Forward-Backward Asymmetry in $B \rightarrow X_d e^+ e^-$

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Abstract

The Forward-backward asymmetry in the angular distribution of e^+e^- is studied in the processes $B \rightarrow X_d e^+ e^-$ and $\bar{B} \rightarrow \bar{X}_d e^+ e^-$. The possibility of observing CP- violation through the asymmetries in these two processes is examined.

The flavor changing neutral current transitions $B \rightarrow \ X_s e^+ e^-$ and $B \rightarrow X_d e^+ e^-$ offer a deeper proble for the weak interaction sector of the standard model since they go through second order weak interactions. The basic quark transition involved in these $b \rightarrow \ s$ and $b \rightarrow \ d$ occur through an intermediate t, c, or u quark. These processes can be described in terms of an effective Hamiltonian which incorporates both the results of short distance expansion techniques as well as the effect of virtual quark-antiquark pairs [1] . For $B \rightarrow X_s e^+ e^-$ the transitions involving intermediate top, charm and u-quarks enter respectively with factors $V_{tb} V^*_{ts},\ V_{cb} V^*_{cs}\ and\ V_{ub} V^*_{us}$. The last of these three is extremely small compared to the other two ; by the unitarity relation between the elements of the CKM matrix, the first two of these become effecively negative of each other. Thus the CKM factors effectively act as an overall factor with the result that there is very little chance of obtaining details of the CKM matrix, in particular its CP-violating phase from the study of this process. Krüger and Sehgal [2] have recently pointed out that the situation is quite different for the process $B \rightarrow \ X_s e^+ e^-$. There the three CKM factors, obtained from above by the replacement s→d , are comparable so that the cross section for the process will have significant interference terms, possibly opening up prospects of meaningful estimation of the complex CKM matrix elements. Krüger and Sehgal [2] have shown that the cross-section for the processes $B \rightarrow \ X_d e^+ e^- and\ \bar{B} \rightarrow \bar{X}_d e^+ e^-$ have differences depending on the value of the CKM matrix elements that may be experimentally significant in the near future.

For inclusive B-decays into lepton pairs, there is another asymmetry, namely the forward-backward (FB) asymmetry, introduced by Ali,Mannel and Morozumi [3],which again is another parameter which is likely to be very useful for comparison of theory with experimental data. For $B \rightarrow \ X_s e^+ e^-$ process, this parameter is again not very sensitive to the CKM matrix elements and furthermore tha magnitude of this parameter is the same for the $\bar{B}$ decay as for the B-decay. The observations made by Krüger and Sehgal[2] however show that we may expect that these parameters will be more sensitive in the inclusive B-decay into lepton pair with non-strange hadrons. In this brief note we obtain quantitative predictions for the FB asymmetry for the processes $B \rightarrow \ X_d\ e^+\ e^-$ and $\bar{B} \rightarrow \ \bar{X}_d\ e^+\ e^-$. The results as expected show considerably more dependence on the value of the CKM matrix elements and also brings out the possibility of observing the CP-violating phase of the CKM matrix elements in a mixture of equal numbers of B and $\bar{B}$ particles.

In the lowest order of the heavy quark effective theory, the process $B \rightarrow X_d\ e^+\ e^-$ can be equated to the QCD corrected matrix element for the process (with the p's and q's representing the on-shell momentum of the particles)

$$b\ (p_{\ b})\ \rightarrow\ d\ (p_d)\ +\ e^+\ (q_{\ 1}\ )\ +\ e^-\ (\ q_{\ 2})$$

The standard kinematical variables for this process are (with all dimensional quantities scaled to the b-quark mass):

$$q = \ q_1\ +\ q_2\ \ ;\ s\ = (\ q^2\ )\ ;\ u = 2\ p_b.\ (q_2\ - q_1)$$

$$u^2(\ s\ )\ =\ (s\ -\ \left(1\ + m_d^2\ \right))\ \left(\ s\ -\ \left(\ 1\ - m_d^2\ \right)\right)\ ;\ z\ =\ u\ /\ u(\ s\ )$$

z is the cosine of the angle between $\vec{q_1}$ and $\vec{p_b}$ in the rest frame of the lepton pair. In terms of these variables and the standard Wilson coefficients C_i 's,[1] the matrix element for the above process can be written as:

$$M\ (p_b, q_1, q_2\ ;\ \lambda\ ) = 2\ \sqrt{2}\ G_F\ V_{tb}\ V_{td}^*\ \ (\alpha\ /4\pi)\ F, \tag{1}$$

where

$$\begin{aligned} F \ &= \ C_9^{eff}\ (\bar{d}\ \gamma_\mu\ b_L)\ (\bar{e}\ \gamma_\mu\ e) + C_{10}\ (\bar{d}\ \gamma_\mu\ b_L)\ (\bar{e}\ \gamma^\mu\ \gamma^5\ e) \\ &- 2\ C_7\ [\bar{d}\ i\ \sigma_{\mu\nu}](\ m_b\ b_R\ +\ m_d\ b_L\ )\ (\ \bar{e}\ \gamma^\mu\ e\ ))(q^\nu\ /q^2) \end{aligned} \tag{2}$$

The constant C's are given by [1]

$$C_1 = -0.249,\ C_2 = 1.108,\ C_3 = 1.112X10^{-2}$$
$$C_4 = -2.569X10^{-2}\ ,\ C_5 = 7.404X10^{-3},\ C^6 = -3.144X10^{-2}$$
$$C_7 = -0.315,\ C_9 = 4.227;$$

C_9^{eff} is given by:

$$C_9^{eff}\ \ =\ \xi_1\ \ +\lambda\ \xi_2, \tag{3}$$

with

$$\begin{aligned} \xi_1 \ &= \ C_9 + g_c\ (\ 3C_1\ \ + C_2\ \ + 3C_3\ \ + C_4 +\ \ 3C_5\ C_6\ ) \\ &- 1/2\ g_d\ (C_3\ \ +\ \ 3C_4)\ \ -\ \ 1/2\ g_b\ (4C_3 +\ \ 4C_4\ \ + 3C_5 +\ \ \ C_6) \\ &+2\ (3C_3 +\ \ C_4 +\ \ 3C_5 +\ \ C_6)/9 \end{aligned} \tag{4}$$

$$\xi_2\ \ =\ \ (g_c\ \ - g_u\ )\ (3\ C_1\ \ +\ \ C_2\ )$$

where

$$\begin{aligned} g_q \ &= \ -8\ ln(m_q)/9\ \ + 8/27\ \ + 4\ y_q/9\ \ - (4 + 2y_q)\ /3 + \\ &+\sqrt{x}\ \theta(x)\ [\ ln(1 + x)\ \ - ln(1 - x)\ \ - i\pi] \\ &+\sqrt{-x}\theta(-x)[\ 2\ arctan(1/\sqrt{-x})], \end{aligned} \tag{5}$$

with $y_q\ \ = 4\ m_q^2\ /s\ ;\ x = 1 -\ \ y_q$. The parameter λ is the ratio $V_{ub}V_{ud}^*\ /V_{tb}V_{td}^*$ and can be expressed in terms of the Wolfenstein parameters as:

$$\lambda = \frac{\rho(1-\rho) - \eta^2\ \ - i\ \eta}{(1-\rho)^2\ \ + \eta^2} \tag{6}$$

Large distance effects can also be included in this scheme by adding to the expression for C_9^{eff} suitable Breit-Wigner forms corresponding to the $J/\psi,\ \psi'$ resonances. However as we will see, the region in which we will be interested in is below the resonance region and we will therefore neglect them. We shall also set the mass of the d-quark and the electron zero. With the matrix elemtent given as above, the differential cross-section for the process $b \rightarrow \ d + e^+ e^-$ can be worked out as

$$\begin{aligned}
\frac{d^2\sigma}{dz\ ds} &= C\ (1\ -\ s\ )\ \Big(\ (|C_9^{eff}|^2\ +\ C_{10}^2\ )\ \ [\ 2\ +2s+\ -\ 2z^2(1-s)\ ] \\
&\quad +8\ C_7^2\ [2\ -(\ 1\ -s\ )\ (1-\ z^2)\ ]\ /s \\
&\quad +8\ Re(C_9^{eff})\ [2\ C_7\ -s\ z\ C_{10}\ ] \\
&\quad -16\ C_{10}\ C_7\ z\Big) \qquad (7)
\end{aligned}$$

where C is an overall constant.
With this expression the normalized FB asymmetry A(s) defines as

$$\begin{aligned}
A(s) &= \frac{[\int_0^1\ -\int_{-1}^0]\ dz\ D(z,s)}{[\int_0^1\ +\int_{-1}^0]\ dz\ D(z,s)} \\
&= \frac{-3[\ s\ C_{10}\ reC_9^{eff}\ \ +2C_{10}\ C_7\ ]}{(1+2s)\ [\ |C_9^{eff}|^2\ \ +C_{10}^2\ ]\ \ +4C_7^2\ (2+1/s)\ \ +12\ C_7\ reC_9^{eff}} \qquad (8)
\end{aligned}$$

where $D(z.s)$ is the left hand side of equation (6).

The above expression refers to the transion $b(p_b) \rightarrow \ d\ e^+(q_1)e^-(q_2)$. By the CPT theorem, the matrix element for the process $\bar{b}(p_b) \rightarrow \ \bar{d}\ e^-(q_1)e^+(q_2)$ is given by $M(\ p_b,\ q_2,\ q_1\ ;\ \lambda^*\ )$. Thus but for the imaginary part part of λ ,equation (6), the FB asymmetry of the process $\bar{B} \rightarrow \ \bar{X}_d\ e^+\ e^-$ would be exactly the negative of B - decay. The difference in the magnitude of the B- and the $\bar{B}$ FB asymmetry would thus directly measure the CP violating phase of the CKM matrix.

Figures (1)-(3) show the calculated values of the two asymmetry parameters for three values of the parameter ρ in the experimentally allowed range for η =0.34. As can be seen , there is some dependence on the value of the parameter ρ. The study of the FB asymmetry in B-decays would thus be a useful confirmatory data in pinning down the value of CKM matrix elements.

The difference between the B and the $\bar{B}$ asymmetry is most pronounced, below the J/ψ threshold, as expected. This raises the possibility of measuring the asymmetry in a beam containing equal number of B and $\bar{B}$ particles; this would then be directly proportional to η. In figures (1)-(3), we have shown the values expected for a mixed system. In Table -I , we show also the value of this mixed asymmetry parameter averaged over a range of s from 0.05 to 0.35, well below the region of the resonances.

The magnitude of the FB asymmetry is of the same of the order as the CP-asymmetry in $B \rightarrow \ X_d\ e^+\ e^-$ and will be within observational range at future colliders. Improvement of statistics would perhaps also the make the asymmetry observable in a beam containing equal numbers of B and $\bar{B}$'s, which experimentally would not require any 'tagging' and is thus an interesting possibility .

Table 1: FB Asymmetry averaged between s=0.05 and s=0.35 for B, $\bar{B}$ and B+$\bar{B}$ system.

ρ	For B	For$B$	For B+B
0.30	0.086	-0.103	-0.014
-0.07	0.086	-0.094	-0.006
-0.30	0.080	-0.086	-0.005

I would like the International Centre for Theoretical Physics at Trieste where this work was done. I would also like to thank Professor L.M.Sehgal for going through the manuscript and for pointing out that very recently completed a similar investigation whose results match the ones quoted here.

References

[1] For an up-to-date review see G.Buchalla,A.J.Buras and M.E.Lautenbacher, Rev. Mod. Phys., 68,1125(1996)

[2] F.Krüger and L.M.Sehgal, Phys. Rev. D55,2799 (1997).

[3] A.Ali, T. Mannel and T. Morozumi, Phys. Lett., B 273, 505 (1991).

Figure 1: FB asymmetry for ρ=0.30

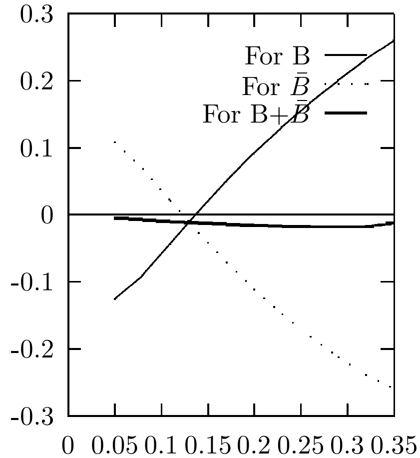

Figure 2: FB asymmetry for ρ=-0.07

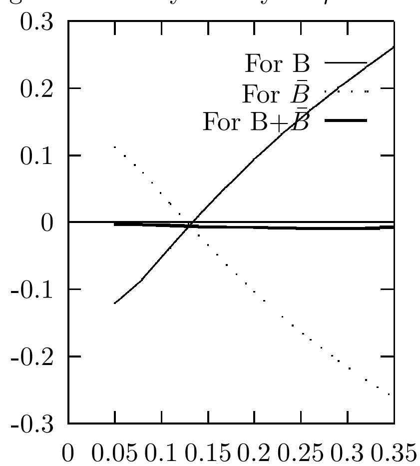

Figure 3: FB asymmetry for ρ=-0.30