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Texture of a Four–Neutrino Mass Matrix

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Abstract

We propose a simple texture of the neutrino mass matrix with one sterile neutrino along with the three standard ones. It gives maximal mixing angles for $\nu_e \to \nu_S$ and $\nu_\mu \to \nu_\tau$ oscillations or vice versa. Thus with only four parameters, this mass matrix can explain the solar neutrino anomaly, atmospheric neutrino anomaly, LSND result and the hot dark matter of the universe, while satisfying all other Laboratory constraints. Depending on the choice of parameters, one can get the vacuum oscillation or the large angle MSW solution of the solar neutrino anomaly.

Recently the super-Kamiokande experiment has confirmed the atmospheric neutrino oscillation result, suggesting nearly maximal mixing of ν_μ with another species of neutrino [1]. The same experiment has also confirmed the solar neutrino oscillation result, which suggests mixing of ν_e with another species of neutrino [2]. Moreover, the energy spectrum of the recoil electron seems to favour the large mixing-angle vacuum oscillation of ν_e over the MSW solutions [2], although this may have limited statistical significance in the global fit to the solar neutrino data [3, 4]. They have led to a flurry of phenomenological models for neutrino mass and mixing which can account for these oscillations [5-8], most of which are focussed on the bi-maximal mixing angles for the atmospheric and the solar neutrinos. However, almost all of these works are based on the three-neutrino formalism, involving the standard left handed neutrinos ν_e, ν_μ and ν_τ [5].

On the other hand the inclusion of the LSND neutrino oscillation result [9] is known to require a fourth neutrino, which has to be a sterile one (ν_S) for consistency with the observed Z–width [10]. Moreover it requires either ν_μ or ν_e to oscillate into ν_S for explaining the atmospheric and solar neutrino anomalies, while requiring $\nu_\mu \to \nu_e$ oscillation for the LSND result. Thus the three-neutrino models for atmospheric and solar neutrino anomalies, based on a $\nu_e - \nu_\mu - \nu_\tau$ mixing, are in direct conflict with the LSND result. While the LSND result has not been corroborated by the preliminary KARMEN data [11], the statistical significance of the latter is limited by its lower sensitivity in the relevant region of parameter space. Indeed, with the standard statistical method the 90 % c.l. limit of KARMEN excludes only half the parameter space of the LSND data in the $\Delta m^2 \leq 2eV^2$ region [12]. Hopefully, this issue will be resolved by the proposed mini-BOONE experiment at Fermilab along with more data from KARMEN. It seems to us premature, however, to rule out the LSND result at present. Therefore we have tried to construct a four-neutrino mass matrix, which can account for the present solar and atmospheric neutrino data along with the LSND result. It can also account for the hot dark matter content of the universe [13], while satisfying all laboratory and astrophysical constraints [14, 15, 16].

Table 1 summarises the experimental constraints on neutrino mass and mixing parameters, which are relevant for our model. The large angle MSW and the vacuum oscillation solutions to the solar neutrino data [2, 17, 18] are taken from a recent fit to the ν_e suppression rates along with the recoil electron spectrum by Bahcall, Krastev and Smirnov [3]. For both the solutions the fit favours the oscillation of ν_e into a doublet neutrino over $\nu_e \to \nu_S$. The reason is that in the former case the NC scattering of this doublet neutrino

Table 1: Present experimental constraints on neutrino masses and mixing

Solar Neutrino [3]	:	$\Delta m^2 \sim (0.8-2)\times 10^{-5} eV^2$
(Large angle MSW)		$\sin^2 2\theta \sim 1$
Solar Neutrino [3]	:	$\Delta m^2 \sim (0.5-6)\times 10^{-10} eV^2$
(Vacuum oscillation)		$\sin^2 2\theta \sim 1$
Atmospheric Neutrino [1]	:	$\Delta m^2 \sim (0.5-6)\times 10^{-3} eV^2$
		$\sin^2 2\theta > 0.82$
LSND [9]	:	$\Delta m^2_{e\mu} \sim (0.4-2) eV^2$
		$\sin^2 2\theta_{e\mu} \sim 10^{-3} - 10^{-2}$
Hot Dark Matter [13]	:	$\sum_i m_i \sim (4-5) eV$
Neutrinoless Double Beta Decay [14]	:	$m_{\nu_e} < 0.46 eV$
CHOOZ [15]	:	$\Delta m^2_{eX} < 10^{-3} eV^2$
		(or $\sin^2 2\theta_{eX} < 0.2$)

with electron can partly account for the discrepancy between the observed suppression rates in super-Kamiokande and the Homestake experiments. On the other hand one can get acceptable solutions with $\nu_e \to \nu_S$ oscillation if one makes allowance for a 20 % normalisation uncertainty for the Homestake experiment. This will also enlarge the acceptable range of Δm^2. Therefore we shall consider both the oscillation scenarios $\nu_e \to \nu_S$ and $\nu_e \to \nu_\tau$ in our model. It should be added here that the best value of $\sin^2 2\theta$ for the large angle MSW solution is slightly less than 1; and even there the quality of fit is rather poor when all the experimental data are put together [3]. However one can get acceptable fit with the large angle MSW solution, including the $\sin^2 2\theta = 1$ boundary, if one makes reasonable allowance for the uncertainty in the Boron neutrino flux [3, 4]. Finally, the global fits [3, 4] have also found acceptable small angle MSW solutions for both these oscillation scenarios. But we do not consider them here, since the texture of our mass matrix naturally leads to bimaximal mixing as we shall see below.

The dark matter constraint on the sum of neutrino masses comes from a recent global fit to the spectrum of density perturbation in the universe using various cosmological models [13]. The best fit is obtained with a hot and cold dark matter model, where the former constitutes 20 % of the critical density. Besides there is an astrophysical upper bound on the number of neutrino species from nucleosynthesis, which allows 1 or atmost 2 sterile neutrinos

[19].

We shall consider a four-neutrino mass matrix for the three doublet neutrinos and a singlet (sterile) neutrino. We shall present two scenarios, where the solar and atmospheric neutrino anomalies are explained by the oscillations (A) $\nu_e \to \nu_S$ and $\nu_\mu \to \nu_\tau$ and (B) $\nu_e \to \nu_\tau$ and $\nu_\mu \to \nu_S$. The corresponding mass matrices will be related to one another by suitable permutation of neutrino indices. In each case maximal mixing between the oscillating neutrino pairs will be ensured by the texture of the mass matrix. Moreover we shall obtain the vacuum oscillation and the large angle MSW solutions in each case depending on the choice of parameters.

(A) $\nu_e \to \nu_S$ and $\nu_\mu \to \nu_\tau$ Oscillations :

In this case the texture of our neutrino mass matrix in the basis $[\nu_e \;\; \nu_\mu \;\; \nu_\tau \;\; \nu_S]$ is

$$m_\nu = \begin{pmatrix} 0 & 0 & a & d \\ 0 & c & b & 0 \\ a & b & 0 & 0 \\ d & 0 & 0 & 0 \end{pmatrix}. \tag{1}$$

Note that it has only 4 parameters. In comparison the earlier mass matrices considered had at least 5 parameters [20, 21, 22]. Moreover, the above mass matrix is minimal in the sense that it has only one diagonal element. The mass matrix of [20] has effectively 4 parameters in the case of maximal vacuum oscillation solution of the solar neutrino. However it contains two equal diagonal elements, which could in general be different from one another. It is clear from the mass matrix that the neutrinoless double beta decay vanishes because $\sum_i U_{ei}^2 m_i = m_{ee} = 0$; so that the corresponding constraint [14] is automatically satisfied.

For the 3×3 submatrix of doublet neutrinos, we shall assume the hierarchy

$$b \gg a, c. \tag{2}$$

Consequently the ν_μ and ν_τ will form a nearly degenerate pair with maximal mixing and small mass squared difference ($\sim 2bc$) to explain the atmospheric neutrino anomaly. Moreover, the remaining eigenvalue of this 3×3 submatrix gets a tiny double see-saw contribution $2a^2c/b^2$, which will be much smaller than d over a wide range of the latter parameter. Consequently, the ν_e and ν_S will form a nearly degenerate pair with maximal mixing and small mass squared difference to explain the solar neutrino anomaly. The vacuum oscillation and the large angle MSW solutions will correspond to the choices $d < a, c$ and $d \sim b$ respectively.

I – Vacuum Oscillation Solution $(b >> a, c > d)$ **:** In this approximation the mass eigenvalues are given by,

$$
\begin{aligned}
m_1 &= d + \frac{a^2 c}{2b^2} \\
m_2 &= b + \frac{c}{2} + \frac{a^2}{2b} + \frac{c^2}{8b} - \frac{a^2 c}{2b^2} \\
m_3 &= -b + \frac{c}{2} - \frac{a^2}{2b} - \frac{c^2}{8b} - \frac{a^2 c}{2b^2} \\
m_4 &= -d + \frac{a^2 c}{2b^2}
\end{aligned}
\qquad (3)
$$

and the corresponding mass eigenstates ($\nu_i^T \equiv \{\nu_1 \quad \nu_2 \quad \nu_3 \quad \nu_4\}$) are related to the weak eigenstates ($\nu_\alpha^T \equiv \{\nu_e \quad \nu_\mu \quad \nu_\tau \quad \nu_S\}$) through the mixing matrix $U_{i\alpha}$ as,

$$
\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_S \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -s_1 & s_1 & \frac{1}{\sqrt{2}} \\ s_1 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & s_1 \\ s_2' & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -s_2' \\ -\frac{1}{\sqrt{2}} & s_2 & s_2 & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix} \qquad (4)
$$

where, $s_1 = \frac{a}{\sqrt{2}b}$ and we can neglect the terms s_2 and s_2', which are of the order of $\sim O(\frac{ac}{\sqrt{2}b^2}, \frac{ad}{b^2}) \sim 10^{-5}$ for our choice of parameters. For the given 4×4 mixing matrix $U_{i\alpha}$ the probability of two flavour oscillation is given by,

$$
P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right), \qquad (5)
$$

where, $\Delta m_{ij}^2 = m_i^2 - m_j^2$. For our mixing matrix the flavour oscillation in each case is dominated by one mixing angle which can be determined by comparing the expression (5) with the effective 2×2 flavour oscillation formula

$$
P_{\nu_\alpha \nu_\beta} = \sin^2 2\theta_{\alpha\beta} \ \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right). \qquad (6)
$$

Thus we get an expression for $\sin^2 2\theta_{\alpha\beta}$ in terms of parameters of the mixing matrix $U_{i\alpha}$.

For illustration we shall now present the solution for a specific set of parameters, *i.e.*, $a = 0.05eV, b = 1.5eV, c = 0.001eV$ and $d = 0.0001eV$. There are two pairs of nearly degenerate eigenvalues

$$
\begin{aligned}
m_{\nu_1} \simeq -m_{\nu_4} &\simeq d = 0.0001eV \\
m_{\nu_2} \simeq -m_{\nu_3} &\simeq b = 1.5eV.
\end{aligned}
\qquad (7)
$$

The LSND experiment can be explained by the oscillations between states with mass squared difference of the order eV^2 which means that it can be explained by the oscillations between the $\nu_{1,4}$ and $\nu_{2,3}$ states. To explain LSND as an oscillation between the ν_e and ν_μ the effective mixing angle $\sin^2 2\theta_{e\mu}$ is obtained by comparing (5) and (6) and reading off the mixing matrix elements from (4),

$$
\begin{aligned}
sin^2 2\theta_{e\mu} &= -4\{U_{e1}U_{e2}U_{\mu1}U_{\mu2} + U_{e1}U_{e3}U_{\mu1}U_{\mu3} + U_{e4}U_{e2}U_{\mu4}U_{\mu2} + U_{e4}U_{e3}U_{\mu4}U_{\mu3}\} \\
&= -4 \times 4(-s_1^2)(\frac{1}{\sqrt{2}})^2 = \frac{4a^2}{b^2}. \qquad (8)
\end{aligned}
$$

Similarly the other masses and the relevant mass squared differences and the corresponding mixing angles for the experiments listed in Table 1 are given by,

$$
\begin{aligned}
\Delta m^2_{sol} = m^2_{\nu_1} - m^2_{\nu_4} &= \frac{2a^2cd}{b^2} = 2.2 \times 10^{-10} eV^2 \\
\sin^2 2\theta_{eS} &= 1 \\
\Delta m^2_{atm} = m^2_{\nu_2} - m^2_{\nu_3} &= 2bc = 0.003 eV^2 \\
\sin^2 2\theta_{\mu\tau} &= 1 \\
\Delta m^2_{LSND} = m^2_{\nu_1} - m^2_{\nu_2} &= b^2 - d^2 = 2.2 eV^2 \\
\sin^2 2\theta_{e\mu} &= 8s_1^2 = \frac{4a^2}{b^2} = 0.004 \\
m_{DM} = \sum_i |m_i| &= 3eV. \qquad (9)
\end{aligned}
$$

The $\nu_e \to \nu_S$ oscillation gives the vacuum oscillation solution to the solar neutrino anomaly, while the $\nu_\mu \to \nu_\tau$ oscillation explains the atmospheric neutrino anomaly. The LSND result is explained by $\nu_\mu \to \nu_e$ oscillation. The contribution to dark matter is 3 eV. The CHOOZ [15] and other laboratory constraints are satisfied.

Note that the above solution consists of two nearly degenerate pairs of maximally mixed neutrinos, separated by a realatively large mass squared gap. This is known to be the favoured mass configuration for satisfying the various laboratory constraints [20, 23]. The four model parameters are used to ensure that the three mass squared gaps correspond to the required values of Δm^2 for the solar, atmospheric and LSND neutrino oscillations, and $\theta_{e\mu}$ corresponds to the required mixing angle for LSND. The hot dark matter prediction comes out as a bonus. All these features are natural predictions of our mass matrix; and as such they will be shared by each of the alternative solutions discussed below. It should also be noted that the underlying double see-saw mechanism is responsible for generating mass squared gaps differing by 10 orders of magnitude starting with mass parameters, which differ by only 3–4 orders of magnitude (*i.e.*, similar to the case of the up type quark mass matrix).

II – Large Angle MSW Solution ($b > d >> a, c$) : In this approximation ($b \neq d$)the mass eigenstates are given by,

$$\begin{aligned}
m_1 &= d - \frac{a^2 d}{2(b^2-d^2)} + \frac{a^2 b^2 c}{2(b^2-d^2)^2} \\
m_2 &= b + \frac{c}{2} + \frac{c^2}{8b} + \frac{a^2 b}{2(b^2-d^2)} - \frac{a^2 b^2 c}{2(b^2-d^2)^2} \\
m_3 &= -b + \frac{c}{2} - \frac{c^2}{8b} - \frac{a^2 b}{2(b^2-d^2)} - \frac{a^2 b^2 c}{2(b^2-d^2)^2} \\
m_4 &= -d + \frac{a^2 d}{2(b^2-d^2)} + \frac{a^2 b^2 c}{2(b^2-d^2)^2} \qquad (10)
\end{aligned}$$

and the mixing matrix has the same form as (4), with $s_1 = \frac{a}{(b^2+d^2)^{1/2}}$, and $s_2 = s_2' = \frac{ab}{d(b^2+d^2)^{1/2}}$. Hence like before $\nu_e \to \nu_S$ mixing and the $\nu_\mu \to \nu_\tau$ mixing are maximal, where as the $\nu_e \to \nu_\mu$ mixing is given by the small parameter s_1. It may be added here that one gets a smooth numerical solution at $b = d$, although the approximation (10) breaks down there.

In this case let us consider a choice, $a = 0.025eV, b = 1.5eV, c = 0.0015eV$ and $d = 1.25eV$. Then the different masses and the relevant mass squared differences and the corresponding mixing angles are given by,

$$\begin{aligned}
m_{\nu_1} \simeq -m_{\nu_4} &\simeq d = 1.25eV \\
m_{\nu_2} \simeq -m_{\nu_3} &\simeq b = 1.5eV \\
\Delta m^2_{sol} = m^2_{\nu_1} - m^2_{\nu_4} &= \frac{2a^2 b^2 c d}{(b^2-d^2)^2} = 1.1 \times 10^{-5} eV^2 \\
\sin^2 2\theta_{eS} &= 1 \\
\Delta m^2_{atm} = m^2_{\nu_2} - m^2_{\nu_3} &= 2bc = 0.004 eV^2 \\
\sin^2 2\theta_{\mu\tau} &= 1 \\
\Delta m^2_{LSND} = m^2_{\nu_1} - m^2_{\nu_2} &= b^2 - d^2 = 0.69 eV^2 \\
\sin^2 2\theta_{e\mu} &= 8 s_1^2 = 8\frac{a^2}{(b^2+d^2)} = 1.3 \times 10^{-3} \\
m_{DM} = \sum_i |m_i| &= 5.5eV. \qquad (11)
\end{aligned}$$

The numerical values of the mass square differences have been calculated using the exact solutions of for the masses. The analytical expressions for the mass square differences are from the polynomial approximation (10). The numerical agreement for the mass square differences between the exact solutions and the polynomial approximation agrees upto 6 decimals in this range of parameters.

Thus the $\nu_e \rightarrow \nu_S$ oscillation provides the large angle MSW solution to the solar neutrino problem. The $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_e$ oscillations explain the atmospheric neutrino anomaly and the LSND result respectively as before. The contribution to dark matter is 5 eV. It may be added here that with these parameters $s_1 < s_2$; so the effective mass of ν_e is slightly lower than that of ν_S.

(B) $\nu_e \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_S$ Oscillations :

In this case we can use the same mass matrix as before if we make the following change of basis,

$$(\nu_e \;\; \nu_\mu \;\; \nu_\tau \;\; \nu_S) \longrightarrow (\nu_e \;\; \nu_\mu \;\; \nu_S \;\; \nu_\tau). \tag{12}$$

Note that with this change of basis the diagonal element for the sterile neutrino m_{SS} continues to remain zero, which is an important condition as we shall see below. Moreover the change of basis does not affect the mass eigenvalues m_1, m_2, m_3 and m_4. But now the nearly degenerate pair m_1 and m_4 represent the two maximally mixed states of ν_e and ν_τ, while m_2 and m_3 represent similar admixtures of ν_μ and ν_S. Thus the solutions (9) and (11) will continue to hold with the exchange of the neutrino flavour indices τ and S in θ_{eS} and $\theta_{\mu\tau}$. Consequently they will represent the vacuum oscillation and large angle MSW solutions to the solar neutrino anomaly via $\nu_e \rightarrow \nu_\tau$ oscillation, while the atmospheric neutrino anomaly is explained via $\nu_\mu \rightarrow \nu_S$ oscillation. The rest of the results remain the same as before.

Let us briefly discuss the possible mechanisms underlying the above mass matrix. Consider first the 3×3 submatrix corresponding to the three left-handed doublet neutrinos. This sub-eV scale mass matrix could arise from the standard see-saw mechanism with three heavy right-handed singlet neutrinos. Alternatively, one can get it without any right-handed neutrino but with an expanded higgs sector via a radiative mechanism [24] or Majorana coupling of the left-handed neutrino pairs to a heavy Higgs triplet [25]. In both cases one can naturally obtain a sub-eV scale mass matrix. The extension of the mass matrix to include a light singlet neutrino has been tried recently in each of the above three models [6, 7, 8]. In the standard see-saw model it is assumed to be one of the right-handed singlets while one adds a singlet neutrino in the other two models. In each case one has to impose a zero Majorana mass for this singlet, as otherwise it will naturally assume a high mass value. This is the reason why we have set m_{SS} to zero in our mass matrix (1). In the standard see-saw model this has been done by assuming a singular Majorana mass matrix for the singlet neutrinos, so that one of the eigenvalues (m_{SS}) is zero [8]. In the other two models the m_{SS} is made to vanish by imposing an additional symmetry [6, 7]. Finally one asks if this singlet neutrino can naturally have Dirac masses in the ≤ 1 eV scale? It seems possible to get it in the Zee model [6] and the triplet higgs model [7] via the same suppression mechanisms which keep the 3×3 doublet mass matrix in the sub-eV range. But in the case of the standard see-saw model it had to be put in by hand [8]. We feel it is important to look for a more natural way to keep the Dirac masses small in this model.

To summarize, we have presented a texture of four-neutrino mass matrix which automatically ensures bi-maximal mixing between $\nu_e \to \nu_S$ and $\nu_e \to \nu_\tau$ or vice versa. Thus with only four parameters it can account for the solar, atmospheric and LSND neutrino anomalies while remaining consistent with other experimental constraints. The prediction of the desired hot dark matter density comes out as bonus. Depending on the choice of parameters we can get both the vacuum oscillation and the large angle MSW solutions to the solar neutrino anomaly. Thanks to the underlying double see-saw mechanism, one can generate the desired mass squared gaps differing by 10 orders of magnitude starting with the four mass parameters which differ by only 3–4 orders of magnitude.

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