tilde{\theta})\, d\,\Omega/4\pi = (\mathscr{L}/4\pi) Q\,(\tilde{\theta}) \sin\tilde{\theta}\, d\tilde{\theta}\, d\phi, \tag{29}$$

where $\tilde{\theta}$ measures the angle with the *backward* direction of motion in the rest frame of the quasar. Although $Q\,(\tilde{\theta}) \neq$ constant indicates anisotropic emission, it still preserves axi-symmetry about the direction of motion.

We now give quantitative expression to Hoyle's hypothesis that a quasar emits preferentially within $\theta \leqslant \tilde{\theta}_{\mathrm{H}}$. It is of course possible to choose a function $Q\,(\tilde{\theta})$ which vanishes beyond this range. However, a very sharp cut-off is unlikely to arise in a physical theory of light propagation from a rapidly moving source, so we prefer an exponential form which would tend to damp out beyond $\tilde{\theta} = \theta_{\mathrm{H}}$. A form for $Q\,(\tilde{\theta})$ which naturally suggests itself is

$$Q\,(\tilde{\theta}) = Q_0 \exp\left[-\frac{(1-\cos\tilde{\theta})}{2n^2\,(1-\cos\tilde{\theta}_{\mathrm{H}})}\right], \tag{30}$$

where Q_0 and n are constants. The similarity of equation (30) with the Gaussian form is more apparent if we write it as

$$Q\,(\tilde{\theta}) = Q_0 \exp\left[-\frac{\sin^2(\tilde{\theta}/2)}{2n^2 \sin^2(\tilde{\theta}_{\mathrm{H}}/2)}\right]. \tag{31}$$

In a crude sense the above expression suggests that the damping at $\tilde{\theta} = \tilde{\theta}_H$ corresponds to an attenuation of the Gaussain profile at a distance σ/n from its mean. This comparison is, however, not exact since the apparent Gaussian form of equation (31) is with respect to sin $\tilde{\theta}/2$ and not with respect to $\tilde{\theta}$. Thus the damping for $\tilde{\theta} > \tilde{\theta}_H$ is not as rapid as it would have been in a Gaussian profile with respect to $\tilde{\theta}$.

The constant Q_0 is determined by integrating $Q\ (\tilde{\theta}\ )$ over all solid angles and requiring the integral to be unity. Assuming that n is large enough to ensure that

$$\exp\left[-\{n^2(1-\cos\tilde{\theta}_H)\}^{-1}\right] \ll 1 \tag{32}$$

we get

$$Q_0 = \frac{1}{4\pi n^2(1-\cos\tilde{\theta}_H)}\ . \tag{33}$$

Note that at this empirical level it is equally possible to postulate anisotropic emission which is preferentially in the *forward* direction. This would correspond to $n^2 < 0$. While the form favouring backward emission (equation 30) is

$$Q(\theta) = Q_B(\tilde{\theta}) = \frac{1}{4\pi n^2(1-\cos\tilde{\theta}_H)}\exp\left[-\frac{(1-\cos\tilde{\theta})}{2n^2(1-\cos\tilde{\theta}_H)}\right], \quad n^2 > 0\ , \tag{34}$$

a preferentially forward emission function would look like

$$Q(\theta) = Q_F(\tilde{\theta}) = \frac{1}{4\pi\,|n^2|\,(1-\cos\tilde{\theta}_H)}\exp\left[\frac{1+\cos\tilde{\theta}}{2n^2(1-\cos\tilde{\theta}_H)}\right], \quad n^2 < 0. \tag{35}$$

We will consider both Q_B and Q_F.

Our testing procedure is now as follows. We make the assumption that the two quasars L and N are ejected from M with equal luminosities and that their emission functions satisfy the form given by equation (34) with the same value of n^2. The Hoyle angles for the two quasars are however different, as are their speeds of ejection and redshifts. Their apparent brightness at O will not therefore be the same. Since the apparent magnitudes of all quasars in the four triplets are given we can use them to determine n^2. If the underlying assumptions of high velocity and backward emission are correct, we expect n^2 to be positive and small ($n^2 < 1$). If the quasars are emitting preferentially in the forward direction then n^2 should come out negative and equation (35) should provide the correct fit to the data.

A few kinematic corrections are needed before the emission functions can be compared directly with the data. Strittmatter (1967) has discussed, within the framework of a purely local quasar ejection theory how the flux densities are modified by the relativistic Doppler effect and aberration. His calculation has to be modified to include the effect on the flux densities by the expansion of the universe. The calculation is straightforward, though somewhat tedious. We quote the answer in the following form.

Suppose the emission function of quasar L in its rest frame has the form

$$f(\tilde{\alpha}_L, \nu) = \mathscr{L} Q(\tilde{\alpha}_L) J(\nu), \tag{36}$$

where $J(\nu)d\nu$ is the fraction of energy emitted in the frequency range $(\nu, \nu + d\nu)$. Thus

$$\int_0^\infty J(\nu)\, d\nu = 1. \tag{37}$$

Then the flux density at O in the frequency range $(\nu_0, \nu_0 + d\nu_0)$ is given by $S_L(\nu_0)\, d\nu_0$ where

$$S_L(\nu_0) = \frac{\mathscr{L}}{r_M^2 S_0^2} Q(\tilde{\alpha}_L) J[\nu_0(1 + z_L)] \frac{(1 + z_M)^2}{(1 + z_L)^3}. \tag{38}$$

Therefore we have

$$\frac{S_L(\nu_0)}{S_N(\nu_0)} = \frac{1 - \cos\tilde{\alpha}_{NH}}{1 - \cos\tilde{\alpha}_{LH}} \exp\left[\frac{-1}{2n^2}\left\{\frac{1 - \cos\tilde{\alpha}_L}{1 - \cos\tilde{\alpha}_{LH}} - \frac{1 - \cos\tilde{\alpha}_N}{1 - \cos\tilde{\alpha}_{NH}}\right\}\right]$$
$$\times \frac{J[\nu_0(1 + z_L)]}{J[\nu_0(1 + z_N)]} \frac{(1 + z_N)^3}{(1 + z_L)^3}. \tag{39}$$

To fix ideas we have taken

$$J(\nu) \propto \nu^{-\beta}, \quad 0 \leqslant \beta \leqslant 1 \tag{40}$$

in the relevant frequency of observation. Since the V_{5000} magnitudes of L and N are known from the work of Arp and Hazard for the four triplets, it is now a simple matter to compute n^2 from the formula (40). Table 2 gives the values of n^2 for the three cases $\beta = 0$, $\beta = 0{\cdot}5$ and $\beta = 1$ for each of the four triplets.

Table 2. Emission parameters of quasar triplets I, II, III and IV.

Triplet	Observed apparent magnitude			Attenuation parameter n^2		
No.	M	L	N	$\beta=0$	$\beta=0{\cdot}5$	$\beta=1$
I	17·6	19·4	19·0	0·27	0·23	0·20
II	16·9	19·0	19·9	0·075	0·075	0·07
III	19·5	18·9	18·9	0·16	0·135	0·115
IV	18·9	18·2	18·1	0·17	0·165	0·155

Note that the expectations of the model are borne out in all cases. We have $n^2 > 0$ and $n^2 < 1$. If ballistic, the quasars *are* preferentially emitting backwards within their Hoyle-cones. If we try to fit equation (35) to the data we do not get a similar consistent picture.

If we further assume that the quasar emission mechanism recognizes a universal value of n^2, then we can use the least square method to determine the 'best fit' value of n^2 for all four triplets taken together. Such an analysis assigns different weights to the different triplets depending on residual errors and leads to the value

$$n^2 \simeq 0{\cdot}125. \tag{41}$$

Recalling our interpretation of n from the pseudo-Gaussian form in equation (31) we note that the damping at the Hoyle-angle is as at $\sim 3\sigma$ away from the mean of $\sin(\theta/2)$.

It would be interesting to apply the above analysis to any other triplet that might be discovered in future and to see whether n^2 does vary around the value given by equation (41). In what follows we will use the value of 0·125.

5. The possibility of observing blueshifts

Since the model holds good so far, it is worth considering its further consequences. The most important of these is whether we are likely to observe any blueshifted quasars. In Section 1 we had mentioned that the original Burbidge-Hoyle ballistic model predicted an embarrassingly large preponderance of blueshifted quasars over the redshifted ones. The possibility of seeing blueshifted quasars can be entirely eliminated in Hoyle's new model by assuming that quasars do not emit at all outside their Hoyle-cone. However, we will consider the emission function of Section 4 in which radiation outside the Hoyle-cone is permitted but with damped intensity.

First we note that if a quasar L has been ejected out of a source M which itself has cosmological redshift z_M, then in order for us to see L as blueshifted, its Doppler blueshift must be larger than the redshift of M. More precisely, from equation (14) we must have

$$\frac{1 + V_L \cos\alpha}{(1 - V_L^2)^{1/2}}(1 + z_M) < 1. \tag{42}$$

In order to be seen blueshifted, L must therefore emit well outside its Hoyle-cone. The exponential damping may therefore render such a quasar too faint to be readily observable.

This raises an interesting possibility. Suppose a case where the factor multiplying $(1 + z_M)$ in the expression (42) is less than unity but not small enough to satisfy the inequality. In this case L may be seen with a redshift smaller than z_M. Now in the case of a triplet, α the direction of ejection could have any value between 0 and π. What is the probability that α lies in a small enough range so that

$$(1 + z_M)^{-1} < \frac{1 + V_L \cos\alpha}{(1 - V_L^2)^{1/2}} < 1\ ? \tag{43}$$

A simple calculation shows the answer to be

$$\rho(Z_{\mathrm{M}}, V_{\mathrm{L}}) = \frac{(1 - V_{\mathrm{L}}^2)^{1/2}}{2V_{\mathrm{L}}}\left[1 - \frac{1}{1 + Z_{\mathrm{M}}}\right]. \tag{44}$$

For the triplet I of Table 1, this value is $\sim$ 1/8. Similarly for quasar N of triplet I this value is $\sim$ 4/27. Since the cos α values in, the two ranges do not overlap the total probability that one or the other end quasar will have a lower redshift than the middle quasar is $\sim$ 0·28. However, if the triplet arose out of accidental projection of three quasars on a straight-line the probability that the middle quasar will have a larger redshift than at least one end quasar would be simply 2/3. Thus, provided a large number of photographic plates are examined for such triplets, the distribution of quasar redshifts will provide a test between this theory and the cosmological hypothesis. On the basis of this theory the expected proportion of cases with $z_{\mathrm{M}} > z_{\mathrm{L}}$ or $z_{\mathrm{M}} > z_{\mathrm{N}}$ should be significantly *less* than 2/3.

Since the observation of *net* blueshifts is easier for small z_{M}, let us first set $z_{\mathrm{M}} = 0$. We are then following Hoyle in considering quasars ejected in our local region of space. Hoyle and Burbidge (1966) had considered sources of quasars lying within a distance of $\lesssim$ 100 Mpc. Arp's claim of observations of anomalous redshifts usually relate to the presence of high redshift quasars near low redshift (NGC) galaxies. Thus setting $z_{\mathrm{M}} = 0$ will give a good approximation of such scenarios.

Suppose from such a source M a quasar L is fired in an arbitrary direction making angle α with the radial direction OM. Let V be the speed of the quasar L relative to M, and $\tilde{\alpha}$ denote the angle with the backward direction of motion of L made by the light ray from L to O, as measured in the rest frame of L. α and $\tilde{\alpha}$ are related by the equation

$$\cos\tilde{\alpha} = \frac{\cos\alpha + V}{1 + V\cos\alpha}, \tag{45}$$

and the redshift/blueshift factor $1 + z$ is given by

$$1 + z = \frac{1 + V\cos\alpha}{(1 - V^2)^{1/2}} = \frac{(1 - V^2)^{1/2}}{1 - V\cos\tilde{\alpha}}. \tag{46}$$

For blueshift, $z < 0$. The maximum redshift ($z_{\max}$) is obtained in the case $\alpha = 0$ and the maximum blueshift ($z_{\min}$) in the case $\alpha = \pi$. Let us compare the flux density $S(z)$ for different values of z. Thus if we fix z we know $\tilde{\alpha}$: from equation (46). Then from equations (38) and (40) we can compute $S(z)$. It is convenient to express $S(z)$ as the fraction of the flux level $S(z_{\max})$ corresponding to the maximum redshift. We get

$$\frac{S(z)}{S(z_{\max})} = \exp\left[-\frac{(1 - \cos\tilde{\alpha})}{2n^2(1 - \cos\tilde{\alpha}_{\mathrm{H}})}\right]\frac{(1 + z_{\max})^{3+\beta}}{(1 + z)^{3+\beta}}. \tag{47}$$

The above result can be expressed in the form

$$\frac{S(z)}{S(z_{\text{max}})} = \left(\frac{1 + z_{\text{max}}}{1 + z}\right)^{3+\beta} \exp\left[-\frac{(z_{\text{max}} - z)}{2n^2\, z_{\text{max}}\, (1 + z)}\right]. \tag{48}$$

The magnitude difference corresponding to equation (48) is plotted in Fig. 3 against $(1 + z)$, for $z_{\max} = 1, 2, 3$ (*i.e.* for $V = \cdot 6, \cdot 8, \cdot 88$), $\beta = 1$ for a range of values of n^2. It is easy to verify that $S(z)$ has a maximum at $\bar{z}$ given by

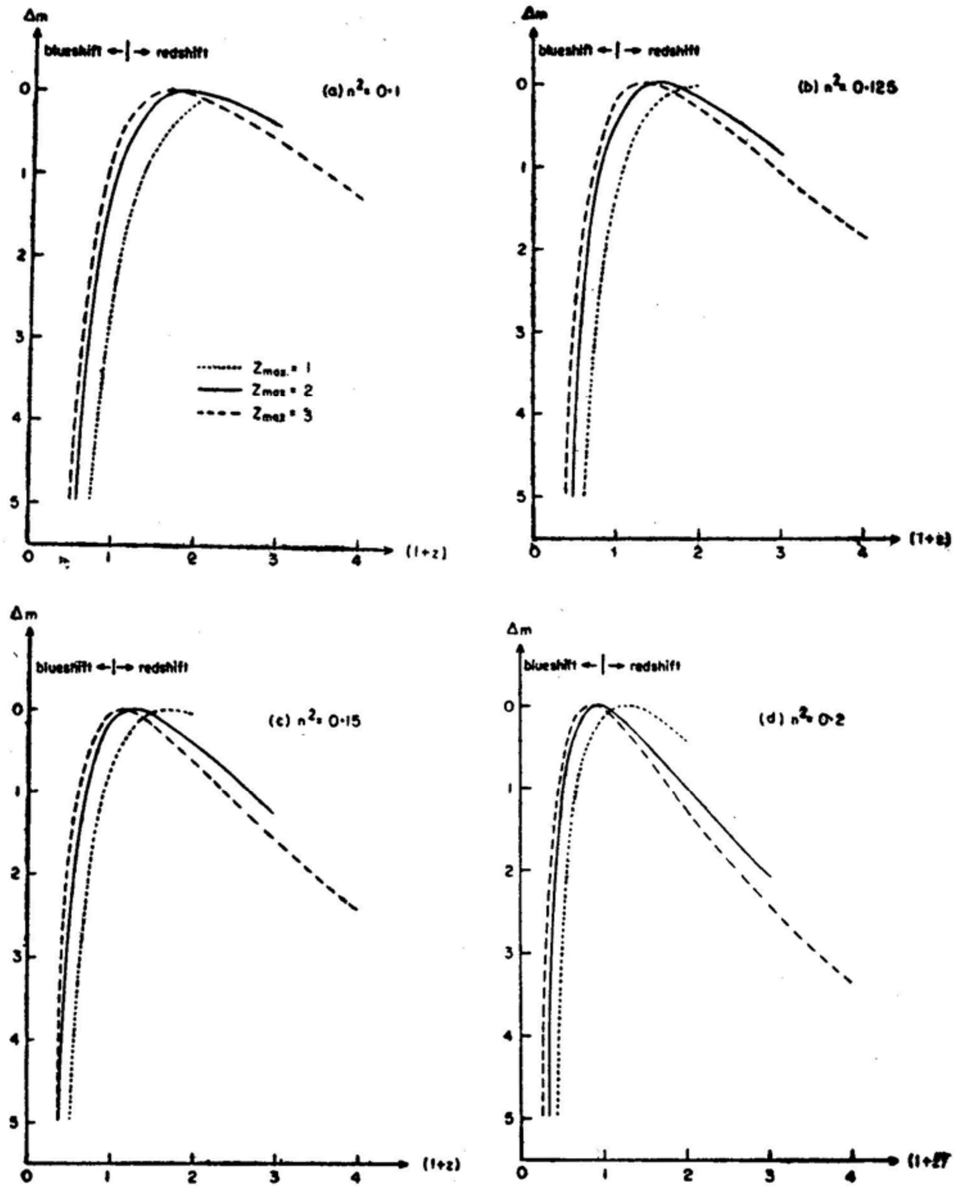

Figure 3. Redshift-apparent magnitude curves for different values of the parameters $z_{\max}$ and n^2. The zero magnitude for each curve is arbitrarily set at the maximum value of $S(z)$.

$$1+\bar{z}=\frac{1+z_{\max}}{2n^2(3+\beta)\,z_{\max}}. \tag{49}$$

For $\beta = 1$, the maximum flux is in the blueshift range ($\bar{z} < 0$) if

$$n^2 > \frac{1+z_{\max}}{8\,z_{\max}}.$$

Thus small values of n^2 make it easier to observe redshifted objects than blueshifted ones. For our canonical value of n^2 = 0·125, and for $z_{\max}$ = 2, $\bar{z}$ = 0·5; *i.e.* redshifted-quasars are generally brighter.

Note further that from equation (48)

$$\frac{S(z_{\max})}{S(\bar{z})}=\left[\frac{e}{2n^2(3+\beta)\,z_{\max}}\right]^{3+\beta}\exp\left[-\frac{1}{2n^2\,z_m}\right]. \tag{50}$$

Thus as the ejection speed is increased, the flux level at maximum redshift decreases in relation to the maximum flux level. We have therefore an. effect working against the observation of very large redshift quasars. We will return to this point in our final discussion.

If the quasar L is ejected randomly in any direction with a given velocity V, the probability that z lies in a range dz is

$$P(z)=\frac{dz}{z_{\max}-z_{\min}},\quad (z_{\min}\leqslant z\leqslant z_{\max}).$$

The probability of observing redshift is, however, higher than this because the life-time of a redshifted quasar is higher in the rest frame of O. The appropriate weighting factor is $(1 + z)$, as was pointed out by Strittmatter (1967).

If we consider Strittmatter's calculation, the ratio of the number of redshifted quasars to the number of blueshifted quasars is now modified to

$$\frac{N_r}{N_b}=\frac{\phi\,[1+z_{\max},1]}{\phi\,[1,1+z_{\min}]}, \tag{51}$$

where

$$\phi(z_1,z_2)=\int_{z_2}^{z_1}(1+z)^{-(3\cdot5+1\cdot5\beta)}\exp\left[-\frac{(z_{\max}-z)}{4n^2 z_{\max}(1+z)}\right]dz. \tag{52}$$

It is now easy to estimate the ratio N_b/N_r in a given flux limited sample. The ratio is given in Table 3, column 2 for various values of n^2. Table 3 also gives the ratio

Table 3. The ratio of blueshifted to redshifted quasars in a flux-limited quasar sample calculated for ejection speed $V = 0{\cdot}8$.

Attenuation parameter n^2	Maximum redshift of the ejecting source $z = 0$	$z = 0{\cdot}2$	$z = 0{\cdot}5$
0·1	0·008	0·005	0·003
0·125	0·03	0·02	0·01
0·15	0·08	0·04	0·02
0·20	0·27	0·145	0·07
0·25	0·58	0·32	0·15

Notes: (1) Zero redshift corresponds to local ejection of quasars.
(2) The ratios N_b/N_r for redshifts 0·2 and 0·5 have been computed for the Friedmann model with $q_0 = 0$.

N_b/N_r, for our mixed theory in which the ejection centre has a nonzero cosmological redshift z. The values of N_b/N_r given in each column represent those expected in a flux-limited sample with the maximum value of z given at the top of the column. We have used the empty Friedmann model ($q_0 = 0$) for making these estimates.

It is immediately clear that the ratio N_b/N_r is dramatically reduced from the high value of equation (1). Further, the larger the value of z and smaller the value of n^2 the smaller is the value of N_b/N_r. Thus the theory no longer predicts a preponderance of blueshifts.

However, so long as the radiation outside the Hoyle-cone is allowed, the possibility of observing blueshifted quasars exists. The calculations presented here therefore provide a potential test of this theory. The following points may be made.

(a) From equation (42) it is clear that relative blueshifts can be seen in triplets with $z_M > z_L$ or z_N. Arp (personal communication) has reported that he has found one such triplet.

(b) If z is very small say, $z \simeq 0$, then $N_b/N_r \simeq 0{\cdot}03$ for $n^2 = 0{\cdot}125$ (Table 3). Corresponding to a total of $\sim$ 1500 redshifted quasars listed by Hewitt and Burbidge (1980), we should be seeing $\sim$ 45 blueshifted quasars. This number drops to $\sim$ 30 for $z=0{\cdot}2$ and to $\sim$ 15 for $z = 0{\cdot}5$. On the other hand all these numbers increase if n^2 increases above the canonical value of 0·125. For example, for $n^2 = 0{\cdot}2$, the number of blueshifted quasars in a purely local theory ($z = 0$) should be as high as $\sim$ 400.

(c) The above estimates in (b) are reduced if we assume that a substantial part of the quasar population has only cosmological redshift. Thus if only a fraction $f(<1)$ of all quasars have large Doppler components then the numbers estimated in (b) must be multiplied by f. Although a small value of f may help in explaining why no blueshifted quasars are found, this alternative is not attractive if the theory is claimed to offer a serious alternative to the cosmological hypothesis.

(d) Although increasing z reduces N_b/N_r, there is a limit on how high z can be. This limit arises from the V/V_m test of radio quasars made by Wills and Lynds (1978). These authors find that the V/V_m test is consistent with an entirely local theory of quasars ($z = 0$). As z is increased the average value of V/V_m calculated for the quasars in the Wills and Lynds samples rises above 0·5, thus implying evolution. However, for $z \lesssim 0{\cdot}2$ the departure from 0·5 may be statistically insignificant. Taking $z = 0{\cdot}2$ we find from Table 3, N_b/N_r. = 0·02 for $n^2 = 0{\cdot}125$. Thus corresponding to the 226 quasars in Wills and Lynds analysis, we should be seeing ~5 blueshifted quasars.

(e) For calculations of Table 3 the velocity of ejection was taken as $V = 0{\cdot}8$. Larger values of V can lead to larger N_b/N_r. However, if V is large, the maximum redshift observed also increases. In a purely local theory, the least value of V which can generate a redshift of 3·5 is ~ 0·905. If, however, z can be as high as 0.5, $V \lesssim 0{\cdot}8$. Thus the paucity of redshifts higher than 3·5 sets limits on z and V. We have already seen in equation (50) how the very high redshift quasars become relatively faint and difficult to observe.

(f) Finally we consider the old question again: 'Why are no blueshifts seen?' Although this question can be answered by following Hoyle's hypothesis strictly and permitting no radiation beyond the Hoyle-cone, we find it more attractive to take the more vulnerable option of Section 4. Apart from the indirect evidence of the type mentioned in (a) it is tempting to speculate whether some of the lineless objects might not in fact be blueshifted quasars. The BL Lac objects listed in the Hewitt-Burbidge catalogue contain a large fraction for which no spectral lines are known. We end this section with the provocative conjecture that although we have no reason to believe that the line spectrum of blueshifted quasars would be radically different from that of redshifted ones, it remains possible that some of the lineless objects could be blueshifted.

6. Conclusion

Having applied the available data on aligned triplets of quasars to a Doppler theory of quasars we find that a self-consistent solution emerges. In a typical triplet, the middle quasar is assumed to have a wholly cosmological redshift while the end quasars have substantial Doppler redshift components. Hoyle's hypothesis that quasars emit predominantly backwards is quantified by the assumption of an exponential drop off in emission outside the Hoyle-cone. The parameters of the model can be fitted uniquely to the four triplets known.

We have investigated the implications of our model for the detection of blueshifted quasars. Unlike the original Doppler theory of Burbidge and Hoyle, the theory predicts only a small (but nonzero) number. of observable blueshifted quasars. If a Doppler theory is to survive, examples of at least a few of these blueshifted quasars must be found.

Acknowledgement

One of us (J. V. N.) thanks the Science Research Council for a Senior Visiting Fellowship and the Department of Applied Mathematics and Astronomy, University College, Cardiff for hospitality which made this work possible. We also thank Sir Fred Hoyle for critical comments.

References

Arp, H. 1966, *Science,* **151**, 1214.

Arp, H., Hazard, C. 1980, *Astrophys. J.,* **240**, 726.

Burbidge, G., Burbidge, M. 1967, *Quasi-Stellar Objects,* Freeman, San Francisco, p. 173.

Edmunds, M. G., George, G. H. 1981, *Nature,* **290**, 481.

Hewitt, A., Burbidge, G. 1980, *Astrophys. Suppl. Ser.,* **43**, 57.

Hoyle, F. 1980, *Preprint.*

Hoyle, F., Burbidge, G. R. 1966, *Astrophys. J.,* **144**, 534.

Narlikar, J. V. 1981, *Preprint.*

Narlikar, J. V., Das, P. K. 1980, *Astrophys. J.,* **240**, 401.

Stockton, A. 1978, *Astrophys. J.,* **223**, 747.

Stockton, A. 1980, in *IAU Symp. 92: Objects of High Redshift,* Eds G. O. Abell and P. J. E. Peebles, D. Reidel, Dordrecht, p. 89.

Strittrnatter, P. 1967 in *Quasi-Stellar Objects,* G. Burbidge and M. Burbidge, Freeman, San Francisco, p. 164.

Terrell, J. 1964, *Science,* **145**, 918.

Wills, D., Lynds, R. 1978, *Astrophys. J. Suppl. Ser.,* **36**, 317.

Wyckoff, S., Wehinger, P. A., Gehren, T., Morton, D. C., Boksenberg, A. Albrecht, R. 1980, *Astrophys. J.,* **242**, 159.