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Astroparticle physics: Working group report

Coordinators: SUBHENDRA MOHANTY[1] and VARUN SAHNI[2]

Working Group Members: S Chakraborty[3], Ashok Goyal[4], Sukanta Dutta[5], Debajyoti Choudhury[6], Shobhit Mahajan[4], Amitabha Mukherjee[4], Hatem Widyan[4], Urjit Yajnik[7], S Mohanty[1], J A Grifols[8], Uma Mahanta[6], Sarira Sahu[1], Varun Sahni[2], Tarun Souradeep, Albert Stebbins[9], Ioav Waga[10], Vikram Soni[11], N Panchapakesan[4], R Brandenberger[12]

[1] Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India
[2] Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune 411 007, India
[3] Department of Physics, University of Kalyani, Kalyani, West Bengal 741 235, India
[4] Department of Physics and Astrophysics, University of Delhi, Delhi 110 007, India
[5] Department of Physics/Electronics, SGTB Khalsa College, University of Delhi, New Delhi 110 085, India
[6] Mehta Research Institute, Chhatnag Road, Jhusi, Allahabad 221 019, India
[7] Indian Institute of Technology, Mumbai 400 076, India
[8] Department de Fisica, Universitat Autonoma de Barcelona, Edifici C, 08193 Bellaterra, Spain
[9] Fermi Lab, Box 500, Batavia IL 60510, USA
[10] Universidade Federal do Rio de Janeiro, Instituto de Fisica, Ilha do Fundao, Blocoa, CEP21945-970, C.P. 68528, Brasil
[11] National Physical Laboratory, New Delhi 110 012, India
[12] Physics Department, Brown University, Providence, R1, USA

Abstract. The astroparticle physics working group witnessed intense discussion and activity covering a broad range of topics ranging from supergravity and baryogenesis to compact stars and the large scale structure of the Universe. A summary of some of the subject areas in which collaborations were initiated during WHEPP-5 is presented below.

Keywords. Cosmology; neutron stars; compact objects; supergravity; baryogenesis; large scale structure; cosmic microwave background.

1. Compact stars

S Chakraborty and A Goyal

The study of neutrino transport in dense hadronic/quark matter of astrophysical importance in presence of a strong magnetic field is an extremely interesting open problem. The effect of strong magnetic fields on neutrino opacities is also another open problem and has not been studied in detail. The pulsar observations give estimates of surface magnetic field of neutron stars $\sim 10^{13}$–10^{14}G. The field strength at the core region is expected to be much higher. For new born proto-neutron star it may go up to 10^{18}–10^{19}G. The presence of such strong magnetic field should affect the bulk properties, e.g. the equation of states of dense hadronic/quark matter and cooling of compact stellar objects. It should also modify significantly some of the gross properties of neutron stars/proto-neutron stars.

In the recent WHEPP-5, S Chakrabarty and A Goyal have undertaken a project to investigate the effect of strong magnetic fields of neutron stars with a dense quark matter core, on neutrino emissivity and transport of neutrinos in dense astrophysical matter. They have considered the quantum mechanical effect of strong magnetic field on the motion charged particles (protons, electrons and quarks). The preliminary calculations show that the results change significantly in presence of a strong magnetic field with respect to the field free case. The cooling rate of neutron star increases in presence of a strong magnetic field. In absence of magnetic field, the most important neutrino emission process in a neutron star is the modified URCA process. In presence of a strong magnetic field, the kinematical condition shows that the direct URCA process becomes the most significant one.

In a separate calculation S Chakrabarty showed that in presence of a strong magnetic field the emission time scale for neutrino from a neutron star modifies significantly. The time scale becomes anisotropic in presence of a strong magnetic field. The time scale is a function of polar angle θ (assuming the direction of field along Z-axis). The emission rate is maximum on the magnetic equator and minimum on the magnetic meridian. He has also showed that the neutrino transport coefficients also become anisotropic in presence of a strong magnetic field. The presence of strong magnetic fields modifies the mean free path or the collision frequency of neutrinos in neutron stars/quark stars. He has showed in a preliminary calculation that from the anisotropy of neutrino emission from a neutron star one can infer the strength of magnetic field and also the composition of the star. According to his calculations (i) if the star is non-magnetized the emission of neutrino is isotropic irrespective of its composition. (ii) If there is a phase transition to quark matter at the core region and the field strength is low enough so that only the electrons are affected quantum mechanically, the emission of neutrino is again isotropic. (iii) On the other hand if the field strength is large enough so that quarks (only u and d) are also affected, the emission of neutrino becomes anisotropic (see ref. [1,2]).

The effect of strong magnetic field on baryon condensates and on the self energy of scalar meson σ is another project undertaken by H Mishra, A Mishra and S Chakrabarty. They have considered a $\sigma - \omega - \rho$ mesons exchange type of mean field model for dense hadronic matter in presence of a strong magnetic field. The effect of strong magnetic field on the equation of state of dense hadronic matter is in progress. They have also planned to include the effect of magnetic field on baryon condensates and self energy of σ mesons and consider these terms in the equation of state. Using the modified form of equation of state for nuclear matter, they have planned to study some of the gross properties of magnetized neutron stars.

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2. Astrophysical bounds on superlight gravitinos, scalars and pseudoscalars in a class of supergravity models

Ashok Goyal and Sukanta Dutta

In a class of super gravity models with a low lying supersymmetry breaking scale, the gravitino ($\tilde{G}$) can be superlight and may or may not be accompanied by light scalar (S) and pseudoscalar (P) particles. In such cases gravitino or P/S emission may be relevant for stellar cooling and would provide constraints on the masses and interactions of superlight particles. In the early universe they will affect the primordial helium production in the era of the big-bang nucleosynthesis. These considerations have led to lower bounds on the SUSY breaking scale. Some of these results rely on the amplitudes involving two goldstinos and two photons and the dominant process considered is gravitino pair emission through

$$\gamma\gamma \longrightarrow \tilde{G}\tilde{G}$$

process in the case of proto-neutron star. The same process is considered to be the dominant one in the early universe for the calculation of gravitino decoupling temperature and thereby affecting the primordial He abundance. Recently gravitino coupling to the matter was re-derived by Luty and Ponton through a dimension six operator of the form

$$\frac{\tilde{M}^2}{F^2}\,\partial^\nu \lambda \sigma^\mu \bar{\lambda} F_{\mu\nu}$$

which contributes to the single photon gravitino pair interaction. Based on this dimension six operator lower bounds on the supersymmetry breaking scale Λ_s from 160–500 GeV and lower limit on gravitino mass $\approx (0.6-6)\times 10^{-6}$ eV were obtained principally through the compton process $\gamma + e \longrightarrow e + \tilde{G}\tilde{G}$ and through electron positron annihilation $e^+e^- \longrightarrow \tilde{G}\tilde{G}$.

These couplings have been re-examined recently by a number of authors, Clark *et al* using correct low energy effective lagrangian obtained an additional suppression factor $\approx S/\tilde{M}^2$ in the leading gravitino single photon coupling thereby making the astrophysical bounds on the SUSY breaking scale untenable in models where gravitino is the only light super particle. Further it has been argued that the amplitudes involving the gravitinos, two matter fermions and a vector boson e.g. $\tilde{G}\tilde{G}f\bar{f}\gamma$, $\tilde{G}\tilde{G}f\bar{f}g$ involving a photon or gluon are the most important ones for phenomenology.

It is now generally believed that there is a large abundance of thermal pions/meson condensates in the core of the neutron stars, thereby giving rise to pion mediated new and faster cooling mechanisms. It is also possible that the nuclear matter in the core of a neutron star may have undergone a phase transition to deconfined strange quark matter and thereby giving rise to new processes involving quarks and gluons. Goyal and Dutta plan to revisit these constraints in the likely presence of the above mentioned exotic core. Specifically in models which allow light scalar and pseudoscalar SUSY particles with coupling to matter through two photons, Goyal and Dutta calculate the emissivity through Primakoff scattering on pions

$$\pi^- + \gamma \longrightarrow \pi^- + P/S,$$
$$\pi^- + \gamma \longrightarrow \pi^- + \tilde{G}\tilde{G}.$$

Reexamination of one photon effective coupling to $\tilde{G}\tilde{G}$ results in the following processes:

$$\pi^- + p \longrightarrow n + \tilde{G}\tilde{G},$$
$$\pi^- + \gamma \longrightarrow \pi^- + \tilde{G}\tilde{G}.$$

The phenomenologically important processes for gravitino pair production in stars give rise to

$$e^- + \gamma \longrightarrow e^- + \tilde{G}\tilde{G},$$
$$q + g \longrightarrow q + \tilde{G}\tilde{G}$$

whose importance in hadron colliders has recently been emphasized.

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3. Baryogenesis

Debajyoti Choudhury, Shobhit Mahajan, Amitabha Mukherjee,
Hatem Widyan and Urjit Yajnik

Motivation

The possibility that the baryon number of the universe is generated at the electroweak (EW) scale has generated a lot of interest in recent years. However, EW baryogenesis in the standard model seems to be ruled because of constraints on the Higgs mass and the smallness of CP violation [1, 2]. The known constraints on mass parameters also seem to rule out a first order phase transition. This gives the motivation to search for other unification models with a potential for baryogenesis from low energy physics.

In the working group at WHEPP-5, two possibilities came up for exploration:

(a) left-right symmetric models
(b) general supersymmetric models.

It was decided to concentrate on the former.

4. Left-right symmetric (L-R) models

In L-R models, there are two ways of generating baryons. The first is by bubble nucleation. This relies on a first-order phase transition, and the problems with such transitions in the standard model probably carry over to L-R models. The second way, which presents no such problems, is through topological defects. Though these topologically stable objects are very heavy, their presence in the early universe is natural because of the high temperature and slow cooling during that epoch [3]. Some of these objects, viz, cosmic strings have been investigated for their role in structure formation and baryogenesis [4–6]. On the other hand, monopole and domain wall solutions [6–8] have undesirable cosmological consequences and their absence puts stringent limits on the theory. However, there is very little existing work on defects in L-R models. (The exception is $SO(10)$ models, in which a rich variety of cosmic string solutions had earlier been demonstrated [9, 10].) Thus it was decided to concentrate on searching for the existence of topological defects.

To facilitate the search for topologically nontrivial classical solutions, the potential has to be minimized. This was investigated for a particular ('minimal') L-R model with two triplet Higgs fields, and exact minima of the potential were found. We found that a cosmic string solution exists in the high temperature phase of the theory where the electroweak symmetry is restored. These defects may either be destabilized at the electroweak phase transition or may acquire additional condensates and continue to enjoy topological stability. The model also contains domain walls (DWs) which are stable only above the electroweak scale.

5. Cosmic string

The left-right symmetric unification group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ possesses a $U(1)$ factor whose gauge charge is the $B-L$ number of the observed fermions. We begin with the phase in which only the first stage of symmetry breaking $SU(2)_R \otimes U(1)_{B-L} \rightarrow U(1)_Y$ has occurred. In the conventions of Mohapatra [11], the field signalling this breakdown is the $(1,0,2)$ field Δ_R which acquires the vacuum expectation value (vev) with the $(2,1)$ entry of the matrix being the only non-trivial component, $\langle\Delta_R\rangle_{21} = v_R$. A cosmic string ansatz can be constructed by selecting a map $U(1)^\infty$ from the circle S^∞ at infinity into some broken $U(1)$ subgroup of the original group. Clearly, a proper $U(1)$ subgroup of the $SU(2)_R$ component would lead only to a topologically trivial configuration due to the simply connected nature of $SU(2)_R$ [12]. Since $Y = T_R^3 + X$ (with $X = 2(B-L)$) is unbroken, we propose a cosmic string ansatz using the $U(1)$ generated by $\tilde{Y} = T_R^3 - X$. Furthermore, we select the internal parameter to be one-half times the spatial cylindrical angle θ. Thus, $U^\infty(\theta) = \exp\{i(T_R^3 - X)\theta/2\}$. The $SU(2)$ acts on Δ_R by similarity transformation, so $\langle\Delta_R(\infty,\theta)\rangle_{21} = e^{i\theta}v_R$, so that the vev remains single valued; however,

$$U^\infty(2\pi) = e^{-i\pi}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \neq U^\infty(0).$$

Since the inequality above is detected by general matrix Δ_R but not by the vev, we have a discrete stability group Z_2, a remnant from the symmetry breakdown. The corresponding ansatz for the gauge field is $W^3_{R\theta}(\infty,\theta) = 1/4gr$. The stability criterion based on the Z_2 identified above does not survive the subsequent phase transition. The low energy vevs of the $(1,0,2)$ field Δ_L and the $(1/2,1/2,0)$ field ϕ are, respectively, $\langle\Delta_L\rangle_{21} = v_L$ and diag $(\kappa, -\tilde{\kappa})$ which are not invariant under the action of $U^\infty(2\pi)$. However, one may think of the above curve $U^\infty(\theta)$ as a projection to the subspace $SU(2)_R \otimes U(1)_{B-L}$ of the more general curve $\tilde{U}^{\infty}(\theta) = \exp\{i(T_R^3 + T_L^3 - X)\theta/2\}$. This leaves $\Delta_R(\infty, \theta)$ to be as above and leaves the ϕ vev invariant, but makes $\langle\Delta_L(\infty, \theta)\rangle_{21} = e^{i\theta}v_L$. Thus the new vevs also possess a discrete stability group Z_2', a simple generalization of the earlier Z_2. If such cosmic strings form, they should exist as relics at the present epoch.

At the electroweak phase transition, if the vevs of Δ_L and ϕ in the domains around an existing vortex are not too different from each other, they will destabilize the vortex. If the new vevs wind nontrivially in the internal space while traversing a closed physical path around the existing vortex, then a stable string forms. It may be noted that the stable strings necessarily contain $SU(2)_R$ charged condensates. No new strings can arise at the electroweak phase transition since the latter is unlikely to release latent heat sufficient to excite $SU(2)_R$ charged field condensates.

6. Domain walls

The theory also possesses more than one kind of domain wall (DW) solution. Introducing ansatz functions $R(x)$, $L(x), f(x), \tilde{f}(x)$ for the nonzero components of Δ_R, Δ_L, ϕ and $\tilde{\phi}$ respectively, the boundary conditions, as $x \longrightarrow \pm\infty$, are

$$R(x) \longrightarrow \pm v_R, \quad L(x) \longrightarrow \pm v_L, \quad f(x) \longrightarrow \pm\kappa, \quad \tilde{f}(x) \longrightarrow \pm\tilde{\kappa}.$$

To have the observed parity violation at low energies, we must have $\kappa \ll v_R$. Also to avoid fine tuning in the potential we must have $v_L \ll \kappa$. Thus we shall set $v_L = 0$; moreover, for the simplest forms of the Higgs potential one must take $\tilde{\kappa} = 0$ [13].

In terms of the nonvanishing ansatz functions, the equations of motion are:

$$\frac{d^2R(x)}{dx^2} = C(4\rho C^{-2}(R(x)^3 - R(x)) + 2\alpha R(x) f(x)^2),$$
$$\frac{d^2f(x)}{dx^2} = C(4\lambda(f(x)^3 - f(x)) + 2\alpha C^{-2} f(x)R(x)^2),$$

where $C = \kappa/v_R$, ρ, λ and α are parameters in the potential [11]. The boundary condition is such that $R(x) \longrightarrow \pm v_R$ as $x \longrightarrow \pm\infty$. We choose the origin of x, such that $R(0) = 0$. It can be seen that nontrivial solutions exist in which $f(x)$ is peaked at $x = 0$. Subsequently (after WHEPP), we have solved the equations numerically. We have also analysed the behaviour of the equations at small and large values of x, and verified that they possess a nontrivial solution.

Another kind of DW solution is obtained by 'switching off' the fields ϕ and $\tilde{\phi}$. At tree level the Lagrangian density is symmetric under the exchange of Δ_L and Δ_R. If the vacuum values for these two Higgs fields are assumed to be as above, it can be shown [11] that their potential assumes the form

$$V(v_L, v_R) = -\mu^2(v_L^2 + v_R^2) + (\rho_1 + \rho_2)(v_L^4 + v_R^4) + \rho_3 v_L^2 v_R^2,$$

where the parameters are inherited from the original form of the potential [11]. Upon parametrizing $v_R = v\cos\alpha$ and $v_L = v\sin\alpha$, the conditions for extrema become

$$-2\mu^2 + 4v^2[(\rho_1 + \rho_2) + \tfrac{1}{4}(\rho_3 - 2(\rho_1 + \rho_2))\sin^2 2\alpha] = 0,$$
$$v^4[\rho_3 - 2(\rho_1 + \rho_2)]\sin 2\alpha\cos 2\alpha = 0.$$

The points $(v^2, \alpha) = (\mu^2/2(\rho_1 + \rho_2), 0)$ and $(\mu^2/2(\rho_1 + \rho_2), \pi/2)$ are the minima, and $(2\mu^2/(\rho_3 + 2(\rho_1 + \rho_2)), \pi/4)$ a saddle point, provided $\rho_3 > 2(\rho_1 + \rho_2) > 0$. In the translation invariant Minkowski vacuum, one may select $\alpha = 0$ which keeps $SU(2)_L$ unbroken. We assume that, in the early Universe, (i.e. above the electroweak scale), the temperature-dependent parameters of the potential were such that this was permitted. However, as the universe cools from the completely $L-R$ symmetric phase, there should be causally disconnected regions that select either $\alpha = 0$ or $\alpha = \pi/2$. We assume that the DW is centred on the plane $x = 0$, and that there is translational invariance in the y and z directions. In terms of the ansatz functions $R(x)$ and $L(x)$, we can write down the equations of motion obeyed by the domain wall. We further define

$$\sigma(x) = \sqrt{R(x)^2 + L(x)^2}, \quad \xi(x) = \tan^{-1}\frac{L(x)}{R(x)}.$$

Then the equations of motion take the form

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x^2} = -2\mu^2\sigma + 4\sigma^3[(\rho_1 + \rho_2) + \frac{1}{4}(\rho_3 - 2(\rho_1 + \rho_2))\sin^2 2\xi],$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sigma^2\frac{\mathrm{d}\xi}{\mathrm{d}x}\right) = \sigma^4[\rho_3 - 2(\rho_1 + \rho_2)]\frac{1}{2}\sin 4\xi.$$

The boundary conditions appropriate to the DW are

$$\sigma(x) \to v \quad \text{as } x \to \pm\infty, \quad \xi(x) \to 0 \quad \text{as } x \to +\infty,$$
$$\xi(x) \to \pi/2 \quad \text{as } x \to -\infty$$

or alternatively,

$$R(x) \to v, \quad L(x) \to 0 \quad \text{as } x \to -\infty; \quad R(x) \to 0, L(x) \to v \quad \text{as } x \to +\infty.$$

That the required solution exists can be seen by observing that if the values of v are not too different at $\alpha = 0$ and at the saddle point $\alpha = \pi/4$, for example if $\beta \equiv \rho_3 - 2(\rho_1 + \rho_2)$ is small, then we get the approximate solution $v^2 = \mu^2/(2(\rho_1 + \rho_2))$, and

$$\alpha(x) = \tan^{-1}[\exp\{\mu x\sqrt{\beta/2(\rho_1 + \rho_2)}\}].$$

Numerical solutions also can be obtained.

7. Plan of future work

Investigation of the equations of motion under the string ansatz, both numerical and analytical, is in progress. The stability of the solutions will be studied, as it may be important in cosmological considerations.

Assuming that stable string solutions are found, the detailed mechanism of string-mediated baryogenesis will have to be investigated.

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8. Particle properties from astrophysics

S Mohanty, J A Grifols, Uma Mahanta and Sarira Sahu

The working group started work on few ideas of which two are likely to materialize in the form of papers:

(1) *Astrophysical constraints on light gravitinos (S Mohanty, Tony Grifols, Uma Mahanta and Sarira Sahu)*

An upper bound was put on the gravitino mass by calculating the (a) gravitino relic density from which it was found that the gravitino mass is less than 0.6 keV, (b) from the supernova cooling rates from which it was seen that the gravitino mass is greater than 10^{-7} eV. Some other calculations like the diffuse x-ray background flux from relic gravitinos annhilations were also done but they did not lead to any useful bounds.

(2) *Pair production of neutral fermions in external electro-magentic fields by the Schwinger mechanism (J A Grifols, Eduard Masso and Subhendra Mohanty)*

The Schwinger mechanism is a non-perturbative process where a electric field produces can produce charged particle pairs. This method was extended to the case fermions with zero electric charge but which have non-zero dipole and anapole moments. The magnetic dipole form factor of neutral fermions (or the anamalous part of the dipole form factor for charged fermions) is given by

$$\mathcal{L}(\psi,\bar{\psi},F) = \bar{\psi}\left(\not{\partial} - m - \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\psi. \tag{1}$$

It was seen that in an inhomogenous magnetic field the probability of magnetic dipole pair production per unit volume per unit time is given by

$$w = \frac{1}{4\pi^4}(\mu B')^2 \sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^2 \exp\left(\frac{-n\pi m^2}{\mu B'}\right). \tag{2}$$

Similarly the expresssion for pair production of anapoles with form factor is given by

$$L[F] = -\lambda\bar{\psi}\gamma_\nu\gamma_5\psi\partial_\mu F^{\mu\nu}. \tag{3}$$

This lagrangian is odd under parity and in the standard model neutrinos it arises at one loop. Using the equation of motion of the external fields $\partial_\mu F^{\mu\nu} = J^\nu$ to write the interaction term (3) as

$$L[J] = -\lambda\bar{\psi}(x)\ \not{J}(x)\gamma_5\psi(x). \tag{4}$$

It can be seen that the anapole coupling is a contact interaction between the anapoles and the source of the electromagnetic fields and it vanishes outside the source current density $J(x)$. It was seen that the expression for the pair production of anapoles in an inhomogenous charge distribution is given by

$$w = -2\,\mathrm{Im}\,L_{\mathrm{eff}}[J] = \frac{1}{4\pi^4}(\lambda J_0')^2 \sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^2 \exp\left(\frac{-n\pi m^2}{\lambda J_0'}\right). \tag{5}$$

Applications of these formulae for the case of neutrinos and the generic goldistinos is being studied.

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9. Large scale structure and the cosmic microwave background

Varun Sahni, Tarun Souradeep, Albert Stebbins and Ioav Waga

9.1 *Did the Universe loiter?*

Astrophysical observations are imposing ever tightening constraints on the viability of cosmological models. Observations of gravitational lensing, high-redshift supernovae and the cosmic microwave background have in recent years provided very strong observational criteria against which to test cosmological scenario's attempting to explain the formation of large scale structure of the Universe. The present collaboration critically examines a promising cosmological scenario known as the 'loitering model'. In this scenario the Universe undergoes a stage of extremely slow expansion or 'loitering' during which the expansion factor remains approximately constant so that $H^2 \ll 8\pi G\rho$ where H is the Hubble parameter and ρ is the cosmological matter density. As shown in Sahni *et al* (1992) loitering can occur generically in a closed Universe in the presence of normal matter and matter behaving like a time dependent cosmological term $\Lambda \propto a^{-m}$ where a is the scale factor of the Universe and $0 \leq m < 2$. From the Einstein equations it follows that such a Universe will undergo accelerated expansion or 'late inflation' after loitering, some evidence for a non-zero cosmological term has recently been advanced by Perlmutter (1998).

The loitering redshift z_l and the dimensionless present value of the time-dependent cosmological constant $\Omega_X = \Lambda/3H^2$ can be found by solving the Einstein equations, as a result

$$\Omega_X = \frac{(1+z_l)^{3-m}}{(1+z_l)^{3-m} - (3-m)z_l - 1}, \tag{6}$$

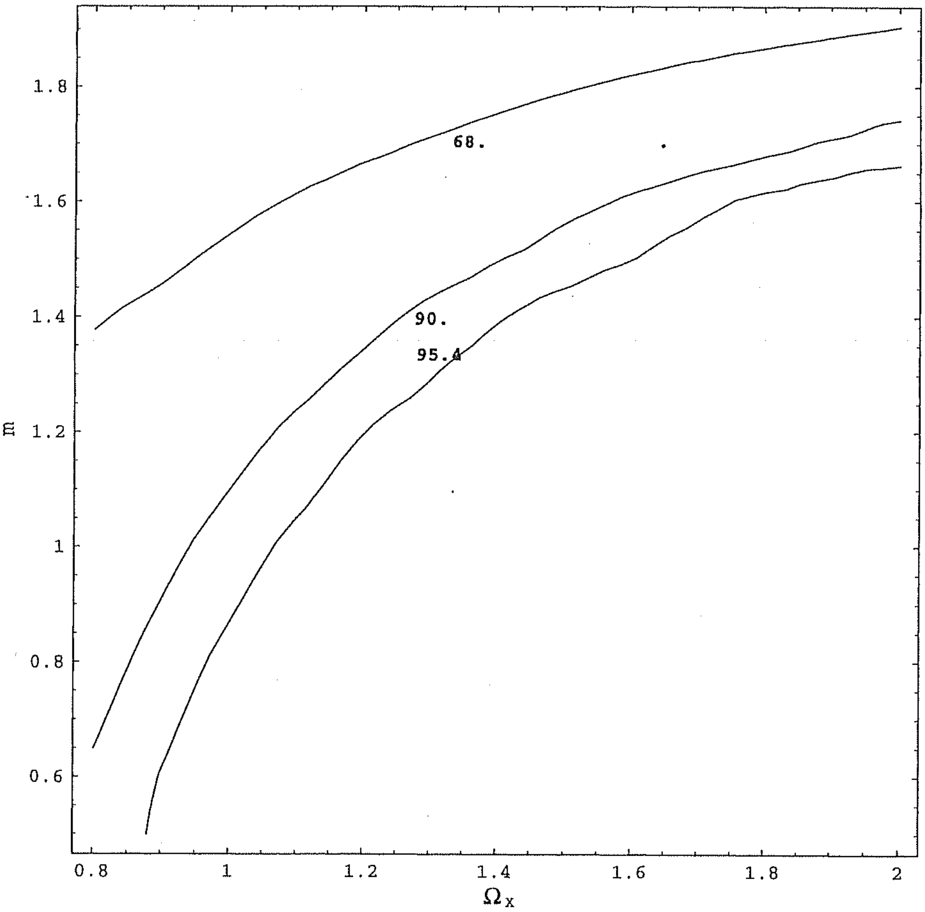

Figure 1. Gravitational lensing constraints on the parameter space $\{\Omega_X, m\}$ in loitering models.

the loitering redshift z_l is determined from

$$\Omega_{0m} = \frac{2-m}{(1+z_l)^{3-m} - (3-m)z_l - 1}, \tag{7}$$

where $m = 3(1+w)$, $p_X = w\rho_X$ is the equation of state of the cosmological term and Ω_{0m} is the present density of normal (pressureless) matter. From (6) and (7) one finds $2 < z_l < 7$ for the loitering redshift if $\Omega_{0m} = 0.1$.

A Universe which loitered in the past has several advantages over more conventional scenarios:

(i) A loitering Universe has a long age which can be much larger than H_0^{-1}. This ameliorates the age problem which arises in conventional flat models because of the conflict between ages of globular clusters and the age of the Universe $t_U = \frac{2}{3}H_0^{-1}$.

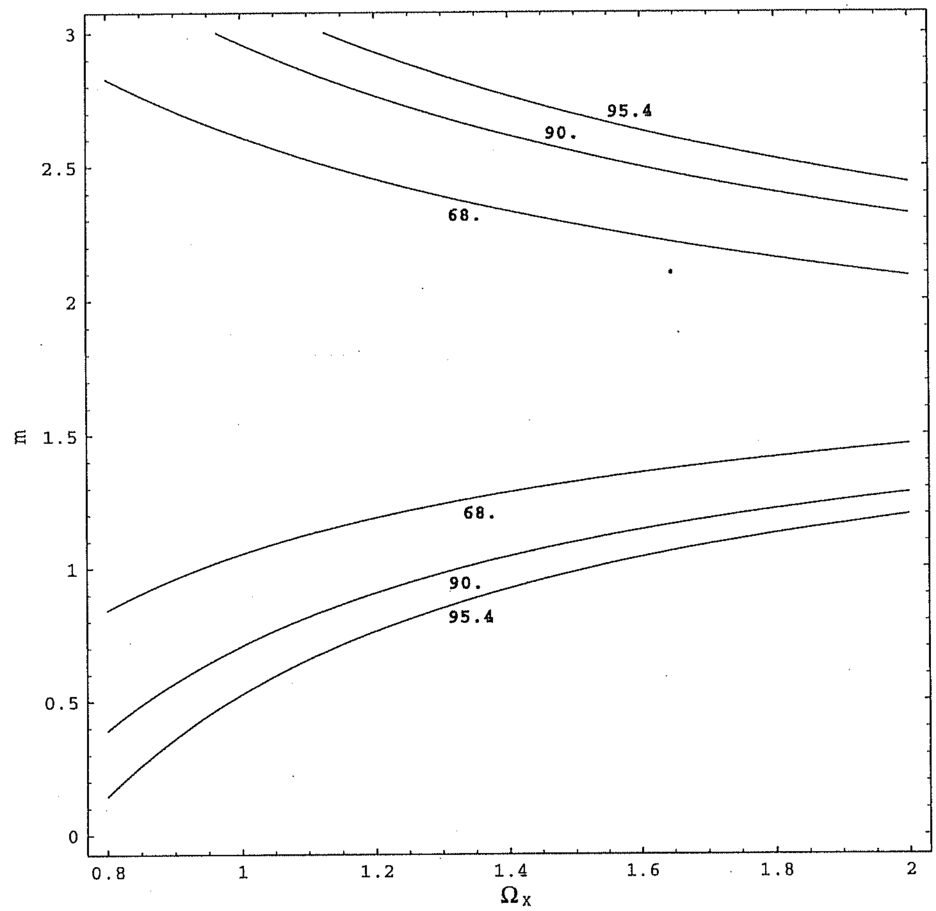

Figure 2. Constraints on loitering models from observations of high redshift supernovae.

(ii) Fluctuations in the cosmological density field grow extremely rapidly during loitering and one can grow large inhomogeneities (galaxies, clusters of galaxies) while producing small anisotropies in the cosmic microwave background (CMB).

The present collaboration between Waga, Stebbins, souradeep and Sahni was begun during WHEPP-5 and examines the loitering scenario in view of the most recent observational results relating to high redshift supernovae, gravitational lensing and both large and small angular fluctuations in the CMB. Our preliminary results are shown in figures 1–4. Figures 1 and 2 show constraints on the parameter space $\{\Omega_X, m\}$ obtained using gravitational lensing and supernovae data (assuming $\Omega_m = 0.2$) while figure 4 shows the angular spectrum of the cosmic microwave background for different loitering models. From figure 4 one finds that a distinctive feature of loitering models is that the power spectrum is strongly suppressed on small angular scales (on larger scales C_l has been normalized to COBE observations). The corresponding Hubble parameter is shown in figure 3. For details see [2].

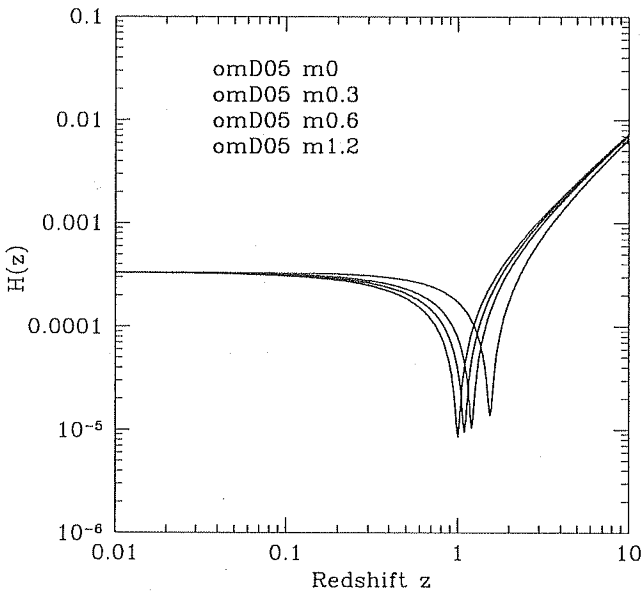

Figure 3. Hubble parameter shown as a function of cosmological redshift in a cold dark matter Universe with loitering: $\Omega_{0m} = \Omega_{cdm} + \Omega_b = 0.5$, $\Omega_b = 0.0125$. Four different loitering models are considered with $m = 0.0, 0.3, 0.6, 1.2$.

9.2 *Loitering in an open Universe*

Having demonstrated some effects of loitering in closed Universes it is interesting to consider the possibility of loitering in open Universes. Consider the 0–0 component of the Einstein equations

$$3H^2 = 8\pi G\left[\frac{\rho_{0m}}{a^3} + \frac{\rho_{0X}}{a^m} + \frac{\rho_{0r}}{a^4} - \frac{3k}{a^2}\right]. \tag{8}$$

If $k = -1$ and $\rho_{0X} < 0$ then ρ_{0X}/a^m can be balanced by the curvature term $3k/a^2$ and ρ_m leading to a *loitering* phase during which $\dot{a} = \ddot{a} = 0$ and the Universe ceases to expand for a while. Such a phase can also arise in spatially flat models containing matter and radiation if $4 < m < 3$.

The idea of matter with a negative energy density is discussed by Parker and Wang (1990), more recently low energy effective string theories appear to give a stress tensor for the dilaton field which could have $T_{00} < 0$, in addition non-minimal scalars have terms in the stress tensor which could be negative.

Setting $H \equiv \dot{a}/a = 0$ in (8) and $\ddot{a} = 0$ in

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\Sigma_i(\rho_i + 3p_i) \tag{9}$$

(the summation is taken over all matter fields including strange matter) one obtains:

(1) For a spatially flat Universe and $1/3 > w > 0$ ($3 < m < 4$):

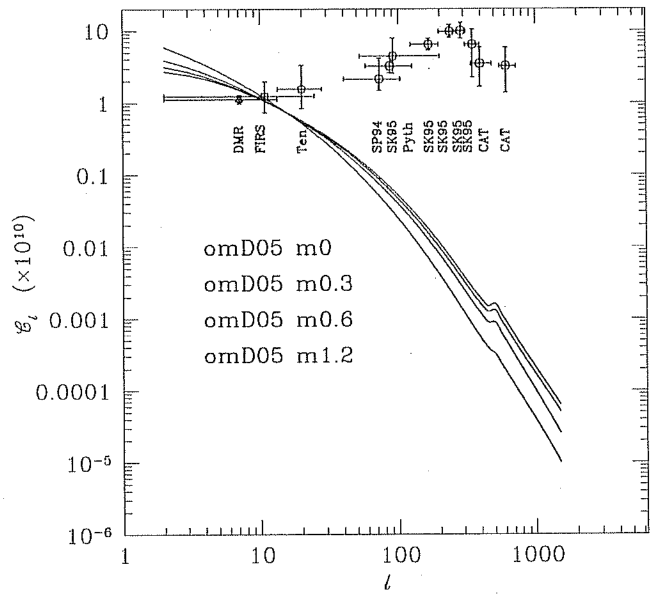

Figure 4. Angular power spectrum for a loitering Universe with cold dark matter, $\Omega_{0m} = 0.5$, four different loitering models are considered with $m = 0, 0.3, 0.6, 1.2$.

$$(1+z_l)^{(1-3w)/3w} = \frac{\Omega_m}{\Omega_r}\frac{w}{1/3 - w} \tag{10}$$

which relates the loitering redshift z_l to the present-day values of Ω_m, Ω_r. The corresponding energy density in strange matter is

$$\Omega_X = \frac{\Omega_m}{3w-1}(1+z_l)^{-3w} < 0. \tag{11}$$

Clearly in this case loitering will occur at fairly high redshifts $z_l \sim 100$–1000.

(2) For $-1/3 < w < 0$ $(2 < m < 3)$ a loitering epoch can occur if the Universe is open, in which case

$$(1+z_l)^{1+3w} = \frac{\Omega_{\text{curv}}}{3w\Omega_X}, \tag{12}$$

where

$$\Omega_X = -\frac{\Omega_m}{1+3w}(1+z_l)^{-3w} < 0 \tag{13}$$

and $\Omega_{\text{curv}} = 1 - \Omega_m - \Omega_X \equiv -3k/a_0^2 H_0^2 > 0$. Substituting (13) in (12) one also gets

$$1 + z_l = -\left(1 + \frac{1}{3w}\right)\frac{\Omega_{\text{curv}}}{\Omega_m} \tag{14}$$

for the loitering redshift. We can also eliminate Ω_{curv} from (14) and recast z_l solely in terms of Ω_m as follows:

$$3w(1+z_l) + (1+z_l)^{-3w} = (1+3w)\frac{\Omega_m - 1}{\Omega_m}, \tag{15}$$

where $-1/3 < w < 0$ $(2 < m < 3)$, which is the analog of (3.7) in Sahni *et al* [1]. Astrophysical implications of loitering in open/flat Universes will be examined in Sahni *et al* [2].

References

[1] V Sahni, H Feldman and A Stebbins, *Astrophys. J.* **385**, 1 (1992)
[2] V Sahni, T Souradeep, A Stebbins and I Waga, in preparation (1998)
[3] L Parker and Y Wang, *Phys. Rev.* **D42**, 1877 (1990)
[4] S Perlmutter *et al*, *Nature* **391**, 51 (1998)