# Direct Measurement of the Top Quark Mass 

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#### Abstract

We measure the top quark mass $m_{t}$ using $t \bar{t}$ pairs produced in the $\mathrm{D} \emptyset$ detector by $\sqrt{s}=1.8 \mathrm{TeV} p \bar{p}$ collisions in a $125 \mathrm{pb}^{-1}$ exposure at the Fermilab Tevatron. We make a two constraint fit to $m_{t}$ in $t \bar{t} \rightarrow b W^{+} \bar{b} W^{-}$final states with one $W$ decaying to $q \bar{q}$ and the other to $e \nu$ or $\mu \nu$. Events are binned in fit mass versus a measure of probability for events to be signal rather than background. Likelihood fits to the data yield $m_{t}=173.3 \pm 5.6$ (stat) $\pm 6.2$ (syst) $\mathrm{GeV} / \mathrm{c}^{2}$.


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[^1]The top quark has a large mass $m_{t}$ that can be determined to greater fractional precision than is possible for the lighter quarks, which decay after they form hadrons. Since $m_{t}$ is large, it controls the strength of quark-loop corrections to tree-level relations among electroweak parameters. If these parameters and $m_{t}$ are measured precisely, the Standard Model Higgs boson mass can be constrained.

Direct measurements of $m_{t}$ have been published as part of the initial observations [1] of $t \bar{t}$ production in $\sqrt{s}=1.8 \mathrm{TeV} p \bar{p}$ collisions. At present, the best accuracy in $m_{t}$ is achieved for lepton + jets ( $\ell+$ jets) final states in which one $W$ boson (from $t \rightarrow b W$ ) decays to $e \nu$ or $\mu \nu$ and the other $W$ decays to a $q \bar{q}$ pair that forms jets. We report a measurement of $m_{t}$ in the $\ell+$ jets channel using the $\approx 125 \mathrm{pb}^{-1}$ exposure of the $\mathrm{D} \emptyset$ detector during the 1992-96 Fermilab Tevatron runs. Since Ref. [1] appeared, our data sample has doubled, and for a fixed sample size our error on $m_{t}$ has halved.

The D $\varnothing$ detector and our basic methods for triggering, reconstructing events, and identifying particles are described elsewhere [2]. Recent advances include enhanced triggering and reconstruction efficiency for $\mu+$ jets events, due in part to better use of calorimeter data. As a signature of $W \rightarrow \ell \nu$, we require missing energy transverse to the beam $\left(E_{T}\right)>20 \mathrm{GeV}$, and one isolated $e$ or $\mu(\ell)$ with $E_{T}^{\ell}>20 \mathrm{GeV}$ and pseudorapidity $\left|\eta_{e}\right|<2$ or $\left|\eta_{\mu}\right|<1.7$. We also demand $E_{T}^{\text {cal }}>25(20) \mathrm{GeV}$ for $e+$ jets $(\mu+$ jets $)$ events, where $E_{T}^{\text {cal }}$ is $E_{T}$ measured only in the calorimeter. As signatures of the $q \bar{q}$ from $W$ decay and the $b$ and $\bar{b}$ from $t$ and $\bar{t}$ decay, we require $\geq 4$ jets reconstructed with cones of half-angle $\Delta \mathcal{R} \equiv\left(\Delta \phi^{2}+\Delta \eta^{2}\right)^{1 / 2}=0.5$, having $E_{T}>15 \mathrm{GeV}$ and $|\eta|<2$.

Within $\Delta \mathcal{R}=0.5$ of a jet axis, additional muons ( $\mu$ tags) satisfying $p_{T}^{\mu}>4 \mathrm{GeV} / c$ and $\left|\eta_{\mu}\right|<1.7$ arise mainly from $b$ and $c$ quark semileptonic decay. These occur in $\approx 20 \%$ of $t \bar{t}$ events but only $\approx 2 \%$ of background events [2]. In untagged events, to suppress background we require $E_{T}^{L}\left(\equiv\left|E_{T}^{\ell}\right|+\left|E_{T}\right|\right)>60 \mathrm{GeV}$ and $\left|\eta_{W}\right|<2$ for the $W \rightarrow \ell \nu$. The latter cut, exhibited in Fig. ⿴囗 (a), reduces the difference in $\eta_{W}$ distributions between data and Monte Carlo (MC) simulated background. We use the HERWIG MC [3] to simulate top signal, and the vecbos MC [7] (with HERWIG fragmentation of partons into jets) to simulate (but not to normalize) the dominant $W$ +multijet background. The $\approx 20 \%$ of background events from non- $W$ sources are modeled by multijet data that barely fail the lepton identification criteria.

To each event passing the above cuts, we make a two constraint (2C) kinematic fit [5] to the $t \bar{t} \rightarrow \ell+$ jets hypothesis by minimizing a $\chi^{2}=\left(\mathbf{v}-\mathbf{v}^{*}\right)^{T} G\left(\mathbf{v}-\mathbf{v}^{*}\right)$, where $\mathbf{v}\left(\mathbf{v}^{*}\right)$ is the vector of measured (fit) variables and $G^{-1}$ is its error matrix. Both reconstructed $W$ masses are constrained to equal the $W$ pole mass, and the same fit mass $m_{\text {fit }}$ is assigned to both the $t$ and $\bar{t}$ quarks. If the event contains $>4$ accepted jets, only the four jets with highest $E_{T}$ are used. In $\approx 50 \%$ of MC top events, these jets correspond to the $b, \bar{b}, q$, and $\bar{q}$. With (without) a $\mu$ tag in the event, there are 6 (12) possible fit assignments of these jets to the quarks, each having two solutions to the $\nu$ longitudinal momentum $p_{z}^{\nu}$. We use $m_{\text {fit }}$ only from the permutation with lowest $\chi^{2}$, the correct choice for $\approx 20 \%$ of MC top events. Because of the ambiguities, $m_{\text {fit }}$ is not the same as $m_{t}$, though they are strongly correlated. Our best estimate of $m_{t}$ is obtained from the best match between MC samples and the data.

From the 90 -event distribution shown in Fig. [(b) we select 77 events with a 2 C fit satisfying $\chi^{2}<10$. Of these, 5 are $\mu$ tagged and $\approx 65 \%$ are background. Further separation of signal and background events is based on four kinematic variables $\mathbf{x} \equiv\left\{x_{1}, x_{2}, x_{3}, x_{4}\right\}$
chosen to have small correlation with $m_{\text {fit }}$. On average, all are larger for MC top events than for background events, selected to have the same $\left\langle m_{\mathrm{fit}}\right\rangle$ as the top events [6]. The simpler variables are $x_{1} \equiv \nabla_{T}$ and $x_{2} \equiv \mathcal{A}$, where aplanarity $\mathcal{A}$ is $\frac{3}{2} \times$ the least eigenvalue of the normalized laboratory momentum tensor of the jets and the $W$ boson. The third variable $x_{3} \equiv H_{T 2} / H_{z}$ measures the event's centrality, where $H_{z}$ is the sum of $\left|p_{z}\right|$ of $\ell, \nu$, and the


FIG. 1. Events per bin vs. event selection variables defined in the text, plotted for (a-b, g-h) top quark mass analysis samples, and (c-f) $W+3$ jet control samples. Histograms are data, filled circles are expected top + background mixture, and open triangles are expected background only. Solid arrows in (a-b) show cuts applied to all events; the open arrow in (g) illustrates the LB cut. The nonuniform bin widths in ( $\mathrm{g}-\mathrm{h}$ ) are chosen to yield uniform bin populations.
jets, and $H_{T 2}$ is the sum of all jet $\left|E_{T}\right|$ except the highest. Finally, $x_{4} \equiv \Delta \mathcal{R}_{j j}^{\min } E_{T}^{\min } / E_{T}^{L}$ measures the extent to which jets are clustered together, where $\Delta \mathcal{R}_{j j}^{\min }$ is the minimum $\Delta \mathcal{R}$ of the six pairs of four jets, and $E_{T}^{\min }$ is the smaller jet $E_{T}$ from the minimum $\Delta \mathcal{R}$ pair. As shown for the background dominated $W+3$ jet sample in Fig. $\mathbb{1}(\mathrm{c}-\mathrm{f}), x_{1}-x_{4}$ are reasonably well modeled by MC; this is true also for the $W+2$ jet and top mass samples (not shown).

We bin events in a two-dimensional array with abscissa $m_{\text {fit }}$ and ordinate $D(\mathbf{x})$, where $D$ is a multivariate discriminant. To show that our results are robust, we use two methods for which the definition of $D$, the granularity with which it is binned, and the additional requirements are different. In our "low bias" (LB) method, we first parametrize $\mathcal{L}_{i}\left(x_{i}\right) \equiv$ $s_{i}\left(x_{i}\right) / b_{i}\left(x_{i}\right)$, where $s_{i}$ and $b_{i}$ are the top signal and background densities in each variable, integrating over the others. We form the $\log$ likelihood $\ln \mathcal{L} \equiv \sum_{i} \omega_{i} \ln \mathcal{L}_{i}$, where the weights $\omega_{i}$ are adjusted slightly away from unity to nullify the average correlation ("bias") of $\mathcal{L}$ with $m_{\text {fit }}$, and for each event we set $D_{\mathrm{LB}}=\mathcal{L} /(1+\mathcal{L})$. Finally, we divide the ordinate coarsely into signal- and background-rich bins according to whether the LB cut is passed. This cut is satisfied if a $\mu$ tag exists; otherwise it is not satisfied if $D_{\mathrm{LB}}<0.43$ (Fig. [1(g)) or if $H_{T 2}<90$ GeV.

Our neural network (NN) method is sensitive to the correlations among the $x_{i}$ as well as to their individual densities. We use a three layer feed-forward NN with 4 input nodes fed by


FIG. 2. Events per bin ( $\propto$ areas of boxes) vs. $D_{\mathrm{NN}}$ (ordinate) and $m_{\text {fit }}$ (abscissa) for (a) expected $172 \mathrm{GeV} / \mathrm{c}^{2}$ top signal, (b) expected background, and (c) data. $D_{\mathrm{NN}}$ is binned as in Fig. 1](h).
$\mathbf{x}, 5$ hidden nodes, and 1 output node, trained on samples of top signal (background) with density $s(\mathbf{x})(b(\mathbf{x}))$ [7]. For a given event, the network output $D_{\mathrm{NN}}$ approximates the ratio $s(\mathbf{x}) /(s(\mathbf{x})+b(\mathbf{x}))$. We divide the ordinate finely into ten bins in $D_{\mathrm{NN}}$, independent of $H_{T 2}$ or $\mu$ tagging. Figure $1(\mathrm{~g}-\mathrm{h})$ shows that $D_{\mathrm{LB}}$ and $D_{\mathrm{NN}}$ are distributed as predicted and provide comparable discrimination, as we expect when the $\omega_{i}$ are close to unity and the $\mathcal{L}_{i}$ are not strongly correlated. Figure 2 exhibits the arrays for the NN method. Little correlation between $D_{\mathrm{NN}}$ and $m_{\mathrm{fit}}$ is evident in the expected signal or background distributions, which are distinct; the data clearly reveal contributions from both sources. Figure 3 shows the distributions of $m_{\text {fit }}$ for data (a) passing and (b) failing the LB cut.


FIG. 3. (a-b) Events per bin vs. $m_{\text {fit }}$ for events (a) passing or (b) failing the LB cut. Histograms are data, filled circles are the predicted mixture of top and background, and open triangles are predicted background only. The circles and triangles are the average of the LB and NN fit predictions, which differ by $<10 \%$. (c) Log of arbitrarily normalized likelihood $L$ vs. true top quark mass $m_{t}$ for the LB (filled triangles) and NN (open squares) fits, with errors due to finite top MC statistics. The curves are quadratic fits to the lowest point and its 8 nearest neighbors. In MC studies, $7 \%(27 \%)$ of simulated experiments yield a smaller LB (NN) maximum likelihood.

TABLE I. Results of fits to data and MC events. Fits to data yield values and errors $\sigma$ (stat) for $m_{t}, n_{s}$, and $n_{b}$ (described in the text). Systematic errors are combined in quadrature. The resulting $m_{t}$ and its statistical error $\sigma_{m}$ are the combined LB and NN values. Fits to MC use ensembles of 10,000 simulated experiments composed of top + background, with $m_{t},\left\langle n_{s}\right\rangle$, and $\left\langle n_{b}\right\rangle$ as listed. They yield a mean result $\left\langle m_{t}\right\rangle$, a mean statistical error $\left\langle\sigma_{m}\right\rangle$, and a range $\pm \delta m$ within which $68 \%$ of the results fall. Using the LB (NN) method, $6 \%$ ( $25 \%$ ) of the simulated experiments produce a $\sigma_{m}$ which is smaller than we obtain. For an "accurate subset" of the MC ensembles with mean $\sigma_{m} / m_{t}$ that matches our value, $\delta m$ is smaller.

| Fits to data Quantity fit |  | ---LB fit--- |  |  |  | ---NN fit--- |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | value | $\sigma($ stat $)$ |
| $m_{t}\left(\mathrm{GeV} / \mathrm{c}^{2}\right)$ |  | $174.0 \pm 5.6$ |  |  |  | $171.3 \pm 6.0$ |  |
| $n_{s}$ |  | $23.8+8.3-7.8$ |  |  |  | $28.8+8.4-9.1$ |  |
| $n_{b}$ |  |  | 53.2 | +10.7 | -9.3 | $48.2+11.4-8.7$ |  |
| Systematic error on $\boldsymbol{m}_{\boldsymbol{t}}$ |  | $\begin{array}{cc} \hline \text { energy scale } & \pm 4.0 \\ \text { generator } & \pm 4.1 \\ \text { other } & \pm 2.2 \\ \hline \end{array}$ |  |  |  |  |  |
| Resulting $\boldsymbol{m}_{\boldsymbol{t}}\left(\mathrm{GeV} / \mathrm{c}^{2}\right)$ |  | $173.3 \pm 5.6$ (stat) $\pm 6.2$ (syst) |  |  |  |  |  |
| Fits to MC <br> (top + background) | type <br> of fit | $m_{t}$ | $\begin{gathered} \text {-input } \\ \left\langle n_{s}\right\rangle \end{gathered}$ | $\left\langle n_{b}\right\rangle$ | $\left\langle\sigma_{m}\right\rangle$ | $\begin{gathered} \text { output- } \\ \left\langle m_{t}\right\rangle \end{gathered}$ | $\delta m$ |
| full ensemble | LB | 175 | 24 | 53 | 9.9 | 175.0 | 8.7 |
| " | NN | 172 | 29 | 48 | 8.5 | 171.6 | 8.0 |
| accurate subset | LB | 175 | 24 | 53 | 5.5 | 175.3 | 4.6 |
| " | NN | 172 | 29 | 48 | 5.8 | 172.0 | 6.0 |

To each $m_{t}$ for which we have generated MC, we assign a likelihood $L$ which assumes that all samples obey Poisson statistics. Bayesian integration [8] over possible true signal and background populations in each bin yields

$$
L\left(m_{t}, n_{s}, n_{b}\right)=\prod_{i=1}^{M} \sum_{j=0}^{n_{i}}\binom{n_{s i}+j}{j}\binom{n_{b i}+k}{k} p_{s}^{j}\left(1+p_{s}\right)^{-n_{s i}-j-1} p_{b}^{k}\left(1+p_{b}\right)^{-n_{b i}-k-1}
$$

where $n_{s}\left(n_{b}\right)$ is the expected number of signal (background) events in the data; $n_{i}, n_{s i}$, and $n_{b i}$ are the actual number of data, MC signal, and MC background events in bin $i ; k \equiv n_{i}-j$; $p_{s, b} \equiv n_{s, b} /\left(M+\sum_{i} n_{s i, b i}\right)$; and $M=40$ (200) bins for the LB (NN) methods. Maximizing $L$ for each $m_{t}$ gives the best estimates $n_{s}^{*}\left(m_{t}\right)$ and $n_{b}^{*}\left(m_{t}\right)$ for $n_{s}$ and $n_{b}$. Figure 3(c) displays $\ln L\left(m_{t}, n_{s}^{*}\left(m_{t}\right), n_{b}^{*}\left(m_{t}\right)\right)$ vs. $m_{t}$, where the curves determine the best fit $m_{t}$ and its statistical error $\sigma_{m}$.

Table [] presents the fit results, which are consistent with Ref. [1] and with recent reports [9]. The LB and NN results $m_{t}^{\mathrm{LB}}$ and $m_{t}^{\mathrm{NN}}$ are mutually consistent; in $21 \%$ of MC experiments they are further apart. Nevertheless we include half of $m_{t}^{\mathrm{LB}}-m_{t}^{\mathrm{NN}}$ in the systematic error. To obtain our result, shown in Table $\rrbracket$, we combine $m_{t}^{\mathrm{LB}}$ and $m_{t}^{\mathrm{NN}}$ allowing
for their $(88 \pm 4) \%$ correlation (determined by MC experiments). Figures $3(\mathrm{a}-\mathrm{b})$ show that this result represents the data well. From the MC experiments summarized in Table [] we measure the interval $\pm \delta m$ within which $68 \%$ of the MC estimates fall. For the full ensemble, $\delta m$ is larger than $\sigma_{m}$ from our data. However, for "accurate subsets" of the ensemble for which the average $\sigma_{m} / m_{t}$ is the same as we observe, $\delta m$ is close to $\sigma_{m}$ (10].

A principal systematic error in $m_{t}$ arises from uncertainty in the jet energy scale, which is calibrated in three steps. In step 1, applied before events are selected, the summed energy $E_{\text {jet }}$ of particles emitted within the jet cone is related [11] to the measured energy $E_{\mathrm{m}}$ by $E_{\text {jet }}=\left(E_{\mathrm{m}}-O\right) / R(1-S)$. Here the calorimeter response $R$ is calibrated using $Z \rightarrow e e$ decays and $E_{T}$ balance in $\gamma+$ jet events, the fractional shower leakage $S$ out of the jet cone is set by test beam data, and the energy offset $O$ due to noise and the underlying event is determined using events with multiple interactions. Steps 2 and 3 are applied only to jet energies used to find $m_{\text {fit }}$. In step 2, top MC is used to correct $E_{\text {jet }}$ to the parton energy in both data and MC. This sharpens the resolution in $m_{\text {fit }}$. Step 3 is a final adjustment based on more detailed study of $\gamma+$ jet events in data and MC, particularly focused on the dependence of the $E_{T}$ balance upon $\eta$ of the jet. We assign a jet-scale error of $\pm(2.5 \%+0.5$ GeV ) based on the internal consistency of step 3 , on variations of the $\gamma+$ jet cuts and the model for the underlying event, and on an independent check of the $E_{T}$ balance in $Z+$ jet events. This leads to an error on $m_{t}$ of $\pm 4.0 \mathrm{GeV} / \mathrm{c}^{2}$.

We estimate the uncertainties in modeling of QCD by substituting the ISAJET MC generator [12] for HERWIG, independently for top MC and for VECbOs fragmentation, and by changing the vecbos QCD scale from jet $\left\langle p_{T}\right\rangle^{2}$ to $M_{W}^{2}$. The resulting systematic error due to the generator is $\pm 4.1 \mathrm{GeV} / \mathrm{c}^{2}$. Other effects including noise, multiple $p \bar{p}$ interactions, and differences in fits to $\ln L$ contribute $\pm 2.2 \mathrm{GeV} / \mathrm{c}^{2}$. All systematic errors (Table (I) sum in quadrature to $\pm 6.2 \mathrm{GeV} / \mathrm{c}^{2}$. Therefore our direct measurement of the top quark mass is $m_{t}=173.3 \pm 5.6$ (stat) $\pm 6.2$ (syst) $\mathrm{GeV} / \mathrm{c}^{2}$.

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